

# Computer algebra independent integration tests

6-Hyperbolic-functions/6.5-Hyperbolic-secant/6.5.2-e-x-^m-a+b-sech-c+d-x^n-^p

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July 25, 2021

Compiled on July 25, 2021 at 12:31am

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 84 ]. This is test number [ 178 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric ${}_2F_1$  functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 84 )	% 0.00 ( 0 )
Mathematica	% 95.24 ( 80 )	% 4.76 ( 4 )
Maple	% 59.52 ( 50 )	% 40.48 ( 34 )
Maxima	% 46.43 ( 39 )	% 53.57 ( 45 )
Fricas	% 75.00 ( 63 )	% 25.00 ( 21 )
Sympy	% 40.48 ( 34 )	% 59.52 ( 50 )
Giac	% 52.38 ( 44 )	% 47.62 ( 40 )
Mupad	% 55.95 ( 47 )	% 44.05 ( 37 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

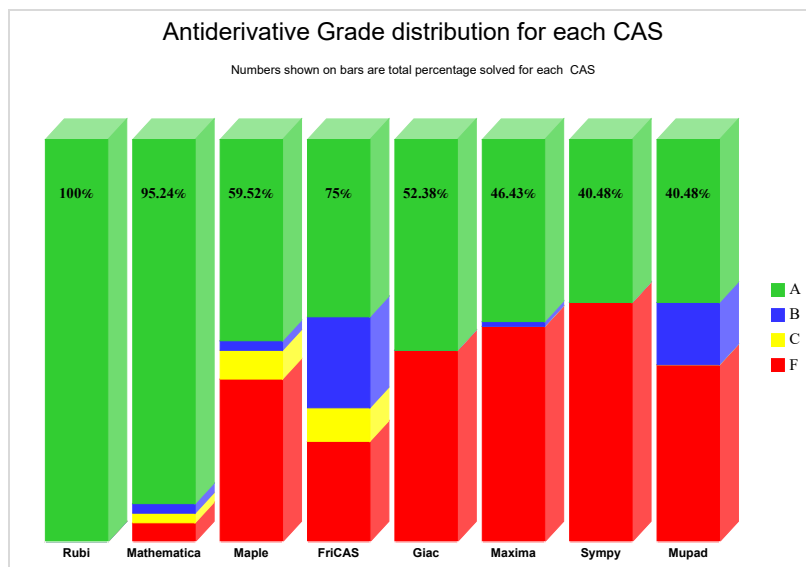
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

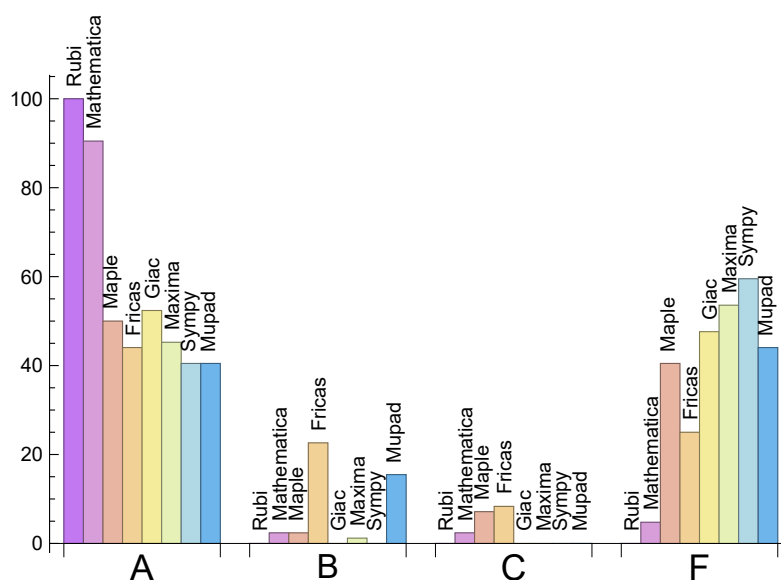
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	90.48	2.38	2.38	4.76
Maple	50.00	2.38	7.14	40.48
Maxima	45.24	1.19	0.00	53.57
Fricas	44.05	22.62	8.33	25.00
Sympy	40.48	0.00	0.00	59.52
Giac	52.38	0.00	0.00	47.62
Mupad	40.48	15.48	0.00	44.05

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	34	100.00 %	0.00 %	0.00 %
Maxima	45	57.78 %	2.22 %	40.00 %
Fricas	21	95.24 %	4.76 %	0.00 %
Sympy	50	98.00 %	2.00 %	0.00 %
Giac	40	95.00 %	0.00 %	5.00 %
Mupad	37	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS



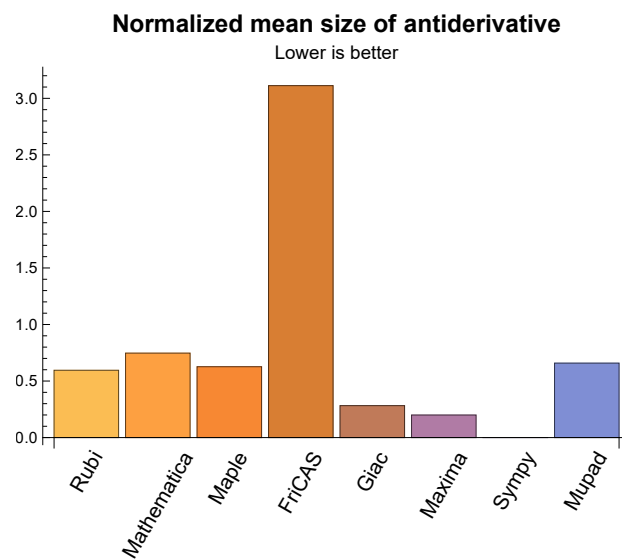
## 1.3 Performance

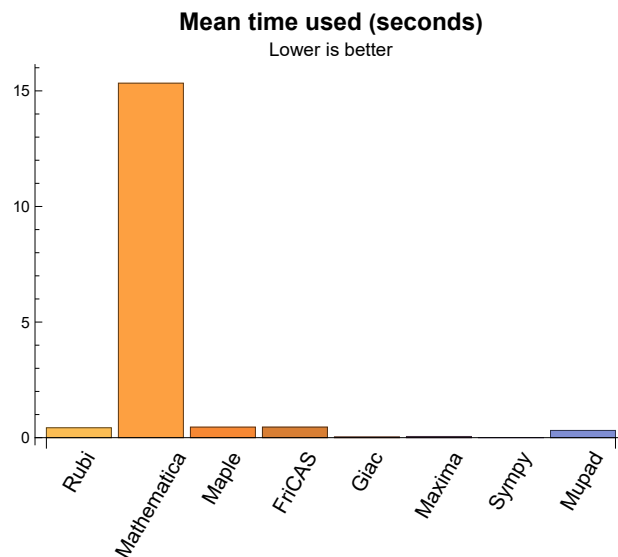
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	274.75	0.60	72.50	1.00
Mathematica	15.33	304.91	0.75	52.50	0.97
Maple	0.46	61.88	0.63	0.00	0.00
Maxima	0.05	8.51	0.20	0.00	0.00
Fricas	0.46	683.63	3.11	0.00	0.00
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.03	16.89	0.28	0.00	0.00
Mupad	0.31	48.40	0.66	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61, 65, 66, 70, 71, 72}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {10, 25, 39, 47, 48, 53, 77}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int', int(expr,x)), output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
    leafCount =1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

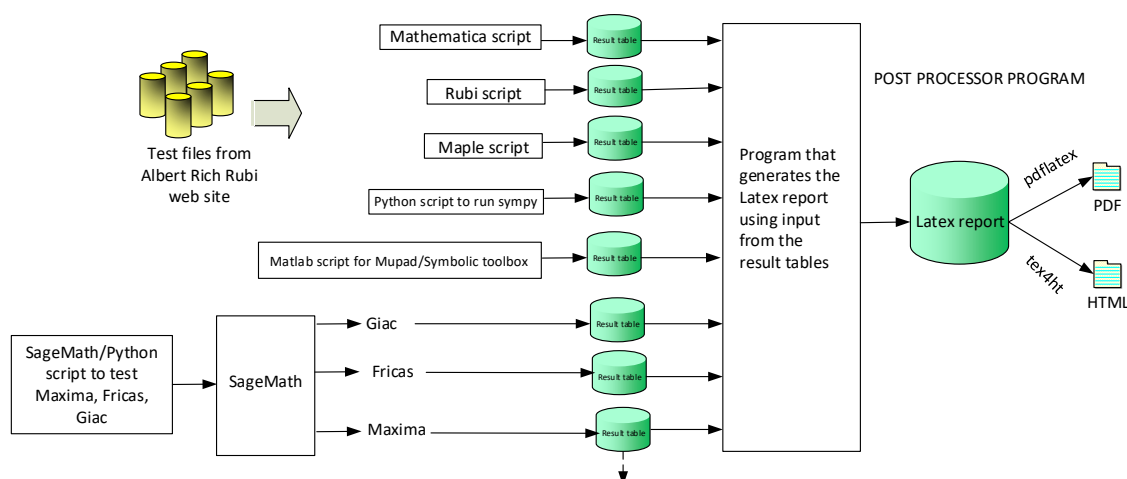
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. This is description of each record**

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

**High level overview of the CAS independent integration test build system**



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 82, 83 }

B grade: { 10, 77 }

C grade: { 18, 80 }

F grade: { 75, 78, 81, 84 }

#### 2.1.3 Maple

A grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 24, 26, 28, 29, 30, 31, 35, 36, 40, 41, 45, 46, 50, 51, 54, 55, 56, 59, 60, 61, 64, 65, 66, 70, 71, 72 }

B grade: { 27, 69 }

C grade: { 73, 74, 76, 79, 80, 82 }

F grade: { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 75, 77, 78, 81, 83, 84 }

#### 2.1.4 Maxima

A grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 31, 35, 36, 40, 41, 45, 46, 50, 51, 54, 55, 56, 59, 60, 65, 66, 70, 71, 72 }

B grade: { 15 }

C grade: { }

F grade: { 1, 3, 8, 10, 16, 18, 20, 23, 25, 27, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 61, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

## 2.1.5 FriCAS

A grade: { 2, 4, 5, 6, 7, 9, 11, 13, 14, 17, 19, 20, 21, 22, 24, 26, 28, 29, 30, 35, 36, 40, 41, 45, 46, 50, 51, 54, 55, 56, 60, 61, 65, 66, 70, 71, 72 }

B grade: { 3, 10, 12, 15, 18, 25, 27, 31, 59, 64, 69, 73, 74, 76, 77, 79, 80, 82, 83 }

C grade: { 1, 8, 16, 23, 75, 78, 81 }

F grade: { 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 84 }

## 2.1.6 Sympy

A grade: { 2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61, 65, 66, 70, 71, 72 }

B grade: { }

C grade: { }

F grade: { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

## 2.1.7 Giac

A grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 15, 17, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 35, 36, 40, 41, 45, 46, 50, 51, 54, 55, 56, 59, 60, 61, 64, 65, 66, 69, 70, 71, 72 }

B grade: { }

C grade: { }

F grade: { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

## 2.1.8 Mupad

A grade: { 2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61, 65, 66, 70, 71, 72 }

B grade: { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69, 73, 76, 79 }

C grade: { }

F grade: { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 74, 75, 77, 78, 80, 81, 82, 83, 84 }



## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	143	0	0	254	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.138	1.412	0.344	0.000	0.424	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	3.895	0.331	0.000	0.389	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	134	0	0	182	0	0	-1
normalized size	1	1.00	1.74	0.00	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.177	0.328	0.000	0.425	0.000	0.000	0.000
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	3.333	0.327	0.000	0.418	0.000	0.000	0.000
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	30	22	33	0	28	42
normalized size	1	1.00	1.00	1.15	0.85	1.27	0.00	1.08	1.62
time (sec)	N/A	0.024	0.021	0.104	0.311	0.424	0.000	0.117	0.070

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	2.677	0.336	0.000	0.418	0.000	0.000	0.000
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	3.490	0.344	0.000	0.482	0.000	0.000	0.000
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	294	0	0	1210	0	0	-1
normalized size	1	1.00	1.35	0.00	0.00	5.58	0.00	0.00	-0.00
time (sec)	N/A	0.358	4.801	0.382	0.000	0.495	0.000	0.000	0.000
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	8.912	0.368	0.000	0.513	0.000	0.000	0.000
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	273	0	0	782	0	0	-1
normalized size	1	1.00	2.29	0.00	0.00	6.57	0.00	0.00	-0.01
time (sec)	N/A	0.159	3.198	0.438	0.000	0.484	0.000	0.000	0.000
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	7.949	0.346	0.000	0.471	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	51	46	194	0	55	77
normalized size	1	1.00	1.00	1.16	1.05	4.41	0.00	1.25	1.75
time (sec)	N/A	0.055	0.099	0.275	0.327	0.507	0.000	0.123	0.100
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	20.681	0.393	0.000	0.726	0.000	0.000	0.000
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	9.854	0.396	0.000	0.547	0.000	0.000	0.000
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	77	83	182	1918	0	146	395
normalized size	1	1.00	0.86	0.92	2.02	21.31	0.00	1.62	4.39
time (sec)	N/A	0.080	0.146	0.251	0.426	0.415	0.000	0.143	0.099
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	376	0	0	731	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.858	1.566	0.342	0.000	0.434	0.000	0.000	0.000
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	8.734	0.319	0.000	0.408	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	843	0	0	536	0	0	-1
normalized size	1	1.00	3.50	0.00	0.00	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.508	1.763	0.316	0.000	0.434	0.000	0.000	0.000
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	7.811	0.315	0.000	0.379	0.000	0.000	0.000
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	95	0	304	0	61	171
normalized size	1	1.00	1.02	1.44	0.00	4.61	0.00	0.92	2.59
time (sec)	N/A	0.103	0.124	0.252	0.000	0.447	0.000	0.170	0.185
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	5.179	0.307	0.000	0.411	0.000	0.000	0.000
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.103	0.012	0.000	0.408	0.000	0.000	0.000
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	994	994	1565	0	0	3918	0	0	-1
normalized size	1	1.00	1.57	0.00	0.00	3.94	0.00	0.00	-0.00
time (sec)	N/A	2.115	12.891	0.445	0.000	0.549	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	59.255	0.436	0.000	0.416	0.000	0.000	0.000
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	755	0	0	2473	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	4.46	0.00	0.00	-0.00
time (sec)	N/A	1.102	6.308	0.467	0.000	0.471	0.000	0.000	0.000
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	52.972	0.397	0.000	0.407	0.000	0.000	0.000
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	220	234	0	1314	0	148	316
normalized size	1	1.00	1.79	1.90	0.00	10.68	0.00	1.20	2.57
time (sec)	N/A	0.250	0.460	0.266	0.000	0.463	0.000	0.150	1.756
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	104.695	0.441	0.000	0.411	0.000	0.000	0.000
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	67.397	0.458	0.000	0.423	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	68.243	0.479	0.000	0.420	0.000	0.000	0.000
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	12	28	0	12	12
normalized size	1	1.00	1.00	1.17	2.00	4.67	0.00	2.00	2.00
time (sec)	N/A	0.022	0.019	0.273	0.296	0.376	0.000	0.114	1.264
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	415	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	1.946	0.574	0.000	0.407	0.000	0.000	0.000
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	311	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	1.813	0.546	0.000	0.431	0.000	0.000	0.000
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	207	0	0	0	0	0	-1
normalized size	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	1.806	0.541	0.000	0.418	0.000	0.000	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	6.225	0.703	0.000	0.421	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.019	7.374	0.641	0.000	0.424	0.000	0.000	0.000
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	677	677	739	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.816	8.993	0.613	0.000	0.422	0.000	0.000	0.000
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	573	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	8.307	0.610	0.000	0.406	0.000	0.000	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	459	0	0	0	0	0	-1
normalized size	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	8.705	0.603	0.000	0.405	0.000	0.000	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	62.225	0.582	0.000	0.417	0.000	0.000	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	28.295	0.581	0.000	0.418	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	961	961	939	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.331	3.250	0.614	0.000	0.459	0.000	0.000	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	721	721	744	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.055	2.180	0.597	0.000	0.427	0.000	0.000	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	508	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.861	21.460	0.587	0.000	0.461	0.000	0.000	0.000
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	5.567	0.581	0.000	0.448	0.000	0.000	0.000
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.018	1.290	0.011	0.000	0.485	0.000	0.000	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	2851	2851	3033	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.782	18.741	0.733	0.000	0.500	0.000	0.000	0.000



Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	2123	2123	2245	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.982	17.375	0.662	0.000	0.500	0.000	0.000	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1395	1395	1393	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.313	17.006	0.651	0.000	0.500	0.000	0.000	0.000
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	120.967	0.640	0.000	0.471	0.000	0.000	0.000
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	71.527	0.645	0.000	0.558	0.000	0.000	0.000
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	288	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	2.052	0.543	0.000	0.437	0.000	0.000	0.000
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	197	0	0	0	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	7.655	0.531	0.000	0.417	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	23	22	33	0	29	43
normalized size	1	1.00	1.15	0.88	0.85	1.27	0.00	1.12	1.65
time (sec)	N/A	0.023	0.036	0.126	0.336	0.520	0.000	0.144	1.341
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	8.916	0.638	0.000	0.549	0.000	0.000	0.000
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.015	9.550	0.642	0.000	0.524	0.000	0.000	0.000
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	487	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	6.340	0.609	0.000	0.500	0.000	0.000	0.000
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	309	0	0	0	0	0	-1
normalized size	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	5.902	0.611	0.000	0.415	0.000	0.000	0.000
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	48	51	48	194	0	55	77
normalized size	1	1.00	1.02	1.09	1.02	4.13	0.00	1.17	1.64
time (sec)	N/A	0.057	0.117	0.411	0.304	0.445	0.000	0.155	1.316

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	27.875	0.580	0.000	0.416	0.000	0.000	0.000
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	27.877	0.575	0.000	0.407	0.000	0.000	0.000
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	626	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.969	2.373	0.592	0.000	0.407	0.000	0.000	0.000
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	390	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.776	8.027	0.589	0.000	0.407	0.000	0.000	0.000
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	95	0	254	0	61	155
normalized size	1	1.00	1.01	1.40	0.00	3.74	0.00	0.90	2.28
time (sec)	N/A	0.092	0.130	0.382	0.000	0.434	0.000	0.140	1.781
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.028	8.830	0.581	0.000	0.404	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	9.032	0.579	0.000	0.438	0.000	0.000	0.000
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1755	1755	1769	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.667	15.434	0.663	0.000	0.429	0.000	0.000	0.000
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1027	1027	986	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.898	13.448	0.656	0.000	0.424	0.000	0.000	0.000
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	232	236	0	1387	0	148	344
normalized size	1	1.00	1.83	1.86	0.00	10.92	0.00	1.17	2.71
time (sec)	N/A	0.200	0.517	0.418	0.000	0.480	0.000	0.155	1.851
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	66.927	0.644	0.000	0.421	0.000	0.000	0.000
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.027	67.060	0.639	0.000	0.427	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.055	18.336	0.704	0.000	0.445	0.000	0.000	0.000
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	155	0	122	0	0	110
normalized size	1	1.00	0.93	3.52	0.00	2.77	0.00	0.00	2.50
time (sec)	N/A	0.056	0.062	0.730	0.000	0.405	0.000	0.000	1.404
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	260	368	0	658	0	0	-1
normalized size	1	1.00	1.93	2.73	0.00	4.87	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.182	0.813	0.000	0.453	0.000	0.000	0.000
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	1072	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	4.94	0.00	0.00	-0.00
time (sec)	N/A	0.183	10.530	1.158	0.000	0.486	0.000	0.000	0.000
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	271	0	646	0	0	158
normalized size	1	1.00	0.72	3.43	0.00	8.18	0.00	0.00	2.00
time (sec)	N/A	0.102	0.205	0.661	0.000	0.420	0.000	0.000	1.419
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	501	0	0	2964	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	14.25	0.00	0.00	-0.00
time (sec)	N/A	0.208	2.877	0.576	0.000	0.535	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	0	0	0	5493	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	15.13	0.00	0.00	-0.00
time (sec)	N/A	0.404	70.762	0.554	0.000	0.607	0.000	0.000	0.000
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	317	0	511	0	0	409
normalized size	1	1.00	0.92	3.64	0.00	5.87	0.00	0.00	4.70
time (sec)	N/A	0.146	0.152	0.562	0.000	0.435	0.000	0.000	2.101
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	859	587	0	1286	0	0	-1
normalized size	1	1.00	2.80	1.91	0.00	4.19	0.00	0.00	-0.00
time (sec)	N/A	0.564	2.100	0.662	0.000	0.418	0.000	0.000	0.000
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	0	0	0	2005	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	4.44	0.00	0.00	-0.00
time (sec)	N/A	0.863	7.220	1.074	0.000	0.445	0.000	0.000	0.000
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	233	491	0	3547	0	0	-1
normalized size	1	1.00	1.48	3.13	0.00	22.59	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.598	0.670	0.000	0.500	0.000	0.000	0.000
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	542	0	0	9020	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	12.58	0.00	0.00	-0.00
time (sec)	N/A	1.220	30.818	1.104	0.000	0.559	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1284	1284	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.258	117.911	1.375	0.000	0.000	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [23] had the largest ratio of [.6667]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	6	1.00	16	0.375
2	A	0	0	0.00	0	0.000
3	A	8	5	1.00	16	0.312
4	A	0	0	0.00	0	0.000
5	A	4	3	1.00	14	0.214
6	A	0	0	0.00	0	0.000
7	A	0	0	0.00	0	0.000
8	A	15	11	1.00	18	0.611
9	A	0	0	0.00	0	0.000
10	A	10	7	1.00	18	0.389
11	A	0	0	0.00	0	0.000
12	A	5	5	1.00	16	0.312
13	A	0	0	0.00	0	0.000
14	A	0	0	0.00	0	0.000
15	A	5	3	1.00	12	0.250
16	A	13	8	1.00	18	0.444
17	A	0	0	0.00	0	0.000
18	A	11	7	1.00	18	0.389
19	A	0	0	0.00	0	0.000
20	A	4	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	0	0	0.00	0	0.000
22	A	0	0	0.00	0	0.000
23	A	31	12	1.00	18	0.667
24	A	0	0	0.00	0	0.000
25	A	22	10	1.00	18	0.556
26	A	0	0	0.00	0	0.000
27	A	6	6	1.00	16	0.375
28	A	0	0	0.00	0	0.000
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	3	3	1.00	10	0.300
32	A	20	7	1.00	18	0.389
33	A	16	7	1.00	18	0.389
34	A	12	7	1.00	16	0.438
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	30	10	1.00	20	0.500
38	A	24	10	1.00	20	0.500
39	A	18	10	1.00	18	0.556
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	23	9	1.00	20	0.450
43	A	19	9	1.00	20	0.450
44	A	15	9	1.00	18	0.500
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	61	11	1.00	20	0.550
48	A	49	11	1.00	20	0.550
49	A	37	11	1.00	18	0.611
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	14	7	1.00	20	0.350
53	A	10	6	1.00	20	0.300
54	A	4	3	1.00	20	0.150
55	A	0	0	0.00	0	0.000
56	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	21	10	1.00	22	0.454
58	A	15	11	1.00	22	0.500
59	A	5	5	1.00	22	0.227
60	A	0	0	0.00	0	0.000
61	A	0	0	0.00	0	0.000
62	A	17	9	1.00	22	0.409
63	A	13	8	1.00	22	0.364
64	A	4	4	1.00	22	0.182
65	A	0	0	0.00	0	0.000
66	A	0	0	0.00	0	0.000
67	A	43	11	1.00	22	0.500
68	A	31	12	1.00	22	0.546
69	A	6	6	1.00	22	0.273
70	A	0	0	0.00	0	0.000
71	A	0	0	0.00	0	0.000
72	A	0	0	0.00	0	0.000
73	A	5	4	1.00	20	0.200
74	A	9	6	1.00	22	0.273
75	A	11	7	1.00	22	0.318
76	A	6	6	1.00	22	0.273
77	A	11	8	1.00	24	0.333
78	A	16	12	1.00	24	0.500
79	A	5	5	1.00	22	0.227
80	A	12	8	1.00	24	0.333
81	A	14	9	1.00	24	0.375
82	A	7	7	1.00	22	0.318
83	A	23	11	1.00	24	0.458
84	A	32	13	1.00	24	0.542



# Chapter 3

## Listing of integrals

### 3.1 $\int x^5 (a + b \operatorname{sech}(c + dx^2)) dx$

Optimal. Leaf size=125

$$\frac{ax^6}{6} + \frac{ib\operatorname{Li}_3(-ie^{dx^2+c})}{d^3} - \frac{ib\operatorname{Li}_3(ie^{dx^2+c})}{d^3} - \frac{ibx^2\operatorname{Li}_2(-ie^{dx^2+c})}{d^2} + \frac{ibx^2\operatorname{Li}_2(ie^{dx^2+c})}{d^2} + \frac{bx^4 \tan^{-1}(e^{c+dx^2})}{d}$$

[Out] 1/6\*a\*x^6+b\*x^4\*arctan(exp(d\*x^2+c))/d-I\*b\*x^2\*polylog(2,-I\*exp(d\*x^2+c))/d^2+I\*b\*x^2\*polylog(2,I\*exp(d\*x^2+c))/d^2+I\*b\*polylog(3,-I\*exp(d\*x^2+c))/d^3-I\*b\*polylog(3,I\*exp(d\*x^2+c))/d^3

**Rubi [A]** time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {14, 5436, 4180, 2531, 2282, 6589}

$$-\frac{ibx^2\operatorname{PolyLog}(2,-ie^{c+dx^2})}{d^2} + \frac{ibx^2\operatorname{PolyLog}(2,ie^{c+dx^2})}{d^2} + \frac{ib\operatorname{PolyLog}(3,-ie^{c+dx^2})}{d^3} - \frac{ib\operatorname{PolyLog}(3,ie^{c+dx^2})}{d^3} + \frac{ax^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*Sech[c + d\*x^2]),x]

[Out] (a\*x^6)/6 + (b\*x^4\*ArcTan[E^(c + d\*x^2)])/d - (I\*b\*x^2\*PolyLog[2, (-I)\*E^(c + d\*x^2)])/d^2 + (I\*b\*x^2\*PolyLog[2, I\*E^(c + d\*x^2)])/d^2 + (I\*b\*PolyLog[3, (-I)\*E^(c + d\*x^2)])/d^3 - (I\*b\*PolyLog[3, I\*E^(c + d\*x^2)])/d^3

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_)+(b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^(c\_)\*((a\_)+(b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -

1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^m\_., x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x^5 (a + b \operatorname{sech}(c + dx^2)) dx &= \int (ax^5 + bx^5 \operatorname{sech}(c + dx^2)) dx \\
 &= \frac{ax^6}{6} + b \int x^5 \operatorname{sech}(c + dx^2) dx \\
 &= \frac{ax^6}{6} + \frac{1}{2} b \operatorname{Subst}\left(\int x^2 \operatorname{sech}(c + dx) dx, x, x^2\right) \\
 &= \frac{ax^6}{6} + \frac{bx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{(ib) \operatorname{Subst}\left(\int x \log(1 - ie^{c+dx}) dx, x, x^2\right)}{d} + \frac{(ib) \operatorname{Subst}\left(\int x \log(1 + ie^{c+dx}) dx, x, x^2\right)}{d} \\
 &= \frac{ax^6}{6} + \frac{bx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{ibx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} + \frac{ibx^2 \operatorname{Li}_2(ie^{c+dx^2})}{d^2} + \frac{(ib) \operatorname{Subst}\left(\int x \log(1 - ie^{c+dx}) dx, x, x^2\right)}{d} \\
 &= \frac{ax^6}{6} + \frac{bx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{ibx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} + \frac{ibx^2 \operatorname{Li}_2(ie^{c+dx^2})}{d^2} + \frac{(ib) \operatorname{Subst}\left(\int x \log(1 + ie^{c+dx}) dx, x, x^2\right)}{d} \\
 &= \frac{ax^6}{6} + \frac{bx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{ibx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} + \frac{ibx^2 \operatorname{Li}_2(ie^{c+dx^2})}{d^2} + \frac{ib \operatorname{Li}_3(-ie^{c+dx^2})}{d^3}
 \end{aligned}$$

**Mathematica [A]** time = 1.41, size = 143, normalized size = 1.14

$$\frac{ax^6}{6} + \frac{ib(d^2x^4 \log(1 - ie^{c+dx^2}) - d^2x^4 \log(1 + ie^{c+dx^2}) - 2dx^2 \operatorname{Li}_2(-ie^{dx^2+c}) + 2dx^2 \operatorname{Li}_2(ie^{dx^2+c}) + 2\operatorname{Li}_3(-ie^{dx^2+c}))}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*Sech[c + d\*x^2]),x]

[Out] (a\*x^6)/6 + ((I/2)\*b\*(d^2\*x^4\*Log[1 - I\*E^(c + d\*x^2)] - d^2\*x^4\*Log[1 + I\*E^(c + d\*x^2)] - 2\*d\*x^2\*PolyLog[2, (-I)\*E^(c + d\*x^2)] + 2\*d\*x^2\*PolyLog[2, I\*E^(c + d\*x^2)] + 2\*PolyLog[3, (-I)\*E^(c + d\*x^2)] - 2\*PolyLog[3, I\*E^(c + d\*x^2)]))/d^3

**fricas** [C] time = 0.42, size = 254, normalized size = 2.03

$$\frac{ad^3x^6 + 6ibdx^2\text{Li}_2(i \cosh(dx^2 + c) + i \sinh(dx^2 + c)) - 6ibdx^2\text{Li}_2(-i \cosh(dx^2 + c) - i \sinh(dx^2 + c))}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sech(d\*x^2+c)),x, algorithm="fricas")

[Out]  $\frac{1}{6}(a*d^3*x^6 + 6*I*b*d*x^2*\text{dilog}(I*\cosh(d*x^2 + c) + I*\sinh(d*x^2 + c)) - 6*I*b*d*x^2*\text{dilog}(-I*\cosh(d*x^2 + c) - I*\sinh(d*x^2 + c)) + 3*I*b*c^2*\log(\cosh(d*x^2 + c) + \sinh(d*x^2 + c) + I) - 3*I*b*c^2*\log(\cosh(d*x^2 + c) + \sinh(d*x^2 + c) - I) + (-3*I*b*d^2*x^4 + 3*I*b*c^2)*\log(I*\cosh(d*x^2 + c) + I*\sinh(d*x^2 + c) + 1) + (3*I*b*d^2*x^4 - 3*I*b*c^2)*\log(-I*\cosh(d*x^2 + c) - I*\sinh(d*x^2 + c) + 1) - 6*I*b*\text{polylog}(3, I*\cosh(d*x^2 + c) + I*\sinh(d*x^2 + c)) + 6*I*b*\text{polylog}(3, -I*\cosh(d*x^2 + c) - I*\sinh(d*x^2 + c)))/d^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sech(d\*x^2+c)),x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)\*x^5, x)

**maple** [F] time = 0.34, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{sech}(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*sech(d\*x^2+c)),x)

[Out] int(x^5\*(a+b\*sech(d\*x^2+c)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}ax^6 + 2b \int \frac{x^5}{e^{(dx^2+c)} + e^{(-dx^2-c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sech(d\*x^2+c)),x, algorithm="maxima")

[Out]  $\frac{1}{6}a*x^6 + 2*b*\text{integrate}(x^5/(e^{(d*x^2 + c)} + e^{(-d*x^2 - c)}), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a + b/cosh(c + d\*x^2)),x)

[Out] int(x^5\*(a + b/cosh(c + d\*x^2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*sech(d*x**2+c)),x)
```

```
[Out] Integral(x**5*(a + b*sech(c + d*x**2)), x)
```

### 3.2 $\int x^4 (a + b \operatorname{sech}(c + dx^2)) dx$

**Optimal.** Leaf size=26

$$b \operatorname{Int}(x^4 \operatorname{sech}(c + dx^2), x) + \frac{ax^5}{5}$$

[Out]  $1/5*a*x^5+b*\operatorname{Unintegrable}(x^4*\operatorname{sech}(d*x^2+c), x)$

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^4 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[x^4*(a + b*\operatorname{Sech}[c + d*x^2]), x]$

[Out]  $(a*x^5)/5 + b*\operatorname{Defer}[\operatorname{Int}[x^4*\operatorname{Sech}[c + d*x^2], x]$

Rubi steps

$$\begin{aligned} \int x^4 (a + b \operatorname{sech}(c + dx^2)) dx &= \int (ax^4 + bx^4 \operatorname{sech}(c + dx^2)) dx \\ &= \frac{ax^5}{5} + b \int x^4 \operatorname{sech}(c + dx^2) dx \end{aligned}$$

**Mathematica [A]** time = 3.90, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[x^4*(a + b*\operatorname{Sech}[c + d*x^2]), x]$

[Out]  $\operatorname{Integrate}[x^4*(a + b*\operatorname{Sech}[c + d*x^2]), x]$

**fricas [A]** time = 0.39, size = 0, normalized size = 0.00

$$\operatorname{integral}(bx^4 \operatorname{sech}(dx^2 + c) + ax^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^4*(a+b*\operatorname{sech}(d*x^2+c)), x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}(b*x^4*\operatorname{sech}(d*x^2 + c) + a*x^4, x)$

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^4*(a+b*\operatorname{sech}(d*x^2+c)), x, \operatorname{algorithm}="giac")$

[Out]  $\operatorname{integrate}((b*\operatorname{sech}(d*x^2 + c) + a)*x^4, x)$

**maple [A]** time = 0.33, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{sech}(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*sech(d*x^2+c)),x)`

[Out] `int(x^4*(a+b*sech(d*x^2+c)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}ax^5 + 2b \int \frac{x^4}{e^{(dx^2+c)} + e^{(-dx^2-c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

[Out] `1/5*a*x^5 + 2*b*integrate(x^4/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^4 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a + b/cosh(c + d*x^2)),x)`

[Out] `int(x^4*(a + b/cosh(c + d*x^2)), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*sech(d*x**2+c)),x)`

[Out] `Integral(x**4*(a + b*sech(c + d*x**2)), x)`



### 3.3 $\int x^3 (a + b \operatorname{sech}(c + dx^2)) dx$

Optimal. Leaf size=77

$$\frac{ax^4}{4} - \frac{ib\operatorname{Li}_2(-ie^{dx^2+c})}{2d^2} + \frac{ib\operatorname{Li}_2(ie^{dx^2+c})}{2d^2} + \frac{bx^2 \tan^{-1}(e^{c+dx^2})}{d}$$

[Out] 1/4\*a\*x^4+b\*x^2\*arctan(exp(d\*x^2+c))/d-1/2\*I\*b\*polylog(2,-I\*exp(d\*x^2+c))/d^2+1/2\*I\*b\*polylog(2,I\*exp(d\*x^2+c))/d^2

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {14, 5436, 4180, 2279, 2391}

$$-\frac{ib\operatorname{PolyLog}(2, -ie^{c+dx^2})}{2d^2} + \frac{ib\operatorname{PolyLog}(2, ie^{c+dx^2})}{2d^2} + \frac{ax^4}{4} + \frac{bx^2 \tan^{-1}(e^{c+dx^2})}{d}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Sech[c + d\*x^2]), x]

[Out] (a\*x^4)/4 + (b\*x^2\*ArcTan[E^(c + d\*x^2)])/d - ((I/2)\*b\*PolyLog[2, (-I)\*E^(c + d\*x^2)])/d^2 + ((I/2)\*b\*PolyLog[2, I\*E^(c + d\*x^2)])/d^2

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 2279

Int[Log[(a\_)+(b\_)\*((F\_)^(e\_)\*((c\_)+(d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_)+(e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4180

Int[csc[(e\_)+Pi\*(k\_)+(Complex[0, fz\_])\*(f\_)\*(x\_)]\*((c\_)+(d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m-1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m-1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 5436

Int[(x\_)^(m\_)\*((a\_)+(b\_)\*Sech[(c\_)+(d\_)\*(x\_)^(n\_)])^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n]-1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m+1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \operatorname{sech}(c + dx^2)) dx &= \int (ax^3 + bx^3 \operatorname{sech}(c + dx^2)) dx \\
&= \frac{ax^4}{4} + b \int x^3 \operatorname{sech}(c + dx^2) dx \\
&= \frac{ax^4}{4} + \frac{1}{2} b \operatorname{Subst} \left( \int x \operatorname{sech}(c + dx) dx, x, x^2 \right) \\
&= \frac{ax^4}{4} + \frac{bx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{(ib) \operatorname{Subst} \left( \int \log(1 - ie^{c+dx}) dx, x, x^2 \right)}{2d} + \frac{(ib) \operatorname{Subst} \left( \int \log(1 + ie^{c+dx}) dx, x, x^2 \right)}{2d} \\
&= \frac{ax^4}{4} + \frac{bx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{(ib) \operatorname{Subst} \left( \int \frac{\log(1-ix)}{x} dx, x, e^{c+dx^2} \right)}{2d^2} + \frac{(ib) \operatorname{Subst} \left( \int \frac{\log(1+ix)}{x} dx, x, e^{c+dx^2} \right)}{2d^2} \\
&= \frac{ax^4}{4} + \frac{bx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{ib \operatorname{Li}_2(-ie^{c+dx^2})}{2d^2} + \frac{ib \operatorname{Li}_2(ie^{c+dx^2})}{2d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 134, normalized size = 1.74

$$\frac{1}{4} \left( ax^4 + \frac{b \left( -2i \left( \operatorname{Li}_2(-ie^{dx^2+c}) - \operatorname{Li}_2(ie^{dx^2+c}) \right) - \left( (-2ic - 2idx^2 + \pi) \left( \log(1 - ie^{c+dx^2}) - \log(1 + ie^{c+dx^2}) \right) \right) \right) + \pi}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Sech[c + d\*x^2]),x]

[Out] (a\*x^4 + (b\*(-((( (-2\*I)\*c + Pi - (2\*I)\*d\*x^2)\*(Log[1 - I\*E^(c + d\*x^2)] - Log[1 + I\*E^(c + d\*x^2)]))) + (( (-2\*I)\*c + Pi)\*Log[Cot[((2\*I)\*c + Pi + (2\*I)\*d\*x^2)/4]] - (2\*I)\*(PolyLog[2, (-I)\*E^(c + d\*x^2)] - PolyLog[2, I\*E^(c + d\*x^2)])))))/d^2)/4

**fricas [B]** time = 0.42, size = 182, normalized size = 2.36

$$\frac{ad^2x^4 - 2ibc \log(\cosh(dx^2 + c) + \sinh(dx^2 + c) + i) + 2ibc \log(\cosh(dx^2 + c) + \sinh(dx^2 + c) - i) + 2ib \operatorname{Li}_2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(d\*x^2+c)),x, algorithm="fricas")

[Out] 1/4\*(a\*d^2\*x^4 - 2\*I\*b\*c\*log(cosh(d\*x^2 + c) + sinh(d\*x^2 + c) + I) + 2\*I\*b\*c\*log(cosh(d\*x^2 + c) + sinh(d\*x^2 + c) - I) + 2\*I\*b\*dilog(I\*cosh(d\*x^2 + c) + I\*sinh(d\*x^2 + c)) - 2\*I\*b\*dilog(-I\*cosh(d\*x^2 + c) - I\*sinh(d\*x^2 + c)) + (-2\*I\*b\*d\*x^2 - 2\*I\*b\*c)\*log(I\*cosh(d\*x^2 + c) + I\*sinh(d\*x^2 + c) + 1) + (2\*I\*b\*d\*x^2 + 2\*I\*b\*c)\*log(-I\*cosh(d\*x^2 + c) - I\*sinh(d\*x^2 + c) + 1))/d^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(d\*x^2+c)),x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)\*x^3, x)

**maple [F]** time = 0.33, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*sech(d*x^2+c)),x)`

[Out] `int(x^3*(a+b*sech(d*x^2+c)),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}ax^4 + 2b \int \frac{x^3}{e^{(dx^2+c)} + e^{(-dx^2-c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

[Out] `1/4*a*x^4 + 2*b*integrate(x^3/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b/cosh(c + d*x^2)),x)`

[Out] `int(x^3*(a + b/cosh(c + d*x^2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*sech(d*x**2+c)),x)`

[Out] `Integral(x**3*(a + b*sech(c + d*x**2)), x)`

### 3.4 $\int x^2 (a + b \operatorname{sech}(c + dx^2)) dx$

**Optimal.** Leaf size=26

$$b \operatorname{Int}(x^2 \operatorname{sech}(c + dx^2), x) + \frac{ax^3}{3}$$

[Out]  $1/3*a*x^3+b*\operatorname{Unintegrable}(x^2*\operatorname{sech}(d*x^2+c), x)$

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[x^2*(a + b*\operatorname{Sech}[c + d*x^2]), x]$

[Out]  $(a*x^3)/3 + b*\operatorname{Defer}[\operatorname{Int}[x^2*\operatorname{Sech}[c + d*x^2], x]$

Rubi steps

$$\begin{aligned} \int x^2 (a + b \operatorname{sech}(c + dx^2)) dx &= \int (ax^2 + bx^2 \operatorname{sech}(c + dx^2)) dx \\ &= \frac{ax^3}{3} + b \int x^2 \operatorname{sech}(c + dx^2) dx \end{aligned}$$

**Mathematica [A]** time = 3.33, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[x^2*(a + b*\operatorname{Sech}[c + d*x^2]), x]$

[Out]  $\operatorname{Integrate}[x^2*(a + b*\operatorname{Sech}[c + d*x^2]), x]$

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}(bx^2 \operatorname{sech}(dx^2 + c) + ax^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^2*(a+b*\operatorname{sech}(d*x^2+c)), x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}(b*x^2*\operatorname{sech}(d*x^2 + c) + a*x^2, x)$

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(x^2*(a+b*\operatorname{sech}(d*x^2+c)), x, \operatorname{algorithm}="giac")$

[Out]  $\operatorname{integrate}((b*\operatorname{sech}(d*x^2 + c) + a)*x^2, x)$

**maple [A]** time = 0.33, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(dx^2 + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*sech(d*x^2+c)),x)`

[Out] `int(x^2*(a+b*sech(d*x^2+c)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}ax^3 + 2b \int \frac{x^2}{e^{(dx^2+c)} + e^{(-dx^2-c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

[Out] `1/3*a*x^3 + 2*b*integrate(x^2/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/cosh(c + d*x^2)),x)`

[Out] `int(x^2*(a + b/cosh(c + d*x^2)), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*sech(d*x**2+c)),x)`

[Out] `Integral(x**2*(a + b*sech(c + d*x**2)), x)`

### 3.5 $\int x (a + b \operatorname{sech}(c + dx^2)) dx$

**Optimal.** Leaf size=26

$$\frac{ax^2}{2} + \frac{b \tan^{-1}(\sinh(c + dx^2))}{2d}$$

[Out]  $1/2*a*x^2+1/2*b*\arctan(\sinh(d*x^2+c))/d$

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {14, 5436, 3770}

$$\frac{ax^2}{2} + \frac{b \tan^{-1}(\sinh(c + dx^2))}{2d}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Sech[c + d\*x^2]),x]

[Out] (a\*x^2)/2 + (b\*ArcTan[Sinh[c + d\*x^2]])/(2\*d)

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 3770

Int[csc[(c\_.) + (d\_)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 5436

Int[(x\_)^(m\_)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_)\*(x\_)^(n\_)])^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned} \int x (a + b \operatorname{sech}(c + dx^2)) dx &= \int (ax + bx \operatorname{sech}(c + dx^2)) dx \\ &= \frac{ax^2}{2} + b \int x \operatorname{sech}(c + dx^2) dx \\ &= \frac{ax^2}{2} + \frac{1}{2} b \operatorname{Subst} \left( \int \operatorname{sech}(c + dx) dx, x, x^2 \right) \\ &= \frac{ax^2}{2} + \frac{b \tan^{-1}(\sinh(c + dx^2))}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 26, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{b \tan^{-1}(\sinh(c + dx^2))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Sech[c + d\*x^2]),x]

[Out] (a\*x^2)/2 + (b\*ArcTan[Sinh[c + d\*x^2]])/(2\*d)

**fricas** [A] time = 0.42, size = 33, normalized size = 1.27

$$\frac{adx^2 + 2b \arctan(\cosh(dx^2 + c) + \sinh(dx^2 + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(d\*x^2+c)),x, algorithm="fricas")

[Out] 1/2\*(a\*d\*x^2 + 2\*b\*arctan(cosh(d\*x^2 + c) + sinh(d\*x^2 + c)))/d

**giac** [A] time = 0.12, size = 28, normalized size = 1.08

$$\frac{(dx^2 + c)a}{2d} + \frac{b \arctan(e^{(dx^2+c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(d\*x^2+c)),x, algorithm="giac")

[Out] 1/2\*(d\*x^2 + c)\*a/d + b\*arctan(e^(d\*x^2 + c))/d

**maple** [A] time = 0.10, size = 30, normalized size = 1.15

$$\frac{ax^2}{2} + \frac{b \arctan(\sinh(dx^2 + c))}{2d} + \frac{ac}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*sech(d\*x^2+c)),x)

[Out] 1/2\*a\*x^2+1/2\*b\*arctan(sinh(d\*x^2+c))/d+1/2/d\*a\*c

**maxima** [A] time = 0.31, size = 22, normalized size = 0.85

$$\frac{1}{2}ax^2 + \frac{b \arctan(\sinh(dx^2 + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(d\*x^2+c)),x, algorithm="maxima")

[Out] 1/2\*a\*x^2 + 1/2\*b\*arctan(sinh(d\*x^2 + c))/d

**mupad** [B] time = 0.07, size = 42, normalized size = 1.62

$$\frac{ax^2}{2} + \frac{\operatorname{atan}\left(\frac{be^{dx^2}e^c\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b/cosh(c + d\*x^2)),x)

[Out] (a\*x^2)/2 + (atan((b\*exp(d\*x^2)\*exp(c)\*(d^2)^(1/2))/(d\*(b^2)^(1/2)))\*(b^2)^(1/2))/(d^2)^(1/2)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{sech}(c + dx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*sech(d*x**2+c)),x)
```

```
[Out] Integral(x*(a + b*sech(c + d*x**2)), x)
```



$$3.6 \quad \int \frac{a+b\operatorname{sech}(c+dx^2)}{x} dx$$

Optimal. Leaf size=22

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+dx^2)}{x}, x\right) + a \log(x)$$

[Out] a\*ln(x)+b\*Unintegrable(sech(d\*x^2+c)/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*x^2])/x,x]

[Out] a\*Log[x] + b\*Defer[Int][Sech[c + d\*x^2]/x, x]

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + dx^2)}{x} dx &= \int \left( \frac{a}{x} + \frac{b\operatorname{sech}(c + dx^2)}{x} \right) dx \\ &= a \log(x) + b \int \frac{\operatorname{sech}(c + dx^2)}{x} dx \end{aligned}$$

Mathematica [A] time = 2.68, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*x^2])/x,x]

[Out] Integrate[(a + b\*Sech[c + d\*x^2])/x, x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b \operatorname{sech}(dx^2 + c) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x,x, algorithm="fricas")

[Out] integral((b\*sech(d\*x^2 + c) + a)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{sech}(dx^2 + c) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)/x, x)

**maple** [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(dx^2 + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(d\*x^2+c))/x,x)

[Out] int((a+b\*sech(d\*x^2+c))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{1}{x(e^{(dx^2+c)} + e^{(-dx^2-c)})} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x,x, algorithm="maxima")

[Out] 2\*b\*integrate(1/(x\*(e^(d\*x^2 + c) + e^(-d\*x^2 - c))), x) + a\*log(x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a + \frac{b}{\cosh(dx^2+c)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^2))/x,x)

[Out] int((a + b/cosh(c + d\*x^2))/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x\*\*2+c))/x,x)

[Out] Integral((a + b\*sech(c + d\*x\*\*2))/x, x)

$$3.7 \quad \int \frac{a+b\operatorname{sech}(c+dx^2)}{x^2} dx$$

Optimal. Leaf size=24

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+dx^2)}{x^2}, x\right) - \frac{a}{x}$$

[Out]  $-a/x + b*\operatorname{Unintegrable}(\operatorname{sech}(d*x^2+c)/x^2, x)$

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*x^2])/x^2, x]$

[Out]  $-(a/x) + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sech}[c + d*x^2]/x^2, x]]$

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b\operatorname{sech}(c + dx^2)}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\operatorname{sech}(c + dx^2)}{x^2} dx \end{aligned}$$

Mathematica [A] time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*x^2])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*x^2])/x^2, x]$

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{sech}(dx^2 + c) + a}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\operatorname{sech}(d*x^2+c))/x^2, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b*\operatorname{sech}(d*x^2 + c) + a)/x^2, x)$

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)/x^2, x)

**maple** [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(dx^2 + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(d\*x^2+c))/x^2,x)

[Out] int((a+b\*sech(d\*x^2+c))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{1}{x^2(e^{(dx^2+c)} + e^{(-dx^2-c)})} dx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x^2,x, algorithm="maxima")

[Out] 2\*b\*integrate(1/(x^2\*(e^(d\*x^2 + c) + e^(-d\*x^2 - c))), x) - a/x

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cosh(dx^2+c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^2))/x^2,x)

[Out] int((a + b/cosh(c + d\*x^2))/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x\*\*2+c))/x\*\*2,x)

[Out] Integral((a + b\*sech(c + d\*x\*\*2))/x\*\*2, x)

### 3.8 $\int x^5 \left( a + b \operatorname{sech} \left( c + dx^2 \right) \right)^2 dx$

**Optimal.** Leaf size=217

$$\frac{a^2 x^6}{6} + \frac{2iab \operatorname{Li}_3(-ie^{dx^2+c})}{d^3} - \frac{2iab \operatorname{Li}_3(ie^{dx^2+c})}{d^3} - \frac{2iabx^2 \operatorname{Li}_2(-ie^{dx^2+c})}{d^2} + \frac{2iabx^2 \operatorname{Li}_2(ie^{dx^2+c})}{d^2} + \frac{2abx^4 \tan^{-1}(e^{c+dx^2})}{d}$$

[Out]  $1/2*b^2*x^4/d+1/6*a^2*x^6+2*a*b*x^4*\arctan(\exp(d*x^2+c))/d-b^2*x^2*\ln(1+\exp(2*d*x^2+2*c))/d^2-2*I*a*b*x^2*\operatorname{polylog}(2,-I*\exp(d*x^2+c))/d^2+2*I*a*b*x^2*\operatorname{polylog}(2,I*\exp(d*x^2+c))/d^2-1/2*b^2*\operatorname{polylog}(2,-\exp(2*d*x^2+2*c))/d^3+2*I*a*b*\operatorname{polylog}(3,-I*\exp(d*x^2+c))/d^3-2*I*a*b*\operatorname{polylog}(3,I*\exp(d*x^2+c))/d^3+1/2*b^2*x^4*\tanh(d*x^2+c)/d$

**Rubi [A]** time = 0.36, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5436, 4190, 4180, 2531, 2282, 6589, 4184, 3718, 2190, 2279, 2391}

$$-\frac{2iabx^2 \operatorname{PolyLog}(2, -ie^{c+dx^2})}{d^2} + \frac{2iabx^2 \operatorname{PolyLog}(2, ie^{c+dx^2})}{d^2} + \frac{2iab \operatorname{PolyLog}(3, -ie^{c+dx^2})}{d^3} - \frac{2iab \operatorname{PolyLog}(3, ie^{c+dx^2})}{d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^5*(a + b*\operatorname{Sech}[c + d*x^2])^2, x]$

[Out]  $(b^2*x^4)/(2*d) + (a^2*x^6)/6 + (2*a*b*x^4*\operatorname{ArcTan}[E^{\wedge}(c + d*x^2)])/d - (b^2*x^2*\operatorname{Log}[1 + E^{\wedge}(2*(c + d*x^2))])/d^2 - ((2*I)*a*b*x^2*\operatorname{PolyLog}[2, (-I)*E^{\wedge}(c + d*x^2)])/d^2 + ((2*I)*a*b*x^2*\operatorname{PolyLog}[2, I*E^{\wedge}(c + d*x^2)])/d^2 - (b^2*\operatorname{PolyLog}[2, -E^{\wedge}(2*(c + d*x^2))])/d^3 + ((2*I)*a*b*\operatorname{PolyLog}[3, (-I)*E^{\wedge}(c + d*x^2)])/d^3 - ((2*I)*a*b*\operatorname{PolyLog}[3, I*E^{\wedge}(c + d*x^2)])/d^3 + (b^2*x^4*\operatorname{Tanh}[c + d*x^2])/(2*d)$

#### Rule 2190

$\operatorname{Int}[\frac{((F_*)^{\wedge}((g_*)*((e_*) + (f_*)*(x_*)))^{\wedge}(n_*)*((c_*) + (d_*)*(x_*)))^{\wedge}(m_*)}{((a_*) + (b_*)*((F_*)^{\wedge}((g_*)*((e_*) + (f_*)*(x_*)))^{\wedge}(n_*)), x\_Symbol]} :> \operatorname{Simp}[\frac{((c + d*x)^{\wedge}m*\operatorname{Log}[1 + (b*(F^{\wedge}(g*(e + f*x)))^{\wedge}n)/a]}{b*f*g*n*\operatorname{Log}[F]}, x] - \operatorname{Dist}[\frac{(d*m)}{b*f*g*n*\operatorname{Log}[F]}, \operatorname{Int}[(c + d*x)^{\wedge}(m-1)*\operatorname{Log}[1 + (b*(F^{\wedge}(g*(e + f*x)))^{\wedge}n)/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_*) + (b_*)*((F_*)^{\wedge}((e_*)*((c_*) + (d_*)*(x_*)))^{\wedge}(n_*)], x\_Symbol]} :> \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{\wedge}(e*(c + d*x)))^{\wedge}n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \&\& \operatorname{GtQ}[a, 0]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol]} :> \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& !\operatorname{MatchQ}[u, (w_*)*((a_*)*(v_*)^{\wedge}(n_*))^{\wedge}(m_*) /; \operatorname{FreeQ}\{a, m, n\}, x \&\& \operatorname{IntegerQ}[m*n]] \&\& !\operatorname{MatchQ}[u, E^{\wedge}((c_*)*((a_*) + (b_*)*x))* (F_*)[v_]] /; \operatorname{FreeQ}\{a, b, c\}, x \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_*)*((d_*) + (e_*)*(x_*)^{\wedge}(n_*))]/(x_), x\_Symbol]} :> -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x \&\& \operatorname{EqQ}[c*d, 1]$

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx &= \frac{1}{2} \operatorname{Subst} \left( \int x^2 (a + b \operatorname{sech}(c + dx))^2 dx, x, x^2 \right) \\
&= \frac{1}{2} \operatorname{Subst} \left( \int (a^2 x^2 + 2abx^2 \operatorname{sech}(c + dx) + b^2 x^2 \operatorname{sech}^2(c + dx)) dx, x, x^2 \right) \\
&= \frac{a^2 x^6}{6} + (ab) \operatorname{Subst} \left( \int x^2 \operatorname{sech}(c + dx) dx, x, x^2 \right) + \frac{1}{2} b^2 \operatorname{Subst} \left( \int x^2 \operatorname{sech}^2(c + dx) dx, x, x^2 \right) \\
&= \frac{a^2 x^6}{6} + \frac{2abx^4 \tan^{-1}(e^{c+dx^2})}{d} + \frac{b^2 x^4 \tanh(c + dx^2)}{2d} - \frac{(2iab) \operatorname{Subst} \left( \int x \log(1 + e^{c+dx^2}) dx, x, x^2 \right)}{d} \\
&= \frac{b^2 x^4}{2d} + \frac{a^2 x^6}{6} + \frac{2abx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{2iabx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} + \frac{2iabx^2 \operatorname{Li}_2(ie^{c+dx^2})}{d^2} \\
&= \frac{b^2 x^4}{2d} + \frac{a^2 x^6}{6} + \frac{2abx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 x^2 \log(1 + e^{2(c+dx^2)})}{d^2} - \frac{2iabx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} \\
&= \frac{b^2 x^4}{2d} + \frac{a^2 x^6}{6} + \frac{2abx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 x^2 \log(1 + e^{2(c+dx^2)})}{d^2} - \frac{2iabx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} \\
&= \frac{b^2 x^4}{2d} + \frac{a^2 x^6}{6} + \frac{2abx^4 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 x^2 \log(1 + e^{2(c+dx^2)})}{d^2} - \frac{2iabx^2 \operatorname{Li}_2(-ie^{c+dx^2})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 4.80, size = 294, normalized size = 1.35

$$\cosh(c + dx^2) (a + b \operatorname{sech}(c + dx^2))^2 \left( a^2 x^6 \cosh(c + dx^2) + \frac{3b \cosh(c + dx^2) (2iad^2 x^4 \log(1 - ie^{c+dx^2}) - 2iad^2 x^4 \log(1 + ie^{c+dx^2}))}{6(a \cosh(c + dx^2) + b \operatorname{sech}(c + dx^2))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] (Cosh[c + d\*x^2]\*(a + b\*Sech[c + d\*x^2])^2\*(a^2\*x^6\*Cosh[c + d\*x^2] + (3\*b\*Cosh[c + d\*x^2]\*((2\*b\*d^2\*E^(2\*c)\*x^4)/(1 + E^(2\*c)) + (2\*I)\*a\*d^2\*x^4\*Log[1 - I\*E^(c + d\*x^2)] - (2\*I)\*a\*d^2\*x^4\*Log[1 + I\*E^(c + d\*x^2)] - 2\*b\*d\*x^2\*Log[1 + E^(2\*(c + d\*x^2))]) - (4\*I)\*a\*d\*x^2\*PolyLog[2, (-I)\*E^(c + d\*x^2)] + (4\*I)\*a\*d\*x^2\*PolyLog[2, I\*E^(c + d\*x^2)] - b\*PolyLog[2, -E^(2\*(c + d\*x^2))] + (4\*I)\*a\*PolyLog[3, (-I)\*E^(c + d\*x^2)] - (4\*I)\*a\*PolyLog[3, I\*E^(c + d\*x^2)]))/d^3 + (3\*b^2\*x^4\*Sech[c]\*Sinh[d\*x^2])/d)/(6\*(b + a\*Cosh[c + d\*x^2])^2)

**fricas [C]** time = 0.50, size = 1210, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 1/6\*(a^2\*d^3\*x^6 - 6\*b^2\*c^2 + (a^2\*d^3\*x^6 + 6\*b^2\*d^2\*x^4 - 6\*b^2\*c^2)\*cosh(d\*x^2 + c)^2 + 2\*(a^2\*d^3\*x^6 + 6\*b^2\*d^2\*x^4 - 6\*b^2\*c^2)\*cosh(d\*x^2 + c)\*sinh(d\*x^2 + c) + (a^2\*d^3\*x^6 + 6\*b^2\*d^2\*x^4 - 6\*b^2\*c^2)\*sinh(d\*x^2 + c)^2 + (12\*I\*a\*b\*d\*x^2 - 6\*(-2\*I\*a\*b\*d\*x^2 + b^2)\*cosh(d\*x^2 + c)^2 - 12\*(-2\*I\*a\*b\*d\*x^2 + b^2)\*cosh(d\*x^2 + c)\*sinh(d\*x^2 + c) - 6\*(-2\*I\*a\*b\*d\*x^2 + b^2)\*sinh(d\*x^2 + c)^2 - 6\*b^2)\*dilog(I\*cosh(d\*x^2 + c) + I\*sinh(d\*x^2 + c)) + (-12\*I\*a\*b\*d\*x^2 - 6\*(2\*I\*a\*b\*d\*x^2 + b^2)\*cosh(d\*x^2 + c)^2 - 12\*(2\*I\*a\*b\*d\*x^2 + b^2)\*cosh(d\*x^2 + c)\*sinh(d\*x^2 + c) - 6\*(2\*I\*a\*b\*d\*x^2 + b^2)\*sinh(d\*x^2 + c)^2)

```

*sinh(d*x^2 + c)^2 - 6*b^2)*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) +
(6*I*a*b*c^2 + 6*b^2*c + (6*I*a*b*c^2 + 6*b^2*c)*cosh(d*x^2 + c)^2 + (12*I
*a*b*c^2 + 12*b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (6*I*a*b*c^2 + 6*b^2
*c)*sinh(d*x^2 + c)^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) + (-6*I*a
*b*c^2 + 6*b^2*c + (-6*I*a*b*c^2 + 6*b^2*c)*cosh(d*x^2 + c)^2 + (-12*I*a*b*
c^2 + 12*b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-6*I*a*b*c^2 + 6*b^2*c)*
sinh(d*x^2 + c)^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - I) + (-6*I*a*b*d
^2*x^4 - 6*b^2*d*x^2 + 6*I*a*b*c^2 - 6*b^2*c + (-6*I*a*b*d^2*x^4 - 6*b^2*d*
x^2 + 6*I*a*b*c^2 - 6*b^2*c)*cosh(d*x^2 + c)^2 + (-12*I*a*b*d^2*x^4 - 12*b^
2*d*x^2 + 12*I*a*b*c^2 - 12*b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-6*I*
a*b*d^2*x^4 - 6*b^2*d*x^2 + 6*I*a*b*c^2 - 6*b^2*c)*sinh(d*x^2 + c)^2)*log(I
*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) + (6*I*a*b*d^2*x^4 - 6*b^2*d*x^2
- 6*I*a*b*c^2 - 6*b^2*c + (6*I*a*b*d^2*x^4 - 6*b^2*d*x^2 - 6*I*a*b*c^2 - 6*
b^2*c)*cosh(d*x^2 + c)^2 + (12*I*a*b*d^2*x^4 - 12*b^2*d*x^2 - 12*I*a*b*c^2
- 12*b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (6*I*a*b*d^2*x^4 - 6*b^2*d*x^
2 - 6*I*a*b*c^2 - 6*b^2*c)*sinh(d*x^2 + c)^2)*log(-I*cosh(d*x^2 + c) - I*si
nh(d*x^2 + c) + 1) + (-12*I*a*b*cosh(d*x^2 + c)^2 - 24*I*a*b*cosh(d*x^2 + c
)*sinh(d*x^2 + c) - 12*I*a*b*sinh(d*x^2 + c)^2 - 12*I*a*b)*polylog(3, I*cos
h(d*x^2 + c) + I*sinh(d*x^2 + c)) + (12*I*a*b*cosh(d*x^2 + c)^2 + 24*I*a*b*
cosh(d*x^2 + c)*sinh(d*x^2 + c) + 12*I*a*b*sinh(d*x^2 + c)^2 + 12*I*a*b)*po
lylog(3, -I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)))/(d^3*cosh(d*x^2 + c)^2 +
2*d^3*cosh(d*x^2 + c)*sinh(d*x^2 + c) + d^3*sinh(d*x^2 + c)^2 + d^3)

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)^2 x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)^2\*x^5, x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{sech}(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(x^5\*(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} a^2 x^6 - \frac{b^2 x^4}{d e^{(2dx^2+2c)} + d} + \int \frac{4 \left( a b d x^5 e^{(dx^2+c)} + b^2 x^3 \right)}{d e^{(2dx^2+2c)} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out] 1/6\*a^2\*x^6 - b^2\*x^4/(d\*e^(2\*d\*x^2 + 2\*c) + d) + integrate(4\*(a\*b\*d\*x^5\*e^(d\*x^2 + c) + b^2\*x^3)/(d\*e^(2\*d\*x^2 + 2\*c) + d), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b/cosh(c + d*x^2))^2,x)`

[Out] `int(x^5*(a + b/cosh(c + d*x^2))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*sech(d*x**2+c))**2,x)`

[Out] `Integral(x**5*(a + b*sech(c + d*x**2))**2, x)`

### 3.9 $\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$

**Optimal.** Leaf size=21

$$\operatorname{Int}\left(x^4 (a + b \operatorname{sech}(c + dx^2))^2, x\right)$$

[Out] Unintegrable(x^4\*(a+b\*sech(d\*x^2+c))^2,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^4\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] Defer[Int][x^4\*(a + b\*Sech[c + d\*x^2])^2, x]

Rubi steps

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

**Mathematica [A]** time = 8.91, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] Integrate[x^4\*(a + b\*Sech[c + d\*x^2])^2, x]

**fricas [A]** time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2 x^4 \operatorname{sech}(dx^2 + c)^2 + 2 abx^4 \operatorname{sech}(dx^2 + c) + a^2 x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^4\*sech(d\*x^2 + c)^2 + 2\*a\*b\*x^4\*sech(d\*x^2 + c) + a^2\*x^4, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)^2 x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)^2\*x^4, x)

**maple [A]** time = 0.37, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{sech}(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*sech(d*x^2+c))^2,x)`

[Out] `int(x^4*(a+b*sech(d*x^2+c))^2,x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{5}a^2x^5 - \frac{b^2x^3}{de^{(2dx^2+2c)} + d} + \int \frac{4abdx^4e^{(dx^2+c)} + 3b^2x^2}{de^{(2dx^2+2c)} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

[Out] `1/5*a^2*x^5 - b^2*x^3/(d*e^(2*d*x^2 + 2*c) + d) + integrate((4*a*b*d*x^4*e^(d*x^2 + c) + 3*b^2*x^2)/(d*e^(2*d*x^2 + 2*c) + d), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int x^4 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a + b/cosh(c + d*x^2))^2,x)`

[Out] `int(x^4*(a + b/cosh(c + d*x^2))^2, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*sech(d*x**2+c))**2,x)`

[Out] `Integral(x**4*(a + b*sech(c + d*x**2))**2, x)`

### 3.10 $\int x^3 \left( a + b \operatorname{sech}(c + dx^2) \right)^2 dx$

**Optimal.** Leaf size=119

$$\frac{a^2 x^4}{4} - \frac{iab \operatorname{Li}_2(-ie^{dx^2+c})}{d^2} + \frac{iab \operatorname{Li}_2(ie^{dx^2+c})}{d^2} + \frac{2abx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 \log(\cosh(c + dx^2))}{2d^2} + \frac{b^2 x^2 \tanh(c + dx^2)}{2d}$$

[Out] 1/4\*a^2\*x^4+2\*a\*b\*x^2\*arctan(exp(d\*x^2+c))/d-1/2\*b^2\*ln(cosh(d\*x^2+c))/d^2-I\*a\*b\*polylog(2,-I\*exp(d\*x^2+c))/d^2+I\*a\*b\*polylog(2,I\*exp(d\*x^2+c))/d^2+1/2\*b^2\*x^2\*tanh(d\*x^2+c)/d

**Rubi [A]** time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {5436, 4190, 4180, 2279, 2391, 4184, 3475}

$$-\frac{iab \operatorname{PolyLog}(2, -ie^{c+dx^2})}{d^2} + \frac{iab \operatorname{PolyLog}(2, ie^{c+dx^2})}{d^2} + \frac{a^2 x^4}{4} + \frac{2abx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 \log(\cosh(c + dx^2))}{2d^2} + \frac{b^2 x^2 \tanh(c + dx^2)}{2d}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] (a^2\*x^4)/4 + (2\*a\*b\*x^2\*ArcTan[E^(c + d\*x^2)])/d - (b^2\*Log[Cosh[c + d\*x^2]])/(2\*d^2) - (I\*a\*b\*PolyLog[2, (-I)\*E^(c + d\*x^2)])/d^2 + (I\*a\*b\*PolyLog[2, I\*E^(c + d\*x^2)])/d^2 + (b^2\*x^2\*Tanh[c + d\*x^2])/(2\*d)

#### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/((f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int x^3 (a + b \operatorname{sech}(c + dx^2))^2 dx &= \frac{1}{2} \operatorname{Subst} \left( \int x (a + b \operatorname{sech}(c + dx))^2 dx, x, x^2 \right) \\ &= \frac{1}{2} \operatorname{Subst} \left( \int (a^2 x + 2abx \operatorname{sech}(c + dx) + b^2 x \operatorname{sech}^2(c + dx)) dx, x, x^2 \right) \\ &= \frac{a^2 x^4}{4} + (ab) \operatorname{Subst} \left( \int x \operatorname{sech}(c + dx) dx, x, x^2 \right) + \frac{1}{2} b^2 \operatorname{Subst} \left( \int x \operatorname{sech}^2(c + dx) dx, x, x^2 \right) \\ &= \frac{a^2 x^4}{4} + \frac{2abx^2 \tan^{-1}(e^{c+dx^2})}{d} + \frac{b^2 x^2 \tanh(c + dx^2)}{2d} - \frac{(iab) \operatorname{Subst} \left( \int \log(1 - e^{-2(c+dx^2)}) dx, x, x^2 \right)}{d} \\ &= \frac{a^2 x^4}{4} + \frac{2abx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 \log(\cosh(c + dx^2))}{2d^2} + \frac{b^2 x^2 \tanh(c + dx^2)}{2d} \\ &= \frac{a^2 x^4}{4} + \frac{2abx^2 \tan^{-1}(e^{c+dx^2})}{d} - \frac{b^2 \log(\cosh(c + dx^2))}{2d^2} - \frac{iab \operatorname{Li}_2(-ie^{c+dx^2})}{d^2} \end{aligned}$$

**Mathematica [B]** time = 3.20, size = 273, normalized size = 2.29

$$\cosh(c + dx^2) (a + b \operatorname{sech}(c + dx^2))^2 \left( dx^2 \cosh(c + dx^2) (a^2 dx^2 + 2b^2 \tanh(c)) + 4ab \cosh(c + dx^2) \left( \frac{\operatorname{csch}(c)}{d} \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*Sech[c + d*x^2])^2,x]
```

```
[Out] (Cosh[c + d*x^2]*(a + b*Sech[c + d*x^2])^2*(4*a*b*Cosh[c + d*x^2]*(-2*ArcTan[Sinh[c] + Cosh[c]*Tanh[(d*x^2)/2]]*ArcTanh[Coth[c]] + (Csch[c]*((d*x^2 + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x^2) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x^2) - ArcTanh[Coth[c]])]) + PolyLog[2, -E^(-(d*x^2) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x^2) - ArcTanh[Coth[c]])]))/Sqrt[-Csch[c]^2]) + 2*b^2*d*x^2*Sech[c]*Sinh[d*x^2] - 2*b^2*d*x^2*Cosh[c + d*x^2]*Tanh[c] + d*x^2*Cosh[c + d*x^2]*(a^2*d*x^2 + 2*b^2*Tanh[c]) - 2*b^2*Cosh[c + d*x^2]*(Log[Cosh[c + d*x^2]] - d*x^2*Tanh[c]))/(4*d^2*(b + a*Cosh[c + d*x^2])^2)
```

**fricas [B]** time = 0.48, size = 782, normalized size = 6.57

$$a^2 d^2 x^4 + 4 b^2 c + (a^2 d^2 x^4 + 4 b^2 dx^2 + 4 b^2 c) \cosh(dx^2 + c)^2 + 2 (a^2 d^2 x^4 + 4 b^2 dx^2 + 4 b^2 c) \cosh(dx^2 + c) \sinh(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d^2*x^4 + 4*b^2*c + (a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*cosh(d*x^2 + c)^2 + 2*(a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*sinh(d*x^2 + c)^2 + (4*I*a*b*cosh(d*x^2 + c)^2 + 8*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + 4*I*a*b*sinh(d*x^2 + c)^2 + 4*I*a*b)*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) + (-4*I*a*b*cosh(d*x^2 + c)^2 - 8*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) - 4*I*a*b*sinh(d*x^2 + c)^2 - 4*I*a*b)*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) + (-4*I*a*b*c - 2*(2*I*a*b*c + b^2)*cosh(d*x^2 + c)^2 - 4*(2*I*a*b*c + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) - 2*(2*I*a*b*c + b^2)*sinh(d*x^2 + c)^2 - 2*b^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) + (4*I*a*b*c - 2*(-2*I*a*b*c + b^2)*cosh(d*x^2 + c)^2 - 4*(-2*I*a*b*c + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) - 2*(-2*I*a*b*c + b^2)*sinh(d*x^2 + c)^2 - 2*b^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - I) + (-4*I*a*b*d*x^2 - 4*I*a*b*c + (-4*I*a*b*d*x^2 - 4*I*a*b*c)*cosh(d*x^2 + c)^2 + (-8*I*a*b*d*x^2 - 8*I*a*b*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-4*I*a*b*d*x^2 - 4*I*a*b*c)*sinh(d*x^2 + c)^2)*log(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) + (4*I*a*b*d*x^2 + 4*I*a*b*c + (4*I*a*b*d*x^2 + 4*I*a*b*c)*cosh(d*x^2 + c)^2 + (8*I*a*b*d*x^2 + 8*I*a*b*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (4*I*a*b*d*x^2 + 4*I*a*b*c)*sinh(d*x^2 + c)^2)*log(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c) + 1))/(d^2*cosh(d*x^2 + c)^2 + 2*d^2*cosh(d*x^2 + c)*sinh(d*x^2 + c) + d^2*sinh(d*x^2 + c)^2 + d^2)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^2 + c) + a)^2*x^3, x)
```

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(dx^2 + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*sech(d*x^2+c))^2,x)
```

```
[Out] int(x^3*(a+b*sech(d*x^2+c))^2,x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} a^2 x^4 + \frac{1}{2} \left( \frac{2x^2 e^{(2dx^2+2c)}}{d e^{(2dx^2+2c)} + d} - \frac{\log\left(\left(e^{(2dx^2+2c)} + 1\right)e^{(-2c)}\right)}{d^2} \right) b^2 + 4ab \int \frac{x^3 e^{(dx^2+c)}}{e^{(2dx^2+2c)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/2*(2*x^2*e^(2*d*x^2 + 2*c)/(d*e^(2*d*x^2 + 2*c) + d) - log((e^(2*d*x^2 + 2*c) + 1)*e^(-2*c))/d^2)*b^2 + 4*a*b*integrate(x^3*e^(d*x^2 + c)/(e^(2*d*x^2 + 2*c) + 1), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b/cosh(c + d*x^2))^2,x)`

[Out] `int(x^3*(a + b/cosh(c + d*x^2))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*sech(d*x**2+c))**2,x)`

[Out] `Integral(x**3*(a + b*sech(c + d*x**2))**2, x)`

### 3.11 $\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx$

**Optimal.** Leaf size=21

$$\operatorname{Int}\left(x^2 (a + b \operatorname{sech}(c + dx^2))^2, x\right)$$

[Out] Unintegrable(x^2\*(a+b\*sech(d\*x^2+c))^2,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] Defer[Int][x^2\*(a + b\*Sech[c + d\*x^2])^2, x]

Rubi steps

$$\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

**Mathematica [A]** time = 7.95, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] Integrate[x^2\*(a + b\*Sech[c + d\*x^2])^2, x]

**fricas [A]** time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2 x^2 \operatorname{sech}(dx^2 + c)^2 + 2 abx^2 \operatorname{sech}(dx^2 + c) + a^2 x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*sech(d\*x^2 + c)^2 + 2\*a\*b\*x^2\*sech(d\*x^2 + c) + a^2\*x^2, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^2 + c) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)^2\*x^2, x)

**maple [A]** time = 0.35, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(dx^2 + c))^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*sech(d*x^2+c))^2,x)`

[Out] `int(x^2*(a+b*sech(d*x^2+c))^2,x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^2 x^3 - \frac{b^2 x}{d e^{(2dx^2+2c)} + d} + \int \frac{4abd x^2 e^{(dx^2+c)} + b^2}{d e^{(2dx^2+2c)} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

[Out] `1/3*a^2*x^3 - b^2*x/(d*e^(2*d*x^2 + 2*c) + d) + integrate((4*a*b*d*x^2*e^(d*x^2 + c) + b^2)/(d*e^(2*d*x^2 + 2*c) + d), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int x^2 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/cosh(c + d*x^2))^2,x)`

[Out] `int(x^2*(a + b/cosh(c + d*x^2))^2, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*sech(d*x**2+c))**2,x)`

[Out] `Integral(x**2*(a + b*sech(c + d*x**2))**2, x)`

### 3.12 $\int x \left( a + b \operatorname{sech} \left( c + dx^2 \right) \right)^2 dx$

**Optimal.** Leaf size=44

$$\frac{a^2 x^2}{2} + \frac{ab \tan^{-1}(\sinh(c + dx^2))}{d} + \frac{b^2 \tanh(c + dx^2)}{2d}$$

[Out]  $1/2*a^2*x^2+a*b*\arctan(\sinh(d*x^2+c))/d+1/2*b^2*\tanh(d*x^2+c)/d$

**Rubi [A]** time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5436, 3773, 3770, 3767, 8}

$$\frac{a^2 x^2}{2} + \frac{ab \tan^{-1}(\sinh(c + dx^2))}{d} + \frac{b^2 \tanh(c + dx^2)}{2d}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] (a^2\*x^2)/2 + (a\*b\*ArcTan[Sinh[c + d\*x^2]])/d + (b^2\*Tanh[c + d\*x^2])/(2\*d)

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3773

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.))^2, x\_Symbol] := Simp[a^2\*x, x] + (Dist[2\*a\*b, Int[Csc[c + d\*x], x], x] + Dist[b^2, Int[Csc[c + d\*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int x (a + b \operatorname{sech}(c + dx^2))^2 dx &= \frac{1}{2} \operatorname{Subst} \left( \int (a + b \operatorname{sech}(c + dx))^2 dx, x, x^2 \right) \\
&= \frac{a^2 x^2}{2} + (ab) \operatorname{Subst} \left( \int \operatorname{sech}(c + dx) dx, x, x^2 \right) + \frac{1}{2} b^2 \operatorname{Subst} \left( \int \operatorname{sech}^2(c + dx) dx, x, x^2 \right) \\
&= \frac{a^2 x^2}{2} + \frac{ab \tan^{-1}(\sinh(c + dx^2))}{d} + \frac{(b^2) \operatorname{Subst} \left( \int 1 dx, x, -i \tanh(c + dx^2) \right)}{2d} \\
&= \frac{a^2 x^2}{2} + \frac{ab \tan^{-1}(\sinh(c + dx^2))}{d} + \frac{b^2 \tanh(c + dx^2)}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 44, normalized size = 1.00

$$\frac{a \left( a \left( c + dx^2 \right) + 2b \tan^{-1} \left( \sinh \left( c + dx^2 \right) \right) \right) + b^2 \tanh \left( c + dx^2 \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Sech[c + d\*x^2])^2,x]

[Out] (a\*(a\*(c + d\*x^2) + 2\*b\*ArcTan[Sinh[c + d\*x^2]]) + b^2\*Tanh[c + d\*x^2])/(2\*d)

**fricas [B]** time = 0.51, size = 194, normalized size = 4.41

$$\frac{a^2 dx^2 \cosh(dx^2 + c)^2 + 2 a^2 dx^2 \cosh(dx^2 + c) \sinh(dx^2 + c) + a^2 dx^2 \sinh(dx^2 + c)^2 + a^2 dx^2 - 2 b^2 + 4 (ab \cosh(dx^2 + c) \sinh(dx^2 + c) + b^2 \tanh(dx^2 + c))}{2 \left( d \cosh(dx^2 + c)^2 + 2 d \cosh(dx^2 + c) \sinh(dx^2 + c) + d \sinh(dx^2 + c)^2 + d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 1/2\*(a^2\*d\*x^2\*cosh(d\*x^2 + c)^2 + 2\*a^2\*d\*x^2\*cosh(d\*x^2 + c)\*sinh(d\*x^2 + c) + a^2\*d\*x^2\*sinh(d\*x^2 + c)^2 + a^2\*d\*x^2 - 2\*b^2 + 4\*(a\*b\*cosh(d\*x^2 + c)^2 + 2\*a\*b\*cosh(d\*x^2 + c)\*sinh(d\*x^2 + c) + a\*b\*sinh(d\*x^2 + c)^2 + a\*b)\*arctan(cosh(d\*x^2 + c) + sinh(d\*x^2 + c))/(d\*cosh(d\*x^2 + c)^2 + 2\*d\*cosh(d\*x^2 + c)\*sinh(d\*x^2 + c) + d\*sinh(d\*x^2 + c)^2 + d)

**giac [A]** time = 0.12, size = 55, normalized size = 1.25

$$\frac{(dx^2 + c)a^2}{2d} + \frac{2ab \arctan \left( e^{(dx^2+c)} \right)}{d} - \frac{b^2}{d \left( e^{(2dx^2+2c)} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] 1/2\*(d\*x^2 + c)\*a^2/d + 2\*a\*b\*arctan(e^(d\*x^2 + c))/d - b^2/(d\*(e^(2\*d\*x^2 + 2\*c) + 1))

**maple [A]** time = 0.28, size = 51, normalized size = 1.16

$$\frac{a^2 x^2}{2} + \frac{b^2 \tanh(dx^2 + c)}{2d} + \frac{2ab \arctan \left( e^{dx^2+c} \right)}{d} + \frac{a^2 c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*sech(d*x^2+c))^2,x)`

[Out]  $\frac{1}{2}a^2x^2 + \frac{1}{2}b^2 \frac{\tanh(dx^2+c)}{d} + \frac{ab \arctan(\exp(dx^2+c))}{d} + \frac{1}{2} \frac{b^2}{d(a^2 - \exp(-2dx^2 - 2c))}$

**maxima** [A] time = 0.33, size = 46, normalized size = 1.05

$$\frac{1}{2}a^2x^2 + \frac{ab \arctan(\sinh(dx^2 + c))}{d} + \frac{b^2}{d(e^{(-2dx^2-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}a^2x^2 + \frac{ab \arctan(\sinh(dx^2 + c))}{d} + \frac{b^2}{d(e^{(-2dx^2 - 2c)} + 1)}$

**mupad** [B] time = 0.10, size = 77, normalized size = 1.75

$$\frac{a^2x^2}{2} + \frac{2 \operatorname{atan}\left(\frac{ab e^{dx^2} e^c \sqrt{d^2}}{d \sqrt{a^2 b^2}}\right) \sqrt{a^2 b^2}}{\sqrt{d^2}} - \frac{b^2}{d(e^{2dx^2+2c} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b/cosh(c + d*x^2))^2,x)`

[Out]  $(a^2x^2)/2 + (2 \operatorname{atan}((ab \exp(dx^2) \exp(c) (d^2)^{(1/2)}) / (d(a^2b^2)^{(1/2)}))) (a^2b^2)^{(1/2)} / (d^2)^{(1/2)} - b^2 / (d(\exp(2c + 2dx^2) + 1))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{sech}(c + dx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sech(d*x**2+c))**2,x)`

[Out] `Integral(x*(a + b*sech(c + d*x**2))**2, x)`

$$3.13 \quad \int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(a+b\operatorname{sech}(c+dx^2))^2}{x}, x\right)$$

[Out] Unintegrable((a+b\*sech(d\*x^2+c))^2/x, x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*x^2])^2/x, x]

[Out] Defer[Int][(a + b\*Sech[c + d\*x^2])^2/x, x]

Rubi steps

$$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx = \int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$$

Mathematica [A] time = 20.68, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*x^2])^2/x, x]

[Out] Integrate[(a + b\*Sech[c + d\*x^2])^2/x, x]

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \operatorname{sech}(dx^2+c)^2 + 2ab \operatorname{sech}(dx^2+c) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))^2/x, x, algorithm="fricas")

[Out] integral((b^2\*sech(d\*x^2 + c)^2 + 2\*a\*b\*sech(d\*x^2 + c) + a^2)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{sech}(dx^2+c)+a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))^2/x, x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)^2/x, x)

**maple** [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(d x^2 + c))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(d\*x^2+c))^2/x,x)

[Out] int((a+b\*sech(d\*x^2+c))^2/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) - \frac{b^2}{dx^2 e^{(2dx^2+2c)} + dx^2} + \int \frac{2(2abd x^2 e^{(dx^2+c)} - b^2)}{dx^3 e^{(2dx^2+2c)} + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))^2/x,x, algorithm="maxima")

[Out] a^2\*log(x) - b^2/(d\*x^2\*e^(2\*d\*x^2 + 2\*c) + d\*x^2) + integrate(2\*(2\*a\*b\*d\*x^2\*e^(d\*x^2 + c) - b^2)/(d\*x^3\*e^(2\*d\*x^2 + 2\*c) + d\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^2))^2/x,x)

[Out] int((a + b/cosh(c + d\*x^2))^2/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x\*\*2+c))\*\*2/x,x)

[Out] Integral((a + b\*sech(c + d\*x\*\*2))\*\*2/x, x)

$$3.14 \quad \int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2}, x\right)$$

[Out] Unintegrable((a+b\*sech(d\*x^2+c))^2/x^2, x)

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*x^2])^2/x^2, x]

[Out] Defer[Int][(a + b\*Sech[c + d\*x^2])^2/x^2, x]

Rubi steps

$$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx = \int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$$

**Mathematica** [A] time = 9.85, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*x^2])^2/x^2, x]

[Out] Integrate[(a + b\*Sech[c + d\*x^2])^2/x^2, x]

**fricas** [A] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \operatorname{sech}(dx^2 + c)^2 + 2ab \operatorname{sech}(dx^2 + c) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))^2/x^2, x, algorithm="fricas")

[Out] integral((b^2\*sech(d\*x^2 + c)^2 + 2\*a\*b\*sech(d\*x^2 + c) + a^2)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{sech}(dx^2 + c) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))^2/x^2, x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)^2/x^2, x)

**maple** [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(d x^2 + c))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(d\*x^2+c))^2/x^2,x)

[Out] int((a+b\*sech(d\*x^2+c))^2/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b^2}{dx^3 e^{(2dx^2+2c)} + dx^3} - \frac{a^2}{x} + \int \frac{4abd x^2 e^{(dx^2+c)} - 3b^2}{dx^4 e^{(2dx^2+2c)} + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))^2/x^2,x, algorithm="maxima")

[Out] -b^2/(d\*x^3\*e^(2\*d\*x^2 + 2\*c) + d\*x^3) - a^2/x + integrate((4\*a\*b\*d\*x^2\*e^(d\*x^2 + c) - 3\*b^2)/(d\*x^4\*e^(2\*d\*x^2 + 2\*c) + d\*x^4), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^2))^2/x^2,x)

[Out] int((a + b/cosh(c + d\*x^2))^2/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x\*\*2+c))\*\*2/x\*\*2,x)

[Out] Integral((a + b\*sech(c + d\*x\*\*2))\*\*2/x\*\*2, x)



### 3.15 $\int x \operatorname{sech}^7(a + bx^2) dx$

**Optimal.** Leaf size=90

$$\frac{5 \tan^{-1}(\sinh(a + bx^2))}{32b} + \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{12b} + \frac{5 \tanh(a + bx^2) \operatorname{sech}^3(a + bx^2)}{48b} + \frac{5 \tanh(a + bx^2)}{32b}$$

[Out] 5/32\*arctan(sinh(b\*x^2+a))/b+5/32\*sech(b\*x^2+a)\*tanh(b\*x^2+a)/b+5/48\*sech(b\*x^2+a)^3\*tanh(b\*x^2+a)/b+1/12\*sech(b\*x^2+a)^5\*tanh(b\*x^2+a)/b

**Rubi [A]** time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5436, 3768, 3770}

$$\frac{5 \tan^{-1}(\sinh(a + bx^2))}{32b} + \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{12b} + \frac{5 \tanh(a + bx^2) \operatorname{sech}^3(a + bx^2)}{48b} + \frac{5 \tanh(a + bx^2)}{32b}$$

Antiderivative was successfully verified.

[In] Int[x\*Sech[a + b\*x^2]^7,x]

[Out] (5\*ArcTan[Sinh[a + b\*x^2]])/(32\*b) + (5\*Sech[a + b\*x^2]\*Tanh[a + b\*x^2])/(32\*b) + (5\*Sech[a + b\*x^2]^3\*Tanh[a + b\*x^2])/(48\*b) + (Sech[a + b\*x^2]^5\*Tanh[a + b\*x^2])/(12\*b)

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_.)]\*(b\_.))^n, x\_Symbol] := -Simp[(b\*Csc[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_.)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned} \int x \operatorname{sech}^7(a + bx^2) dx &= \frac{1}{2} \operatorname{Subst} \left( \int \operatorname{sech}^7(a + bx) dx, x, x^2 \right) \\ &= \frac{\operatorname{sech}^5(a + bx^2) \tanh(a + bx^2)}{12b} + \frac{5}{12} \operatorname{Subst} \left( \int \operatorname{sech}^5(a + bx) dx, x, x^2 \right) \\ &= \frac{5 \operatorname{sech}^3(a + bx^2) \tanh(a + bx^2)}{48b} + \frac{\operatorname{sech}^5(a + bx^2) \tanh(a + bx^2)}{12b} + \frac{5}{16} \operatorname{Subst} \left( \int \operatorname{sech}^3(a + bx) dx, x, x^2 \right) \\ &= \frac{5 \operatorname{sech}(a + bx^2) \tanh(a + bx^2)}{32b} + \frac{5 \operatorname{sech}^3(a + bx^2) \tanh(a + bx^2)}{48b} + \frac{\operatorname{sech}^5(a + bx^2) \tanh(a + bx^2)}{12b} \\ &= \frac{5 \tan^{-1}(\sinh(a + bx^2))}{32b} + \frac{5 \operatorname{sech}(a + bx^2) \tanh(a + bx^2)}{32b} + \frac{5 \operatorname{sech}^3(a + bx^2) \tanh(a + bx^2)}{48b} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 77, normalized size = 0.86

$$\frac{15 \tan^{-1}(\sinh(a + bx^2)) + 8 \tanh(a + bx^2) \operatorname{sech}^5(a + bx^2) + 10 \tanh(a + bx^2) \operatorname{sech}^3(a + bx^2) + 15 \tanh(a + bx^2)}{96b}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sech[a + b\*x^2]^7,x]

[Out] (15\*ArcTan[Sinh[a + b\*x^2]] + 15\*Sech[a + b\*x^2]\*Tanh[a + b\*x^2] + 10\*Sech[a + b\*x^2]^3\*Tanh[a + b\*x^2] + 8\*Sech[a + b\*x^2]^5\*Tanh[a + b\*x^2])/(96\*b)

**fricas [B]** time = 0.41, size = 1918, normalized size = 21.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sech(b\*x^2+a)^7,x, algorithm="fricas")

[Out]  $\frac{1}{48} \cdot (15 \cdot \cosh(bx^2 + a)^{11} + 165 \cdot \cosh(bx^2 + a) \cdot \sinh(bx^2 + a)^{10} + 15 \cdot \sinh(bx^2 + a)^{11} + 5 \cdot (165 \cdot \cosh(bx^2 + a)^2 + 17) \cdot \sinh(bx^2 + a)^9 + 85 \cdot \cosh(bx^2 + a)^9 + 45 \cdot (55 \cdot \cosh(bx^2 + a)^3 + 17 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^8 + 18 \cdot (275 \cdot \cosh(bx^2 + a)^4 + 170 \cdot \cosh(bx^2 + a)^2 + 11) \cdot \sinh(bx^2 + a)^7 + 198 \cdot \cosh(bx^2 + a)^7 + 42 \cdot (165 \cdot \cosh(bx^2 + a)^5 + 170 \cdot \cosh(bx^2 + a)^3 + 33 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^6 + 18 \cdot (385 \cdot \cosh(bx^2 + a)^6 + 595 \cdot \cosh(bx^2 + a)^4 + 231 \cdot \cosh(bx^2 + a)^2 - 11) \cdot \sinh(bx^2 + a)^5 - 198 \cdot \cosh(bx^2 + a)^5 + 90 \cdot (55 \cdot \cosh(bx^2 + a)^7 + 119 \cdot \cosh(bx^2 + a)^5 + 77 \cdot \cosh(bx^2 + a)^3 - 11 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^4 + 5 \cdot (495 \cdot \cosh(bx^2 + a)^8 + 1428 \cdot \cosh(bx^2 + a)^6 + 1386 \cdot \cosh(bx^2 + a)^4 - 396 \cdot \cosh(bx^2 + a)^2 - 17) \cdot \sinh(bx^2 + a)^3 - 85 \cdot \cosh(bx^2 + a)^3 + 3 \cdot (275 \cdot \cosh(bx^2 + a)^9 + 1020 \cdot \cosh(bx^2 + a)^7 + 1386 \cdot \cosh(bx^2 + a)^5 - 660 \cdot \cosh(bx^2 + a)^3 - 85 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^2 + 15 \cdot (\cosh(bx^2 + a)^{12} + 12 \cdot \cosh(bx^2 + a) \cdot \sinh(bx^2 + a)^{11} + \sinh(bx^2 + a)^{12} + 6 \cdot (11 \cdot \cosh(bx^2 + a)^2 + 1) \cdot \sinh(bx^2 + a)^{10} + 6 \cdot \cosh(bx^2 + a)^{10} + 20 \cdot (11 \cdot \cosh(bx^2 + a)^3 + 3 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^9 + 15 \cdot (33 \cdot \cosh(bx^2 + a)^4 + 18 \cdot \cosh(bx^2 + a)^2 + 1) \cdot \sinh(bx^2 + a)^8 + 15 \cdot \cosh(bx^2 + a)^8 + 24 \cdot (33 \cdot \cosh(bx^2 + a)^5 + 30 \cdot \cosh(bx^2 + a)^3 + 5 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^7 + 4 \cdot (231 \cdot \cosh(bx^2 + a)^6 + 315 \cdot \cosh(bx^2 + a)^4 + 105 \cdot \cosh(bx^2 + a)^2 + 5) \cdot \sinh(bx^2 + a)^6 + 20 \cdot \cosh(bx^2 + a)^6 + 24 \cdot (33 \cdot \cosh(bx^2 + a)^7 + 63 \cdot \cosh(bx^2 + a)^5 + 35 \cdot \cosh(bx^2 + a)^3 + 5 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^5 + 15 \cdot (33 \cdot \cosh(bx^2 + a)^8 + 84 \cdot \cosh(bx^2 + a)^6 + 70 \cdot \cosh(bx^2 + a)^4 + 20 \cdot \cosh(bx^2 + a)^2 + 1) \cdot \sinh(bx^2 + a)^4 + 15 \cdot \cosh(bx^2 + a)^4 + 20 \cdot (11 \cdot \cosh(bx^2 + a)^9 + 36 \cdot \cosh(bx^2 + a)^7 + 42 \cdot \cosh(bx^2 + a)^5 + 20 \cdot \cosh(bx^2 + a)^3 + 3 \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^3 + 6 \cdot (11 \cdot \cosh(bx^2 + a)^{10} + 45 \cdot \cosh(bx^2 + a)^8 + 70 \cdot \cosh(bx^2 + a)^6 + 50 \cdot \cosh(bx^2 + a)^4 + 15 \cdot \cosh(bx^2 + a)^2 + 1) \cdot \sinh(bx^2 + a)^2 + 6 \cdot \cosh(bx^2 + a)^2 + 12 \cdot (\cosh(bx^2 + a)^{11} + 5 \cdot \cosh(bx^2 + a)^9 + 10 \cdot \cosh(bx^2 + a)^7 + 10 \cdot \cosh(bx^2 + a)^5 + 5 \cdot \cosh(bx^2 + a)^3 + \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a) + 1) \cdot \arctan(\cosh(bx^2 + a) + \sinh(bx^2 + a)) + 3 \cdot (55 \cdot \cosh(bx^2 + a)^{10} + 255 \cdot \cosh(bx^2 + a)^8 + 462 \cdot \cosh(bx^2 + a)^6 - 330 \cdot \cosh(bx^2 + a)^4 - 85 \cdot \cosh(bx^2 + a)^2 - 5) \cdot \sinh(bx^2 + a) - 15 \cdot \cosh(bx^2 + a)) / (b \cdot \cosh(bx^2 + a)^{12} + 12 \cdot b \cdot \cosh(bx^2 + a) \cdot \sinh(bx^2 + a)^{11} + b \cdot \sinh(bx^2 + a)^{12} + 6 \cdot b \cdot \cosh(bx^2 + a)^{10} + 6 \cdot (11 \cdot b \cdot \cosh(bx^2 + a)^2 + b) \cdot \sinh(bx^2 + a)^{10} + 20 \cdot (11 \cdot b \cdot \cosh(bx^2 + a)^3 + 3 \cdot b \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^9 + 15 \cdot b \cdot \cosh(bx^2 + a)^8 + 15 \cdot (33 \cdot b \cdot \cosh(bx^2 + a)^4 + 18 \cdot b \cdot \cosh(bx^2 + a)^2 + b) \cdot \sinh(bx^2 + a)^8 + 24 \cdot (33 \cdot b \cdot \cosh(bx^2 + a)^5 + 30 \cdot b \cdot \cosh(bx^2 + a)^3 + 5 \cdot b \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a)^7 + 20 \cdot b \cdot \cosh(bx^2 + a)^6 + 4 \cdot (231 \cdot b \cdot \cosh(bx^2 + a)^6 + 315 \cdot b \cdot \cosh(bx^2 + a)^4 + 105 \cdot b \cdot \cosh(bx^2 + a)^2 + 5 \cdot b) \cdot \sinh(bx^2 + a)^6 + 24 \cdot (33 \cdot b \cdot \cosh(bx^2 + a)^7 + 63 \cdot b \cdot \cosh(bx^2 + a)^5 + 35 \cdot b \cdot \cosh(bx^2 + a)^3 + 5 \cdot b \cdot \cosh(bx^2 + a)) \cdot \sinh(bx^2 + a) + 1)$

$$a)^5 + 15*b*\cosh(b*x^2 + a)^4 + 15*(33*b*\cosh(b*x^2 + a)^8 + 84*b*\cosh(b*x^2 + a)^6 + 70*b*\cosh(b*x^2 + a)^4 + 20*b*\cosh(b*x^2 + a)^2 + b)*\sinh(b*x^2 + a)^4 + 20*(11*b*\cosh(b*x^2 + a)^9 + 36*b*\cosh(b*x^2 + a)^7 + 42*b*\cosh(b*x^2 + a)^5 + 20*b*\cosh(b*x^2 + a)^3 + 3*b*\cosh(b*x^2 + a))*\sinh(b*x^2 + a)^3 + 6*b*\cosh(b*x^2 + a)^2 + 6*(11*b*\cosh(b*x^2 + a)^10 + 45*b*\cosh(b*x^2 + a)^8 + 70*b*\cosh(b*x^2 + a)^6 + 50*b*\cosh(b*x^2 + a)^4 + 15*b*\cosh(b*x^2 + a)^2 + b)*\sinh(b*x^2 + a)^2 + 12*(b*\cosh(b*x^2 + a)^11 + 5*b*\cosh(b*x^2 + a)^9 + 10*b*\cosh(b*x^2 + a)^7 + 10*b*\cosh(b*x^2 + a)^5 + 5*b*\cosh(b*x^2 + a)^3 + b*\cosh(b*x^2 + a))*\sinh(b*x^2 + a) + b)$$

**giac** [A] time = 0.14, size = 146, normalized size = 1.62

$$\frac{5\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2bx^2+2a)} - 1\right)e^{(-bx^2-a)}\right)\right)}{64b} + \frac{15\left(e^{(bx^2+a)} - e^{(-bx^2-a)}\right)^5 + 160\left(e^{(bx^2+a)} - e^{(-bx^2-a)}\right)^3 + 528e^{(bx^2+a)} - 528e^{(-bx^2-a)}}{48\left(\left(e^{(bx^2+a)} - e^{(-bx^2-a)}\right)^2 + 4\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sech(b\*x^2+a)^7,x, algorithm="giac")

[Out] 5/64\*(pi + 2\*arctan(1/2\*(e^(2\*b\*x^2 + 2\*a) - 1)\*e^(-b\*x^2 - a)))/b + 1/48\*(15\*(e^(b\*x^2 + a) - e^(-b\*x^2 - a))^5 + 160\*(e^(b\*x^2 + a) - e^(-b\*x^2 - a))^3 + 528\*e^(b\*x^2 + a) - 528\*e^(-b\*x^2 - a))/((e^(b\*x^2 + a) - e^(-b\*x^2 - a))^2 + 4)^3\*b)

**maple** [A] time = 0.25, size = 83, normalized size = 0.92

$$\frac{\operatorname{sech}(bx^2+a)^5 \tanh(bx^2+a)}{12b} + \frac{5 \operatorname{sech}(bx^2+a)^3 \tanh(bx^2+a)}{48b} + \frac{5 \operatorname{sech}(bx^2+a) \tanh(bx^2+a)}{32b} + \frac{5 \arctan\left(\frac{e^{(2bx^2+2a)} - 1}{2} e^{(-bx^2-a)}\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*sech(b\*x^2+a)^7,x)

[Out] 1/12\*sech(b\*x^2+a)^5\*tanh(b\*x^2+a)/b+5/48\*sech(b\*x^2+a)^3\*tanh(b\*x^2+a)/b+5/32\*sech(b\*x^2+a)\*tanh(b\*x^2+a)/b+5/16/b\*arctan(exp(b\*x^2+a))

**maxima** [B] time = 0.43, size = 182, normalized size = 2.02

$$\frac{5 \arctan\left(e^{(-bx^2-a)}\right)}{16b} + \frac{15e^{(-bx^2-a)} + 85e^{(-3bx^2-3a)} + 198e^{(-5bx^2-5a)} - 198e^{(-7bx^2-7a)} - 85e^{(-9bx^2-9a)} - 15e^{(-11bx^2-11a)}}{48b\left(6e^{(-2bx^2-2a)} + 15e^{(-4bx^2-4a)} + 20e^{(-6bx^2-6a)} + 15e^{(-8bx^2-8a)} + 6e^{(-10bx^2-10a)} + e^{(-12bx^2-12a)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sech(b\*x^2+a)^7,x, algorithm="maxima")

[Out] -5/16\*arctan(e^(-b\*x^2 - a))/b + 1/48\*(15\*e^(-b\*x^2 - a) + 85\*e^(-3\*b\*x^2 - 3\*a) + 198\*e^(-5\*b\*x^2 - 5\*a) - 198\*e^(-7\*b\*x^2 - 7\*a) - 85\*e^(-9\*b\*x^2 - 9\*a) - 15\*e^(-11\*b\*x^2 - 11\*a))/(b\*(6\*e^(-2\*b\*x^2 - 2\*a) + 15\*e^(-4\*b\*x^2 - 4\*a) + 20\*e^(-6\*b\*x^2 - 6\*a) + 15\*e^(-8\*b\*x^2 - 8\*a) + 6\*e^(-10\*b\*x^2 - 10\*a) + e^(-12\*b\*x^2 - 12\*a) + 1))

**mupad** [B] time = 0.10, size = 395, normalized size = 4.39

$$\frac{5 \operatorname{atan}\left(\frac{e^a e^{bx^2} \sqrt{b^2}}{b}\right)}{16\sqrt{b^2}} - \frac{8e^{3bx^2+3a}}{3b\left(5e^{2bx^2+2a} + 10e^{4bx^2+4a} + 10e^{6bx^2+6a} + 5e^{8bx^2+8a} + e^{10bx^2+10a} + 1\right)}{b\left(4e^{2bx^2+2a} + 10e^{4bx^2+4a} + 10e^{6bx^2+6a} + 5e^{8bx^2+8a} + e^{10bx^2+10a} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/cosh(a + b\*x^2)^7,x)

[Out] (5\*atan((exp(a)\*exp(b\*x^2)\*(b^2)^(1/2))/b))/(16\*(b^2)^(1/2)) - (8\*exp(3\*a + 3\*b\*x^2))/(3\*b\*(5\*exp(2\*a + 2\*b\*x^2) + 10\*exp(4\*a + 4\*b\*x^2) + 10\*exp(6\*a + 6\*b\*x^2) + 5\*exp(8\*a + 8\*b\*x^2) + exp(10\*a + 10\*b\*x^2) + 1)) - exp(a + b\*x^2)/(b\*(4\*exp(2\*a + 2\*b\*x^2) + 6\*exp(4\*a + 4\*b\*x^2) + 4\*exp(6\*a + 6\*b\*x^2) + exp(8\*a + 8\*b\*x^2) + 1)) + (5\*exp(a + b\*x^2))/(24\*b\*(2\*exp(2\*a + 2\*b\*x^2) + exp(4\*a + 4\*b\*x^2) + 1)) - (16\*exp(5\*a + 5\*b\*x^2))/(3\*b\*(6\*exp(2\*a + 2\*b\*x^2) + 15\*exp(4\*a + 4\*b\*x^2) + 20\*exp(6\*a + 6\*b\*x^2) + 15\*exp(8\*a + 8\*b\*x^2) + 6\*exp(10\*a + 10\*b\*x^2) + exp(12\*a + 12\*b\*x^2) + 1)) + exp(a + b\*x^2)/(6\*b\*(3\*exp(2\*a + 2\*b\*x^2) + 3\*exp(4\*a + 4\*b\*x^2) + exp(6\*a + 6\*b\*x^2) + 1)) + (5\*exp(a + b\*x^2))/(16\*b\*(exp(2\*a + 2\*b\*x^2) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{sech}^7(a + bx^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*sech(b\*x\*\*2+a)\*\*7,x)

[Out] Integral(x\*sech(a + b\*x\*\*2)\*\*7, x)

### 3.16 $\int \frac{x^5}{a+b\operatorname{sech}(c+dx^2)} dx$

**Optimal.** Leaf size=349

$$\frac{b\operatorname{Li}_3\left(-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{b\operatorname{Li}_3\left(-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{bx^2\operatorname{Li}_2\left(-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{bx^2\operatorname{Li}_2\left(-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^4\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}+1\right)}{2ad\sqrt{b^2-a^2}} + \frac{bx^4\log\left(\frac{ae^{c+dx^2}}{b+\sqrt{b^2-a^2}}+1\right)}{2ad\sqrt{b^2-a^2}}$$

```
[Out] 1/6*x^6/a-1/2*b*x^4*ln(1+a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+1/2*b*x^4*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)-b*x^2*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+b*x^2*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+b*polylog(3,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-b*polylog(3,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)
```

**Rubi [A]** time = 0.86, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2531, 2282, 6589}

$$-\frac{bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{bx^2\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{b\operatorname{PolyLog}\left(3,-\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*Sech[c + d*x^2]), x]
```

```
[Out] x^6/(6*a) - (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])])/(2*a*Sqrt[-a^2 + b^2]*d) + (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])])/(2*a*Sqrt[-a^2 + b^2]*d) - (b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (b*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (b*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3)
```

**Rule 2190**

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m-1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2264**

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

**Rule 2282**

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))]
```

$(F\_)[v\_]$  /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] :> Dist[2, Int[((c + d\*x)^m\*E^(-I\*e) + f\*fz\*x)/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/2)) - (b\*E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))], x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx &= \frac{1}{2} \operatorname{Subst} \left( \int \frac{x^2}{a + b \operatorname{sech}(c + dx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \operatorname{Subst} \left( \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b + a \cosh(c + dx))} \right) dx, x, x^2 \right) \\
&= \frac{x^6}{6a} - \frac{b \operatorname{Subst} \left( \int \frac{x^2}{b + a \cosh(c + dx)} dx, x, x^2 \right)}{2a} \\
&= \frac{x^6}{6a} - \frac{b \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{a + 2be^{c+dx} + ae^{2(c+dx)}} dx, x, x^2 \right)}{6a} \\
&= \frac{x^6}{6a} - \frac{b \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, x^2 \right)}{\sqrt{-a^2 + b^2}} + \frac{b \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, x^2 \right)}{\sqrt{-a^2 + b^2}} \\
&= \frac{x^6}{6a} - \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{b \operatorname{Subst} \left( \int x \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right) dx, x, x^2 \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^6}{6a} - \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2} \\
&= \frac{x^6}{6a} - \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2} \\
&= \frac{x^6}{6a} - \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d^2}
\end{aligned}$$

**Mathematica [A]** time = 1.57, size = 376, normalized size = 1.08

$$\frac{d^3 x^6 \sqrt{e^{2c} (b^2 - a^2)} - 3be^c d^2 x^4 \log \left( \frac{ae^{2c+dx^2}}{be^c - \sqrt{e^{2c} (b^2 - a^2)}} + 1 \right) + 3be^c d^2 x^4 \log \left( \frac{ae^{2c+dx^2}}{\sqrt{e^{2c} (b^2 - a^2)} + be^c} + 1 \right) - 6be^c dx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{6ad^3 \sqrt{e^{2c} (b^2 - a^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*Sech[c + d\*x^2]), x]

[Out] (d^3\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^6 - 3\*b\*d^2\*E^c\*x^4\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]]] + 3\*b\*d^2\*E^c\*x^4\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]]] - 6\*b\*d\*E^c\*x^2\*PolyLog[2, -(a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]]] + 6\*b\*d\*E^c\*x^2\*PolyLog[2, -(a\*E^(2\*c + d\*x^2))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]]] + 6\*b\*E^c\*PolyLog[3, -(a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]]] - 6\*b\*E^c\*PolyLog[3, -(a\*E^(2\*c + d\*x^2))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]]])/(6\*a\*d^3\*Sqrt[(-a^2 + b^2)\*E^(2\*c)])

**fricas [C]** time = 0.43, size = 731, normalized size = 2.09

$$\frac{(a^2 - b^2)d^3 x^6 + 6abd x^2 \sqrt{-\frac{a^2 - b^2}{a^2}} \operatorname{Li}_2 \left( -\frac{b \cosh(dx^2 + c) + b \sinh(dx^2 + c) + (a \cosh(dx^2 + c) + a \sinh(dx^2 + c)) \sqrt{-\frac{a^2 - b^2}{a^2}} + a}{a} + 1 \right) - 6abd^3 \sqrt{e^{2c} (b^2 - a^2)}}{6ad^3 \sqrt{e^{2c} (b^2 - a^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sech(d\*x^2+c)),x, algorithm="fricas")

[Out]  $\frac{1}{6} * ((a^2 - b^2) * d^3 * x^6 + 6 * a * b * d * x^2 * \sqrt{-(a^2 - b^2) / a^2} * \text{dilog}(-(b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) + (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2} + a) / a + 1) - 6 * a * b * d * x^2 * \sqrt{-(a^2 - b^2) / a^2} * \text{dilog}(-(b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) - (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2} + a) / a + 1) - 3 * a * b * c^2 * \sqrt{-(a^2 - b^2) / a^2} * \log(2 * a * \cosh(d * x^2 + c) + 2 * a * \sinh(d * x^2 + c) + 2 * a * \sqrt{-(a^2 - b^2) / a^2} + 2 * b) + 3 * a * b * c^2 * \sqrt{-(a^2 - b^2) / a^2} * \log(2 * a * \cosh(d * x^2 + c) + 2 * a * \sinh(d * x^2 + c) - 2 * a * \sqrt{-(a^2 - b^2) / a^2} + 2 * b) - 6 * a * b * \sqrt{-(a^2 - b^2) / a^2} * \text{polylog}(3, -(b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) + (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2})) / a + 6 * a * b * \sqrt{-(a^2 - b^2) / a^2} * \text{polylog}(3, -(b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) - (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2})) / a + 3 * (a * b * d^2 * x^4 - a * b * c^2) * \sqrt{-(a^2 - b^2) / a^2} * \log((b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) + (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2} + a) / a) - 3 * (a * b * d^2 * x^4 - a * b * c^2) * \sqrt{-(a^2 - b^2) / a^2} * \log((b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) - (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2} + a) / a)) / ((a^3 - a * b^2) * d^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{b \operatorname{sech}(dx^2 + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sech(d\*x^2+c)),x, algorithm="giac")

[Out] integrate(x^5/(b\*sech(d\*x^2 + c) + a), x)

**maple** [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + b \operatorname{sech}(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a+b\*sech(d\*x^2+c)),x)

[Out] int(x^5/(a+b\*sech(d\*x^2+c)),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sech(d\*x^2+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{a + \frac{b}{\cosh(dx^2+c)}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(a + b/cosh(c + d*x^2)), x)`

[Out] `int(x^5/(a + b/cosh(c + d*x^2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(a+b*sech(d*x**2+c)), x)`

[Out] `Integral(x**5/(a + b*sech(c + d*x**2)), x)`

$$3.17 \quad \int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx$$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{x^4}{a+b\operatorname{sech}(c+dx^2)}, x\right)$$

[Out] Unintegrable(x^4/(a+b\*sech(d\*x^2+c)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[x^4/(a + b\*Sech[c + d\*x^2]), x]

[Out] Defer[Int][x^4/(a + b\*Sech[c + d\*x^2]), x]

Rubi steps

$$\int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx = \int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx$$

Mathematica [A] time = 8.73, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4/(a + b\*Sech[c + d\*x^2]), x]

[Out] Integrate[x^4/(a + b\*Sech[c + d\*x^2]), x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^4}{b\operatorname{sech}(dx^2+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sech(d\*x^2+c)), x, algorithm="fricas")

[Out] integral(x^4/(b\*sech(d\*x^2 + c) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b\operatorname{sech}(dx^2+c)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sech(d\*x^2+c)), x, algorithm="giac")

[Out] integrate(x^4/(b\*sech(d\*x^2 + c) + a), x)

**maple** [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a + b \operatorname{sech}(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a+b\*sech(d\*x^2+c)), x)

[Out] int(x^4/(a+b\*sech(d\*x^2+c)), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^5}{5a} - 2b \int \frac{x^4 e^{(dx^2+c)}}{a^2 e^{(2dx^2+2c)} + 2abe^{(dx^2+c)} + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sech(d\*x^2+c)), x, algorithm="maxima")

[Out] 1/5\*x^5/a - 2\*b\*integrate(x^4\*e^(d\*x^2 + c)/(a^2\*e^(2\*d\*x^2 + 2\*c) + 2\*a\*b\*e^(d\*x^2 + c) + a^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^4}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a + b/cosh(c + d\*x^2)), x)

[Out] int(x^4/(a + b/cosh(c + d\*x^2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(a+b\*sech(d\*x\*\*2+c)), x)

[Out] Integral(x\*\*4/(a + b\*sech(c + d\*x\*\*2)), x)

$$3.18 \quad \int \frac{x^3}{a+b\operatorname{sech}(c+dx^2)} dx$$

**Optimal.** Leaf size=241

$$-\frac{b\operatorname{Li}_2\left(-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} + \frac{b\operatorname{Li}_2\left(-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} - \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}} + 1\right)}{2ad\sqrt{b^2-a^2}} + \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b} + 1\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^4}{4a}$$

[Out]  $1/4*x^4/a - 1/2*b*x^2*\ln(1+a*\exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2) + 1/2*b*x^2*\ln(1+a*\exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2) - 1/2*b*polylog(2,-a*\exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2) + 1/2*b*polylog(2,-a*\exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)$

**Rubi [A]** time = 0.51, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2279, 2391}

$$-\frac{b\operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}} + \frac{b\operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}\right)}{2ad^2\sqrt{b^2-a^2}} - \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}} + 1\right)}{2ad\sqrt{b^2-a^2}} + \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b} + 1\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^4}{4a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(a + b*\operatorname{Sech}[c + d*x^2]), x]$

[Out]  $x^4/(4*a) - (b*x^2*\operatorname{Log}[1 + (a*E^{(c + d*x^2)})/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d) + (b*x^2*\operatorname{Log}[1 + (a*E^{(c + d*x^2)})/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d) - (b*\operatorname{PolyLog}[2, -((a*E^{(c + d*x^2)})/(b - \operatorname{Sqrt}[-a^2 + b^2]))])/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d^2) + (b*\operatorname{PolyLog}[2, -((a*E^{(c + d*x^2)})/(b + \operatorname{Sqrt}[-a^2 + b^2]))])/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d^2)$

**Rule 2190**

$\operatorname{Int}[(((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)*((c_)+(d_)*(x_))})^{(m_)}]/((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x))))^n)/a]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x))))^n)/a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

**Rule 2264**

$\operatorname{Int}[((F_)^{(u_)*((f_)+(g_)*(x_))})^{(m_)}]/((a_)+(b_)*(F_)^{(u_)}+(c_)*(F_)^{(v_)}), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2 - 4*a*c, 2]\}, \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f+g*x)^m*F^u/(b-q+2*c*F^u), x], x] - \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f+g*x)^m*F^u/(b+q+2*c*F^u), x], x] /; \operatorname{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \operatorname{EqQ}[v, 2*u] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{IGtQ}[m, 0]$

**Rule 2279**

$\operatorname{Int}[\operatorname{Log}[(a_)+(b_)*((F_)^{((e_)*((c_)+(d_)*(x_)))})^{(n_)}], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a+b*x]/x, x], x, (F^{(e*(c+d*x))})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \operatorname{GtQ}[a, 0]$

**Rule 2391**

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_)+(e_)*(x_))^{(n_)}]/(x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x] \&\& \operatorname{EqQ}[c*d, 1]$

Rule 3320

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-(I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/2)) - (b\*E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

Rule 5436

Int[(x\_.)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_.)^(n\_.)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx &= \frac{1}{2} \operatorname{Subst} \left( \int \frac{x}{a + b \operatorname{sech}(c + dx)} dx, x, x^2 \right) \\
 &= \frac{1}{2} \operatorname{Subst} \left( \int \left( \frac{x}{a} - \frac{bx}{a(b + a \cosh(c + dx))} \right) dx, x, x^2 \right) \\
 &= \frac{x^4}{4a} - \frac{b \operatorname{Subst} \left( \int \frac{x}{b + a \cosh(c + dx)} dx, x, x^2 \right)}{2a} \\
 &= \frac{x^4}{4a} - \frac{b \operatorname{Subst} \left( \int \frac{e^{c+dx} x}{a + 2be^{c+dx} + ae^{2(c+dx)}} dx, x, x^2 \right)}{4a} \\
 &= \frac{x^4}{4a} - \frac{b \operatorname{Subst} \left( \int \frac{e^{c+dx} x}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, x^2 \right)}{\sqrt{-a^2 + b^2}} + \frac{b \operatorname{Subst} \left( \int \frac{e^{c+dx} x}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, x^2 \right)}{\sqrt{-a^2 + b^2}} \\
 &= \frac{x^4}{4a} - \frac{bx^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{b \operatorname{Subst} \left( \int \log \left( 1 + \frac{ae^{c+dx}}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} \right) dx, x, x^2 \right)}{2a\sqrt{-a^2 + b^2}} \\
 &= \frac{x^4}{4a} - \frac{bx^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{b \operatorname{Subst} \left( \int \frac{\log \left( 1 + \frac{ae^{c+dx}}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} \right)}{x} dx, x, x^2 \right)}{2a\sqrt{-a^2 + b^2}} \\
 &= \frac{x^4}{4a} - \frac{bx^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} + \frac{bx^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d} - \frac{b \operatorname{Li}_2 \left( -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d^2} + \frac{b \operatorname{Li}_2 \left( -\frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}} \right)}{2a\sqrt{-a^2 + b^2} d^2}
 \end{aligned}$$

**Mathematica [C]** time = 1.76, size = 843, normalized size = 3.50

$$(b + a \cosh(dx^2 + c)) \left( x^4 + \frac{2b \left( 2(dx^2 + c) \tan^{-1} \left( \frac{(a+b) \coth \left( \frac{1}{2}(dx^2 + c) \right)}{\sqrt{a^2 - b^2}} \right) \right) + 2 \left( c - i \cos^{-1} \left( -\frac{b}{a} \right) \right) \tan^{-1} \left( \frac{(a-b) \tanh \left( \frac{1}{2}(dx^2 + c) \right)}{\sqrt{a^2 - b^2}} \right) + \left( \cos^{-1} \left( -\frac{b}{a} \right) + 2 \left( \tan^{-1} \left( \frac{(a-b) \tanh \left( \frac{1}{2}(dx^2 + c) \right)}{\sqrt{a^2 - b^2}} \right) \right) \right)}{(b + a \cosh(dx^2 + c))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Sech[c + d\*x^2]),x]

[Out] ((b + a\*Cosh[c + d\*x^2])\*(x^4 + (2\*b\*(2\*(c + d\*x^2)\*ArcTan[((a + b)\*Coth[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]] + 2\*(c - I\*ArcCos[-(b/a)])\*ArcTan[((a - b)\*Tanh[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] + 2\*(ArcTan[((a + b)\*Coth[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)\*Tanh[(c + d\*x^2)/2])/Sqrt[a^2 - b^2])))\*Log[(Sqrt[a^2 - b^2]\*E^(-1/2\*c - (d\*x^2)/2))/(Sqrt[2]\*Sqrt[a]\*Sqrt[b + a\*Cosh[c + d\*x^2]])] + (ArcCos[-(b/a)] - 2\*(ArcTan[((a + b)\*Coth[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)\*Tanh[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]))\*Log[(Sqrt[a^2 - b^2]\*E^((c + d\*x^2)/2))/(Sqrt[2]\*Sqrt[a]\*Sqrt[b + a\*Cosh[c + d\*x^2]])] - (ArcCos[-(b/a)] + 2\*ArcTan[((a - b)\*Tanh[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]]\*Log[((a + b)\*(-a + b + I\*Sqrt[a^2 - b^2])\*(-1 + Tanh[(c + d\*x^2)/2]))/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^2)/2]))] - (ArcCos[-(b/a)] - 2\*ArcTan[((a - b)\*Tanh[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]]\*Log[((a + b)\*(a - b + I\*Sqrt[a^2 - b^2])\*(1 + Tanh[(c + d\*x^2)/2]))/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^2)/2]))] + I\*(PolyLog[2, ((b - I\*Sqrt[a^2 - b^2])\*(a + b - I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^2)/2]))/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^2)/2]))] - PolyLog[2, ((b + I\*Sqrt[a^2 - b^2])\*(a + b - I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^2)/2]))/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^2)/2]))]))/(Sqrt[a^2 - b^2]\*d^2)\*Sech[c + d\*x^2])/(4\*a\*(a + b\*Sech[c + d\*x^2]))

**fricas [B]** time = 0.43, size = 536, normalized size = 2.22

$$(a^2 - b^2)d^2x^4 + 2abc\sqrt{-\frac{a^2-b^2}{a^2}} \log\left(2a \cosh(dx^2 + c) + 2a \sinh(dx^2 + c) + 2a\sqrt{-\frac{a^2-b^2}{a^2}} + 2b\right) - 2abc\sqrt{-\frac{a^2-b^2}{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(d\*x^2+c)),x, algorithm="fricas")

[Out] 1/4\*((a^2 - b^2)\*d^2\*x^4 + 2\*a\*b\*c\*sqrt(-(a^2 - b^2)/a^2)\*log(2\*a\*cosh(d\*x^2 + c) + 2\*a\*sinh(d\*x^2 + c) + 2\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*b) - 2\*a\*b\*c\*sqrt(-(a^2 - b^2)/a^2)\*log(2\*a\*cosh(d\*x^2 + c) + 2\*a\*sinh(d\*x^2 + c) - 2\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*b) + 2\*a\*b\*sqrt(-(a^2 - b^2)/a^2)\*dilog(-(b\*cosh(d\*x^2 + c) + b\*sinh(d\*x^2 + c) + (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 2\*a\*b\*sqrt(-(a^2 - b^2)/a^2)\*dilog(-(b\*cosh(d\*x^2 + c) + b\*sinh(d\*x^2 + c) - (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 2\*(a\*b\*d\*x^2 + a\*b\*c)\*sqrt(-(a^2 - b^2)/a^2)\*log((b\*cosh(d\*x^2 + c) + b\*sinh(d\*x^2 + c) + (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 2\*(a\*b\*d\*x^2 + a\*b\*c)\*sqrt(-(a^2 - b^2)/a^2)\*log((b\*cosh(d\*x^2 + c) + b\*sinh(d\*x^2 + c) - (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^3 - a\*b^2)\*d^2)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \operatorname{sech}(dx^2 + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(d\*x^2+c)),x, algorithm="giac")

[Out] integrate(x^3/(b\*sech(d\*x^2 + c) + a), x)

**maple** [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \operatorname{sech}(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*sech(d\*x^2+c)),x)

[Out] int(x^3/(a+b\*sech(d\*x^2+c)),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(d\*x^2+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b/cosh(c + d\*x^2)),x)

[Out] int(x^3/(a + b/cosh(c + d\*x^2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*sech(d\*x\*\*2+c)),x)

[Out] Integral(x\*\*3/(a + b\*sech(c + d\*x\*\*2)), x)

$$3.19 \quad \int \frac{x^2}{a+b\operatorname{sech}(c+dx^2)} dx$$

**Optimal.** Leaf size=21

$$\operatorname{Int}\left(\frac{x^2}{a+b\operatorname{sech}(c+dx^2)}, x\right)$$

[Out] Unintegrable(x^2/(a+b\*sech(d\*x^2+c)), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2}{a+b\operatorname{sech}(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[x^2/(a + b\*Sech[c + d\*x^2]), x]

[Out] Defer[Int][x^2/(a + b\*Sech[c + d\*x^2]), x]

Rubi steps

$$\int \frac{x^2}{a+b\operatorname{sech}(c+dx^2)} dx = \int \frac{x^2}{a+b\operatorname{sech}(c+dx^2)} dx$$

**Mathematica [A]** time = 7.81, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\operatorname{sech}(c+dx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/(a + b\*Sech[c + d\*x^2]), x]

[Out] Integrate[x^2/(a + b\*Sech[c + d\*x^2]), x]

**fricas [A]** time = 0.38, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^2}{b\operatorname{sech}(dx^2+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(d\*x^2+c)), x, algorithm="fricas")

[Out] integral(x^2/(b\*sech(d\*x^2 + c) + a), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b\operatorname{sech}(dx^2+c)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(d\*x^2+c)), x, algorithm="giac")



[Out] integrate(x^2/(b\*sech(d\*x^2 + c) + a), x)

**maple** [A] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \operatorname{sech}(dx^2 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*sech(d\*x^2+c)), x)

[Out] int(x^2/(a+b\*sech(d\*x^2+c)), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3}{3a} - 2b \int \frac{x^2 e^{(dx^2+c)}}{a^2 e^{(2dx^2+2c)} + 2abe^{(dx^2+c)} + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(d\*x^2+c)), x, algorithm="maxima")

[Out] 1/3\*x^3/a - 2\*b\*integrate(x^2\*e^(d\*x^2 + c)/(a^2\*e^(2\*d\*x^2 + 2\*c) + 2\*a\*b\*e^(d\*x^2 + c) + a^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b/cosh(c + d\*x^2)), x)

[Out] int(x^2/(a + b/cosh(c + d\*x^2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*sech(d\*x\*\*2+c)), x)

[Out] Integral(x\*\*2/(a + b\*sech(c + d\*x\*\*2)), x)

$$3.20 \quad \int \frac{x}{a+b\operatorname{sech}(c+dx^2)} dx$$

**Optimal.** Leaf size=66

$$\frac{x^2}{2a} - \frac{b \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

[Out]  $1/2*x^2/a-b*\arctan((a-b)^{(1/2)}*\tanh(1/2*d*x^2+1/2*c)/(a+b)^{(1/2)})/a/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {5436, 3783, 2659, 208}

$$\frac{x^2}{2a} - \frac{b \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sech[c + d\*x^2]),x]

[Out]  $x^2/(2*a) - (b*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tanh}[(c + d*x^2)/2])/(\text{Sqrt}[a + b])]/(a*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d)$

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3783

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.) + (a\_)^(-1), x\_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a\*Sin[c + d\*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx &= \frac{1}{2} \operatorname{Subst} \left( \int \frac{1}{a + b \operatorname{sech}(c + dx)} dx, x, x^2 \right) \\
&= \frac{x^2}{2a} - \frac{\operatorname{Subst} \left( \int \frac{1}{1 + \frac{a \cosh(c+dx)}{b}} dx, x, x^2 \right)}{2a} \\
&= \frac{x^2}{2a} + \frac{i \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, i \tanh \left( \frac{1}{2} (c + dx^2) \right) \right)}{ad} \\
&= \frac{x^2}{2a} - \frac{b \tan^{-1} \left( \frac{\sqrt{a-b} \tanh \left( \frac{1}{2} (c + dx^2) \right)}{\sqrt{a+b}} \right)}{a \sqrt{a-b} \sqrt{a+b} d}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 67, normalized size = 1.02

$$\frac{2b \tan^{-1} \left( \frac{(b-a) \tanh \left( \frac{1}{2} (c + dx^2) \right)}{\sqrt{a^2 - b^2}} \right)}{d \sqrt{a^2 - b^2}} + \frac{c}{d} + x^2$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Sech[c + d\*x^2]), x]

[Out] (c/d + x^2 + (2\*b\*ArcTan[((-a + b)\*Tanh[(c + d\*x^2)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]\*d)/(2\*a)

**fricas [A]** time = 0.45, size = 304, normalized size = 4.61

$$\left[ \frac{(a^2 - b^2) dx^2 - \sqrt{-a^2 + b^2} b \log \left( \frac{a^2 \cosh(dx^2+c)^2 + a^2 \sinh(dx^2+c)^2 + 2ab \cosh(dx^2+c) - a^2 + 2b^2 + 2(a^2 \cosh(dx^2+c) + ab) \sinh(dx^2+c)}{a \cosh(dx^2+c)^2 + a \sinh(dx^2+c)^2 + 2b \cosh(dx^2+c) + 2(a \cosh(dx^2+c) + b \sinh(dx^2+c))} \right)}{2(a^3 - ab^2)d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(d\*x^2+c)), x, algorithm="fricas")

[Out] [1/2\*((a^2 - b^2)\*d\*x^2 - sqrt(-a^2 + b^2)\*b\*log((a^2\*cosh(d\*x^2 + c)^2 + a^2\*sinh(d\*x^2 + c)^2 + 2\*a\*b\*cosh(d\*x^2 + c) - a^2 + 2\*b^2 + 2\*(a^2\*cosh(d\*x^2 + c) + a\*b)\*sinh(d\*x^2 + c) + 2\*sqrt(-a^2 + b^2)\*(a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c) + b)))/(a\*cosh(d\*x^2 + c)^2 + a\*sinh(d\*x^2 + c)^2 + 2\*b\*cosh(d\*x^2 + c) + 2\*(a\*cosh(d\*x^2 + c) + b)\*sinh(d\*x^2 + c) + a)))/((a^3 - a\*b^2)\*d), 1/2\*((a^2 - b^2)\*d\*x^2 + 2\*sqrt(a^2 - b^2)\*b\*arctan(-(a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c) + b)/sqrt(a^2 - b^2)))/((a^3 - a\*b^2)\*d)]

**giac [A]** time = 0.17, size = 61, normalized size = 0.92

$$-\frac{b \arctan \left( \frac{ae^{(dx^2+c)} + b}{\sqrt{a^2 - b^2}} \right)}{\sqrt{a^2 - b^2} ad} + \frac{dx^2 + c}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(d\*x^2+c)), x, algorithm="giac")

[Out]  $-b \arctan\left(\frac{a e^{d x^2 + c} + b}{\sqrt{a^2 - b^2}}\right) / (\sqrt{a^2 - b^2} a d) + 1 / 2 (d x^2 + c) / (a d)$

**maple** [A] time = 0.25, size = 95, normalized size = 1.44

$$-\frac{\ln\left(\tanh\left(\frac{d x^2}{2} + \frac{c}{2}\right) - 1\right)}{2 d a} - \frac{b \arctan\left(\frac{(a-b) \tanh\left(\frac{d x^2}{2} + \frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d a \sqrt{(a+b)(a-b)}} + \frac{\ln\left(\tanh\left(\frac{d x^2}{2} + \frac{c}{2}\right) + 1\right)}{2 d a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*sech(d*x^2+c)),x)`

[Out]  $-1/2/d/a \ln(\tanh(1/2*d*x^2+1/2*c)-1) - 1/d/a*b/((a+b)*(a-b))^{(1/2)} \arctan((a-b)*\tanh(1/2*d*x^2+1/2*c)/((a+b)*(a-b))^{(1/2)}) + 1/2/d/a \ln(\tanh(1/2*d*x^2+1/2*c)+1)$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*b^2-4\*a^2>0)', see `assume?` for more details) Is 4\*b^2-4\*a^2 positive or negative?

**mupad** [B] time = 0.18, size = 171, normalized size = 2.59

$$\frac{x^2}{2 a} \frac{\operatorname{atan}\left(\frac{a d \sqrt{b^2}}{\sqrt{a^4 d^2 - a^2 b^2 d^2}} + \frac{b e^{d x^2} e^c \sqrt{a^4 d^2 - a^2 b^2 d^2}}{a^2 d \sqrt{b^2}} + \frac{a^2 b d e^{d x^2} e^c \sqrt{b^2} \sqrt{a^4 d^2 - a^2 b^2 d^2}}{a^6 d^2 - a^4 b^2 d^2}\right) \sqrt{b^2}}{\sqrt{a^4 d^2 - a^2 b^2 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b/cosh(c + d*x^2)),x)`

[Out]  $x^2/(2*a) - (\operatorname{atan}((a*d*(b^2)^{(1/2)})/(a^4*d^2 - a^2*b^2*d^2)^{(1/2)} + (b*\exp(d*x^2)*\exp(c)*(a^4*d^2 - a^2*b^2*d^2)^{(1/2)})/(a^2*d*(b^2)^{(1/2)} + (a^2*b*d*\exp(d*x^2)*\exp(c)*(b^2)^{(1/2)*(a^4*d^2 - a^2*b^2*d^2)^{(1/2)})/(a^6*d^2 - a^4*b^2*d^2)))*(b^2)^{(1/2)})/(a^4*d^2 - a^2*b^2*d^2)^{(1/2)}$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{sech}(c + d x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*sech(d*x**2+c)),x)`

[Out] `Integral(x/(a + b*sech(c + d*x**2)), x)`

$$3.21 \quad \int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+dx^2))}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*sech(d\*x^2+c)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sech[c + d\*x^2])), x]

[Out] Defer[Int][1/(x\*(a + b\*Sech[c + d\*x^2])), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$$

Mathematica [A] time = 5.18, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sech[c + d\*x^2])), x]

[Out] Integrate[1/(x\*(a + b\*Sech[c + d\*x^2])), x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bx \operatorname{sech}(dx^2 + c) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(d\*x^2+c)), x, algorithm="fricas")

[Out] integral(1/(b\*x\*sech(d\*x^2 + c) + a\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(d\*x^2+c)), x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*x^2 + c) + a)\*x), x)

**maple** [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{sech}(dx^2 + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a+b*sech(d*x^2+c)),x)`

[Out] `int(1/x/(a+b*sech(d*x^2+c)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-2b \int \frac{e^{(dx^2+c)}}{a^2 x e^{(2dx^2+2c)} + 2abx e^{(dx^2+c)} + a^2 x} dx + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

[Out] `-2*b*integrate(e^(d*x^2 + c)/(a^2*x*e^(2*d*x^2 + 2*c) + 2*a*b*x*e^(d*x^2 + c) + a^2*x), x) + log(x)/a`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \left( a + \frac{b}{\cosh(dx^2+c)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b/cosh(c + d*x^2))),x)`

[Out] `int(1/(x*(a + b/cosh(c + d*x^2))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*sech(d*x**2+c)),x)`

[Out] `Integral(1/(x*(a + b*sech(c + d*x**2))), x)`

$$3.22 \quad \int \frac{a+b\operatorname{sech}(c+dx^2)}{x^2} dx$$

Optimal. Leaf size=24

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+dx^2)}{x^2}, x\right) - \frac{a}{x}$$

[Out]  $-a/x + b*\operatorname{Unintegrable}(\operatorname{sech}(d*x^2+c)/x^2, x)$

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*x^2])/x^2, x]$

[Out]  $-(a/x) + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sech}[c + d*x^2]/x^2, x]]$

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b\operatorname{sech}(c + dx^2)}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\operatorname{sech}(c + dx^2)}{x^2} dx \end{aligned}$$

Mathematica [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*x^2])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*x^2])/x^2, x]$

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{sech}(dx^2 + c) + a}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\operatorname{sech}(d*x^2+c))/x^2, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b*\operatorname{sech}(d*x^2 + c) + a)/x^2, x)$

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^2 + c) + a)/x^2, x)

**maple** [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(dx^2 + c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(d\*x^2+c))/x^2,x)

[Out] int((a+b\*sech(d\*x^2+c))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{1}{x^2(e^{(dx^2+c)} + e^{(-dx^2-c)})} dx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x^2+c))/x^2,x, algorithm="maxima")

[Out] 2\*b\*integrate(1/(x^2\*(e^(d\*x^2 + c) + e^(-d\*x^2 - c))), x) - a/x

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cosh(dx^2+c)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^2))/x^2,x)

[Out] int((a + b/cosh(c + d\*x^2))/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(d\*x\*\*2+c))/x\*\*2,x)

[Out] Integral((a + b\*sech(c + d\*x\*\*2))/x\*\*2, x)



$$3.23 \quad \int \frac{x^5}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Optimal.** Leaf size=994

$$\frac{x^6}{6a^2} - \frac{b \log\left(\frac{e^{dx^2+c}}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} + \frac{b^3 \log\left(\frac{e^{dx^2+c}}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} + \frac{b \log\left(\frac{e^{dx^2+c}}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} - \frac{b^3 \log\left(\frac{e^{dx^2+c}}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} + \frac{x^6}{6a^2}$$

[Out]  $\frac{1}{2} b^2 x^4 / a^2 (a^2 - b^2) / d + \frac{1}{6} x^6 / a^2 - b^2 x^2 \ln(1 + a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 (a^2 - b^2) / d^2 + \frac{1}{2} b^3 x^4 \ln(1 + a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 (-a^2 + b^2)^{3/2} / d - b^2 x^2 \ln(1 + a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 (a^2 - b^2) / d^2 - \frac{1}{2} b^3 x^4 \ln(1 + a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 (-a^2 + b^2)^{3/2} / d - b^2 \operatorname{polylog}(2, -a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 (a^2 - b^2) / d^3 + b^3 x^2 \operatorname{polylog}(2, -a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 (-a^2 + b^2)^{3/2} / d^2 - b^2 \operatorname{polylog}(2, -a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 (a^2 - b^2) / d^3 - b^3 x^2 \operatorname{polylog}(2, -a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 (-a^2 + b^2)^{3/2} / d^2 - b^3 \operatorname{polylog}(3, -a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 (-a^2 + b^2)^{3/2} / d^3 + b^3 \operatorname{polylog}(3, -a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 (-a^2 + b^2)^{3/2} / d^3 + \frac{1}{2} b^2 x^4 \sinh(dx^2 + c) / a (a^2 - b^2) / d - (b + a \cosh(dx^2 + c)) - b x^4 \ln(1 + a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 / d - (-a^2 + b^2)^{1/2} + b x^4 \ln(1 + a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 / d - (-a^2 + b^2)^{1/2} - 2 b x^2 \operatorname{polylog}(2, -a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 / d^2 - (-a^2 + b^2)^{1/2} + 2 b x^2 \operatorname{polylog}(2, -a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 / d^2 - (-a^2 + b^2)^{1/2} + 2 b \operatorname{polylog}(3, -a \exp(dx^2 + c)) / (b - (-a^2 + b^2)^{1/2}) / a^2 / d^3 - (-a^2 + b^2)^{1/2} - 2 b \operatorname{polylog}(3, -a \exp(dx^2 + c)) / (b + (-a^2 + b^2)^{1/2}) / a^2 / d^3 - (-a^2 + b^2)^{1/2}$

**Rubi [A]** time = 2.11, antiderivative size = 994, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{x^6}{6a^2} - \frac{b \log\left(\frac{e^{dx^2+c}}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} + \frac{b^3 \log\left(\frac{e^{dx^2+c}}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} + \frac{b \log\left(\frac{e^{dx^2+c}}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2 \sqrt{b^2-a^2} d} - \frac{b^3 \log\left(\frac{e^{dx^2+c}}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{2a^2 (b^2-a^2)^{3/2} d} + \frac{x^6}{6a^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b\*Sech[c + d\*x^2])^2, x]

[Out]  $\frac{b^2 x^4}{2 a^2 (a^2 - b^2) d} + \frac{x^6}{6 a^2} - \frac{b^2 x^2 \operatorname{Log}[1 + (a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 (a^2 - b^2) d^2} + \frac{b^3 x^4 \operatorname{Log}[1 + (a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{(2 a^2 (-a^2 + b^2)^{3/2} d)} - \frac{b x^4 \operatorname{Log}[1 + (a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 \operatorname{Sqrt}[-a^2 + b^2] d} - \frac{b^2 x^2 \operatorname{Log}[1 + (a E^{(c + dx^2)}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 (a^2 - b^2) d^2} - \frac{b^3 x^4 \operatorname{Log}[1 + (a E^{(c + dx^2)}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]}{(2 a^2 (-a^2 + b^2)^{3/2} d)} + \frac{b x^4 \operatorname{Log}[1 + (a E^{(c + dx^2)}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 \operatorname{Sqrt}[-a^2 + b^2] d} - \frac{b^2 \operatorname{PolyLog}[2, -(a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{(b - \operatorname{Sqrt}[-a^2 + b^2])} / (a^2 (a^2 - b^2) d^3) + \frac{b^3 x^2 \operatorname{PolyLog}[2, -(a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{(a^2 (-a^2 + b^2)^{3/2} d^2)} - \frac{(2 b x^2 \operatorname{PolyLog}[2, -(a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 \operatorname{Sqrt}[-a^2 + b^2] d^2} - \frac{b^2 \operatorname{PolyLog}[2, -(a E^{(c + dx^2)}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]}{(a^2 (a^2 - b^2) d^3)} - \frac{b^3 x^2 \operatorname{PolyLog}[2, -(a E^{(c + dx^2)}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]}{(a^2 (-a^2 + b^2)^{3/2} d^2)} + \frac{(2 b x^2 \operatorname{PolyLog}[2, -(a E^{(c + dx^2)}) / (b + \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 \operatorname{Sqrt}[-a^2 + b^2] d^2} - \frac{b^3 \operatorname{PolyLog}[3, -(a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{(a^2 (-a^2 + b^2)^{3/2} d^3)} + \frac{(2 b \operatorname{PolyLog}[3, -(a E^{(c + dx^2)}) / (b - \operatorname{Sqrt}[-a^2 + b^2])]}{a^2 (-a^2 + b^2)^{3/2} d^3}$

```

))/((a^2*Sqrt[-a^2 + b^2]*d^3) + (b^3*PolyLog[3, -((a*E^(c + d*x^2))/(b + S
qrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^3) - (2*b*PolyLog[3, -((a*E^(
c + d*x^2))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^3) + (b^2*x^4
*Sinh[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Cosh[c + d*x^2]))

```

#### Rule 2190

```

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

#### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

#### Rule 2279

```

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

#### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

```

#### Rule 2391

```

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

#### Rule 2531

```

Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

#### Rule 3320

```

Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + Pi*(k_) + (Comple
x[0, fz_])*(f_)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

```

#### Rule 3324

```

Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +

```

```
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

#### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

#### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

#### Rule 5562

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]/(Cosh[(c_.) + (d_
.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx &= \frac{1}{2} \operatorname{Subst} \left( \int \frac{x^2}{(a + b \operatorname{sech}(c + dx))^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \operatorname{Subst} \left( \int \left( \frac{x^2}{a^2} + \frac{b^2 x^2}{a^2 (b + a \cosh(c + dx))^2} - \frac{2bx^2}{a^2 (b + a \cosh(c + dx))} \right) dx, x, x^2 \right) \\
&= \frac{x^6}{6a^2} - \frac{b \operatorname{Subst} \left( \int \frac{x^2}{b + a \cosh(c + dx)} dx, x, x^2 \right)}{a^2} + \frac{b^2 \operatorname{Subst} \left( \int \frac{x^2}{(b + a \cosh(c + dx))^2} dx, x, x^2 \right)}{2a^2} \\
&= \frac{x^6}{6a^2} + \frac{b^2 x^4 \sinh(c + dx^2)}{2a(a^2 - b^2)d(b + a \cosh(c + dx^2))} - \frac{(2b) \operatorname{Subst} \left( \int \frac{e^{c+dx^2}}{a + 2be^{c+dx} + ae^{2(c+dx)}} dx, x, x^2 \right)}{a^2} \\
&= \frac{b^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} + \frac{b^2 x^4 \sinh(c + dx^2)}{2a(a^2 - b^2)d(b + a \cosh(c + dx^2))} - \frac{b^3 \operatorname{Subst} \left( \int \frac{1}{a + 2be^{c+dx} + ae^{2(c+dx)}} dx, x, x^2 \right)}{a^2} \\
&= \frac{b^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} - \frac{b^2 x^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} - \frac{bx^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} - \frac{b^3 x^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{b^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} - \frac{b^2 x^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^3 x^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{b^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} - \frac{b^2 x^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^3 x^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{b^2 x^4}{2a^2(a^2 - b^2)d} + \frac{x^6}{6a^2} - \frac{b^2 x^2 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{b^3 x^4 \log \left( 1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}} \right)}{2a^2(-a^2 + b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 12.89, size = 1565, normalized size = 1.57

result too large to display

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b\*Sech[c + d\*x^2])^2,x]

[Out] ((b + a\*Cosh[c + d\*x^2])\*Sech[c + d\*x^2]^2\*(x^6\*(b + a\*Cosh[c + d\*x^2]) - (3\*b\*E^(2\*c)\*(b + a\*Cosh[c + d\*x^2])\*(2\*b\*d^2\*E^(2\*c)\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^4 - 2\*b\*d\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^2\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 2\*b\*d\*E^(2\*c)\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^2\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 2\*a^2\*d^2\*E^c\*x^4\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + b^2\*d^2\*E^c\*x^4\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 2\*a^2\*d^2\*E^(3\*c)\*x^4\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + b^2\*d^2\*E^(3\*c)\*x^4\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 2\*b\*d\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^2\*Log[1 + (a\*E^(2\*c + d\*x^2))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]])



c) + b\*sinh(d\*x^2 + c) - (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a/a + 1) + 3\*(2\*(a^3\*b^2 - a\*b^4)\*c\*cosh(d\*x^2 + c)^2 + 2\*(a^3\*b^2 - a\*b^4)\*c\*sinh(d\*x^2 + c)^2 + 4\*(a^2\*b^3 - b^5)\*c\*cosh(d\*x^2 + c) + 2\*(a^3\*b^2 - a\*b^4)\*c + 4\*((a^3\*b^2 - a\*b^4)\*c\*cosh(d\*x^2 + c) + (a^2\*b^3 - b^5)\*c)\*sinh(d\*x^2 + c) - ((2\*a^4\*b - a^2\*b^3)\*c^2\*cosh(d\*x^2 + c)^2 + (2\*a^4\*b - a^2\*b^3)\*c^2\*sinh(d\*x^2 + c)^2 + 2\*(2\*a^3\*b^2 - a\*b^4)\*c^2\*cosh(d\*x^2 + c) + (2\*a^4\*b - a^2\*b^3)\*c^2 + 2\*((2\*a^4\*b - a^2\*b^3)\*c^2\*cosh(d\*x^2 + c) + (2\*a^3\*b^2 - a\*b^4)\*c^2)\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2))\*log(2\*a\*cosh(d\*x^2 + c) + 2\*a\*sinh(d\*x^2 + c) + 2\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*b) + 3\*(2\*(a^3\*b^2 - a\*b^4)\*c\*cosh(d\*x^2 + c)^2 + 2\*(a^3\*b^2 - a\*b^4)\*c\*sinh(d\*x^2 + c)^2 + 4\*(a^2\*b^3 - b^5)\*c\*cosh(d\*x^2 + c) + 2\*(a^3\*b^2 - a\*b^4)\*c + 4\*((a^3\*b^2 - a\*b^4)\*c\*cosh(d\*x^2 + c) + (a^2\*b^3 - b^5)\*c)\*sinh(d\*x^2 + c) + ((2\*a^4\*b - a^2\*b^3)\*c^2\*cosh(d\*x^2 + c)^2 + (2\*a^4\*b - a^2\*b^3)\*c^2\*sinh(d\*x^2 + c)^2 + 2\*(2\*a^3\*b^2 - a\*b^4)\*c^2\*cosh(d\*x^2 + c) + (2\*a^4\*b - a^2\*b^3)\*c^2 + 2\*((2\*a^4\*b - a^2\*b^3)\*c^2\*cosh(d\*x^2 + c) + (2\*a^3\*b^2 - a\*b^4)\*c^2)\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2))\*log(2\*a\*cosh(d\*x^2 + c) + 2\*a\*sinh(d\*x^2 + c) - 2\*a\*sqrt(-(a^2 - b^2)/a^2) + 2\*b) - 3\*(2\*(a^3\*b^2 - a\*b^4)\*d\*x^2 + 2\*((a^3\*b^2 - a\*b^4)\*d\*x^2 + (a^3\*b^2 - a\*b^4)\*c)\*cosh(d\*x^2 + c)^2 + 2\*((a^3\*b^2 - a\*b^4)\*d\*x^2 + (a^3\*b^2 - a\*b^4)\*c)\*sinh(d\*x^2 + c)^2 + 2\*(a^3\*b^2 - a\*b^4)\*c + 4\*((a^2\*b^3 - b^5)\*d\*x^2 + (a^2\*b^3 - b^5)\*c)\*cosh(d\*x^2 + c) + 4\*((a^2\*b^3 - b^5)\*d\*x^2 + (a^2\*b^3 - b^5)\*c) + ((a^3\*b^2 - a\*b^4)\*d\*x^2 + (a^3\*b^2 - a\*b^4)\*c)\*cosh(d\*x^2 + c))\*sinh(d\*x^2 + c) - ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2 + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2)\*cosh(d\*x^2 + c)^2 + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2)\*sinh(d\*x^2 + c)^2 + 2\*((2\*a^3\*b^2 - a\*b^4)\*d^2\*x^4 - (2\*a^3\*b^2 - a\*b^4)\*c^2)\*cosh(d\*x^2 + c) + 2\*((2\*a^3\*b^2 - a\*b^4)\*d^2\*x^4 - (2\*a^3\*b^2 - a\*b^4)\*c^2) + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2)\*cosh(d\*x^2 + c))\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2))\*log((b\*cosh(d\*x^2 + c) + b\*sinh(d\*x^2 + c) + (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 3\*(2\*(a^3\*b^2 - a\*b^4)\*d\*x^2 + 2\*((a^3\*b^2 - a\*b^4)\*d\*x^2 + (a^3\*b^2 - a\*b^4)\*c)\*cosh(d\*x^2 + c)^2 + 2\*((a^3\*b^2 - a\*b^4)\*d\*x^2 + (a^3\*b^2 - a\*b^4)\*c)\*sinh(d\*x^2 + c)^2 + 2\*(a^3\*b^2 - a\*b^4)\*c + 4\*((a^2\*b^3 - b^5)\*d\*x^2 + (a^2\*b^3 - b^5)\*c)\*cosh(d\*x^2 + c) + 4\*((a^2\*b^3 - b^5)\*d\*x^2 + (a^2\*b^3 - b^5)\*c) + ((a^3\*b^2 - a\*b^4)\*d\*x^2 + (a^3\*b^2 - a\*b^4)\*c)\*cosh(d\*x^2 + c))\*sinh(d\*x^2 + c) + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2 + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2)\*cosh(d\*x^2 + c)^2 + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2)\*sinh(d\*x^2 + c)^2 + 2\*((2\*a^3\*b^2 - a\*b^4)\*d^2\*x^4 - (2\*a^3\*b^2 - a\*b^4)\*c^2)\*cosh(d\*x^2 + c) + 2\*((2\*a^3\*b^2 - a\*b^4)\*d^2\*x^4 - (2\*a^3\*b^2 - a\*b^4)\*c^2) + ((2\*a^4\*b - a^2\*b^3)\*d^2\*x^4 - (2\*a^4\*b - a^2\*b^3)\*c^2)\*cosh(d\*x^2 + c))\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2))\*log((b\*cosh(d\*x^2 + c) + b\*sinh(d\*x^2 + c) - (a\*cosh(d\*x^2 + c) + a\*sinh(d\*x^2 + c))\*sqrt(-(a^2 - b^2)/a^2) + a)/a) + 2\*((a^4\*b - 2\*a^2\*b^3 + b^5)\*d^3\*x^6 + 3\*(a^2\*b^3 - b^5)\*d^2\*x^4 - 6\*(a^2\*b^3 - b^5)\*c^2 + ((a^5 - 2\*a^3\*b^2 + a\*b^4)\*d^3\*x^6 + 6\*(a^3\*b^2 - a\*b^4)\*d^2\*x^4 - 6\*(a^3\*b^2 - a\*b^4)\*c^2)\*cosh(d\*x^2 + c))\*sinh(d\*x^2 + c))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^3\*cosh(d\*x^2 + c)^2 + (a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^3\*sinh(d\*x^2 + c)^2 + 2\*(a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^3\*cosh(d\*x^2 + c) + (a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^3 + 2\*((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^3\*cosh(d\*x^2 + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^3)\*sinh(d\*x^2 + c))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^5/(b\*sech(d\*x^2 + c) + a)^2, x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(x^5/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see 'assume?' for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b/cosh(c + d\*x^2))^2,x)

[Out] int(x^5/(a + b/cosh(c + d\*x^2))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*5/(a + b\*sech(c + d\*x\*\*2))\*\*2, x)

$$3.24 \quad \int \frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Optimal.** Leaf size=21

$$\operatorname{Int}\left(\frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2}, x\right)$$

[Out] Unintegrable(x^4/(a+b\*sech(d\*x^2+c))^2,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[x^4/(a + b\*Sech[c + d\*x^2])^2,x]

[Out] Defer[Int][x^4/(a + b\*Sech[c + d\*x^2])^2, x]

Rubi steps

$$\int \frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2} dx = \int \frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Mathematica [A]** time = 59.26, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4/(a + b\*Sech[c + d\*x^2])^2,x]

[Out] Integrate[x^4/(a + b\*Sech[c + d\*x^2])^2, x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^4}{b^2 \operatorname{sech}(dx^2+c)^2 + 2ab \operatorname{sech}(dx^2+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(x^4/(b^2\*sech(d\*x^2 + c)^2 + 2\*a\*b\*sech(d\*x^2 + c) + a^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b \operatorname{sech}(dx^2+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^4/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^4/(b\*sech(d\*x^2 + c) + a)^2, x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(x^4/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^3de^{2c} - ab^2de^{2c})x^5e^{2dx^2} - 5ab^2x^3 + (a^3d - ab^2d)x^5 - (5b^3x^3e^c - 2(a^2bde^c - b^3de^c)x^5)e^{dx^2}}{5(a^5d - a^3b^2d + (a^5de^{2c} - a^3b^2de^{2c})e^{2dx^2}) + 2(a^4bde^c - a^2b^3de^c)e^{dx^2}} \int \frac{1}{a^5d -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out] 1/5\*((a^3\*d\*e^(2\*c) - a\*b^2\*d\*e^(2\*c))\*x^5\*e^(2\*d\*x^2) - 5\*a\*b^2\*x^3 + (a^3\*d - a\*b^2\*d)\*x^5 - (5\*b^3\*x^3\*e^c - 2\*(a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x^5)\*e^(d\*x^2))/(a^5\*d - a^3\*b^2\*d + (a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*e^(2\*d\*x^2) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*e^(d\*x^2)) - integrate(-(3\*a\*b^2\*x^2 + (3\*b^3\*x^2\*e^c - 2\*(2\*a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x^4)\*e^(d\*x^2))/(a^5\*d - a^3\*b^2\*d + (a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*e^(2\*d\*x^2) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*e^(d\*x^2)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^4}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(a + b/cosh(c + d\*x^2))^2,x)

[Out] int(x^4/(a + b/cosh(c + d\*x^2))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*4/(a + b\*sech(c + d\*x\*\*2))\*\*2, x)

$$3.25 \quad \int \frac{x^3}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Optimal.** Leaf size=555

$$-\frac{b\operatorname{Li}_2\left(-\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} + \frac{b\operatorname{Li}_2\left(-\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} - \frac{b^2\log(a\cosh(c+dx^2)+b)}{2a^2d^2(a^2-b^2)} - \frac{bx^2\log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}+1\right)}{a^2d\sqrt{b^2-a^2}} + \frac{bx^2\log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}\right)}{a^2d\sqrt{b^2-a^2}}$$

[Out]  $1/4*x^4/a^2-1/2*b^2*\ln(b+a*\cosh(d*x^2+c))/a^2/(a^2-b^2)/d^2+1/2*b^3*x^2*\ln(1+a*\exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-1/2*b^3*x^2*\ln(1+a*\exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+1/2*b^3*\operatorname{polylog}(2,-a*\exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-1/2*b^3*\operatorname{polylog}(2,-a*\exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+1/2*b^2*x^2*\sinh(d*x^2+c)/a/(a^2-b^2)/d/(b+a*\cosh(d*x^2+c))-b*x^2*\ln(1+a*\exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+b*x^2*\ln(1+a*\exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-b*\operatorname{polylog}(2,-a*\exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+b*\operatorname{polylog}(2,-a*\exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)$

**Rubi [A]** time = 1.10, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$-\frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} + \frac{b^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}\right)}{2a^2d^2(b^2-a^2)^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}\right)}{a^2d^2\sqrt{b^2-a^2}} - \frac{b^3\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b}\right)}{2a^2d^2(b^2-a^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(a + b*\operatorname{Sech}[c + d*x^2])^2, x]$

[Out]  $x^4/(4*a^2) + (b^3*x^2*\operatorname{Log}[1 + (a*E^(c + d*x^2))/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d) - (b*x^2*\operatorname{Log}[1 + (a*E^(c + d*x^2))/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d) - (b^3*x^2*\operatorname{Log}[1 + (a*E^(c + d*x^2))/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d) + (b*x^2*\operatorname{Log}[1 + (a*E^(c + d*x^2))/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d) - (b^2*\operatorname{Log}[b + a*\operatorname{Cosh}[c + d*x^2]])/(2*a^2*(a^2 - b^2)*d^2) + (b^3*\operatorname{PolyLog}[2, -((a*E^(c + d*x^2))/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d^2) - (b*\operatorname{PolyLog}[2, -((a*E^(c + d*x^2))/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d^2) - (b^3*\operatorname{PolyLog}[2, -((a*E^(c + d*x^2))/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d^2) + (b*\operatorname{PolyLog}[2, -((a*E^(c + d*x^2))/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(a^2*\operatorname{Sqrt}[-a^2 + b^2]*d^2) + (b^2*x^2*\operatorname{Sinh}[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*\operatorname{Cosh}[c + d*x^2]))$

**Rule 31**

$\operatorname{Int}(((a_) + (b_)*(x_))^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

**Rule 2190**

$\operatorname{Int}(((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)*((c_) + (d_)*(x_))^\wedge(m_))/((a_) + (b_)*((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)), x\_Symbol] \rightarrow \operatorname{Simp}(((c + d*x)^\wedge m * \operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n]/a)]/(b*f*g*n*\operatorname{Log}[F]), x) - \operatorname{Dist}((d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}((c + d*x)^\wedge(m - 1)*\operatorname{Log}[1 + (b*(F^\wedge(g*(e + f*x)))^\wedge n]/a), x), x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_)^(m_.)))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3320

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3324

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_)^(n_.)*((c_.) + (d_.)*(x_)^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

Rubi steps



$$2] * \text{Log}[a + 2*b*E^{(c + d*x^2)} + a*E^{(2*(c + d*x^2))}] + \text{Sqrt}[a^2 - b^2]*(-2*a^2 + b^2)*\text{PolyLog}[2, (a*E^{(c + d*x^2)})/(-b + \text{Sqrt}[-a^2 + b^2])] + \text{Sqrt}[a^2 - b^2]*(2*a^2 - b^2)*\text{PolyLog}[2, -((a*E^{(c + d*x^2)})/(b + \text{Sqrt}[-a^2 + b^2]))]/(-((a^2 - b^2)^2)^{(3/2)} + (2*a*b^2*d*x^2*\text{Sinh}[c + d*x^2])/((a - b)*(a + b))))/(4*a^2*d^2*(a + b*\text{Sech}[c + d*x^2])^2)$$

**fricas** [B] time = 0.47, size = 2473, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{4} * ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*\cosh(d*x^2 + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*\sinh(d*x^2 + c)^2 + 2*(2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*\cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*\sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*\cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2}*\text{dilog}(-(b*\cosh(d*x^2 + c) + b*\sinh(d*x^2 + c) + (a*\cosh(d*x^2 + c) + a*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a + 1) - 2*(2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*\cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*\sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*\cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2}*\text{dilog}(-(b*\cosh(d*x^2 + c) + b*\sinh(d*x^2 + c) - (a*\cosh(d*x^2 + c) + a*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a + 1) + 2*((2*a^4*b - a^2*b^3)*d*x^2 + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*\cosh(d*x^2 + c)^2 + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*\sinh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c + 2*((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2}*\log((b*\cosh(d*x^2 + c) + b*\sinh(d*x^2 + c) + (a*\cosh(d*x^2 + c) + a*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a - 2*((2*a^4*b - a^2*b^3)*d*x^2 + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*\cosh(d*x^2 + c)^2 + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*\sinh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c + 2*((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2}*\log((b*\cosh(d*x^2 + c) + b*\sinh(d*x^2 + c) - (a*\cosh(d*x^2 + c) + a*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2} + a)/a) + 4*(a^3*b^2 - a*b^4)*c + 2*((a^4*b - 2*a^2*b^3 + b^5)*d^2*x^4 + 2*(a^2*b^3 - b^5)*d*x^2 + 4*(a^2*b^3 - b^5)*c)*\cosh(d*x^2 + c) - 2*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*\cosh(d*x^2 + c)^2 + (a^3*b^2 - a*b^4)*\sinh(d*x^2 + c)^2 + 2*(a^2*b^3 - b^5)*\cosh(d*x^2 + c) + 2*(a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c) - ((2*a^4*b - a^2*b^3)*c*\cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c*\sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*c*\cosh(d*x^2 + c) + (2*a^4*b - a^2*b^3)*c + 2*((2*a^4*b - a^2*b^3)*c*\cosh(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2}*\log(2*a*\cosh(d*x^2 + c) + 2*a*\sinh(d*x^2 + c) + 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) - 2*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*\cosh(d*x^2 + c)^2 + (a^3*b^2 - a*b^4)*\sinh(d*x^2 + c)^2 + 2*(a^2*b^3 - b^5)*\cosh(d*x^2 + c) + 2*(a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c) + ((2*a^4*b - a^2*b^3)*c*\cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c*\sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*c*\cosh(d*x^2 + c) + (2*a^4*b - a^2*b^3)*c + 2*((2*a^4*b - a^2*b^3)*c*\cosh(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*\sinh(d*x^2 + c))*\sqrt{-(a^2 - b^2)/a^2}*\log(2*a*\cosh(d*x^2 + c) + 2*a*\sinh(d*x^2 + c) - 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d^2*x^4 + 2*(a^2*b^3 - b^5)*d*x^2 + 4*(a^2*b^3 - b^5)*c + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*\cosh(d*x^2 + c))*\sinh(d*x^2 + c)$$

c))/((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^2\*cosh(d\*x^2 + c)^2 + (a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^2\*sinh(d\*x^2 + c)^2 + 2\*(a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^2\*cosh(d\*x^2 + c) + (a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^2 + 2\*((a^7 - 2\*a^5\*b^2 + a^3\*b^4)\*d^2\*cosh(d\*x^2 + c) + (a^6\*b - 2\*a^4\*b^3 + a^2\*b^5)\*d^2)\*sinh(d\*x^2 + c))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^3/(b\*sech(d\*x^2 + c) + a)^2, x)

**maple** [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \operatorname{sech}(d x^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(x^3/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b/cosh(c + d\*x^2))^2,x)

[Out] int(x^3/(a + b/cosh(c + d\*x^2))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*3/(a + b\*sech(c + d\*x\*\*2))\*\*2, x)

$$3.26 \quad \int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Optimal.** Leaf size=21

$$\operatorname{Int}\left(\frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2}, x\right)$$

[Out] Unintegrable(x^2/(a+b\*sech(d\*x^2+c))^2, x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[x^2/(a + b\*Sech[c + d\*x^2])^2, x]

[Out] Defer[Int][x^2/(a + b\*Sech[c + d\*x^2])^2, x]

Rubi steps

$$\int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx = \int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Mathematica [A]** time = 52.97, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2/(a + b\*Sech[c + d\*x^2])^2, x]

[Out] Integrate[x^2/(a + b\*Sech[c + d\*x^2])^2, x]

**fricas [A]** time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^2}{b^2 \operatorname{sech}(dx^2+c)^2 + 2ab \operatorname{sech}(dx^2+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(d\*x^2+c))^2, x, algorithm="fricas")

[Out] integral(x^2/(b^2\*sech(d\*x^2 + c)^2 + 2\*a\*b\*sech(d\*x^2 + c) + a^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b \operatorname{sech}(dx^2+c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(x^2/(b\*sech(d\*x^2 + c) + a)^2, x)

**maple** [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(x^2/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^3 d e^{2c} - a b^2 d e^{2c}) x^3 e^{2dx^2} - 3 a b^2 x + (a^3 d - a b^2 d) x^3 - (3 b^3 x e^c - 2 (a^2 b d e^c - b^3 d e^c) x^3) e^{dx^2}}{3 (a^5 d - a^3 b^2 d + (a^5 d e^{2c} - a^3 b^2 d e^{2c}) e^{2dx^2} + 2 (a^4 b d e^c - a^2 b^3 d e^c) e^{dx^2})} \int \frac{1}{a^5 d - a^3 b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out] 1/3\*((a^3\*d\*e^(2\*c) - a\*b^2\*d\*e^(2\*c))\*x^3\*e^(2\*d\*x^2) - 3\*a\*b^2\*x + (a^3\*d - a\*b^2\*d)\*x^3 - (3\*b^3\*x\*e^c - 2\*(a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x^3)\*e^(d\*x^2))/(a^5\*d - a^3\*b^2\*d + (a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*e^(2\*d\*x^2) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*e^(d\*x^2)) - integrate(-(a\*b^2 + (b^3\*e^c - 2\*(2\*a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x^2)\*e^(d\*x^2))/(a^5\*d - a^3\*b^2\*d + (a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*e^(2\*d\*x^2) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*e^(d\*x^2)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^2}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b/cosh(c + d\*x^2))^2,x)

[Out] int(x^2/(a + b/cosh(c + d\*x^2))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(x\*\*2/(a + b\*sech(c + d\*x\*\*2))\*\*2, x)



$$3.27 \quad \int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Optimal.** Leaf size=123

$$-\frac{b(2a^2 - b^2) \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2 \tanh(c+dx^2)}{2ad(a^2 - b^2)(a+b\operatorname{sech}(c+dx^2))} + \frac{x^2}{2a^2}$$

[Out] 1/2\*x^2/a^2-b\*(2\*a^2-b^2)\*arctan((a-b)^(1/2)\*tanh(1/2\*d\*x^2+1/2\*c)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2\*b^2\*tanh(d\*x^2+c)/a/(a^2-b^2)/d/(a+b\*sech(d\*x^2+c))

**Rubi [A]** time = 0.25, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5436, 3785, 3919, 3831, 2659, 208}

$$-\frac{b(2a^2 - b^2) \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2 \tanh(c+dx^2)}{2ad(a^2 - b^2)(a+b\operatorname{sech}(c+dx^2))} + \frac{x^2}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sech[c + d\*x^2])^2,x]

[Out] x^2/(2\*a^2) - (b\*(2\*a^2 - b^2)\*ArcTan[(Sqrt[a - b]\*Tanh[(c + d\*x^2)/2])/Sqrt[a + b]])/(a^2\*(a - b)^(3/2)\*(a + b)^(3/2)\*d) + (b^2\*Tanh[c + d\*x^2])/(2\*a\*(a^2 - b^2)\*d\*(a + b\*Sech[c + d\*x^2]))

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 2659**

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

**Rule 3785**

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.) + (a\_)^(n\_), x\_Symbol] := Simp[(b^2\*Cot[c + d\*x]\*(a + b\*Csc[c + d\*x])^(n + 1))/(a\*d\*(n + 1)\*(a^2 - b^2)), x] + Dist[1/(a\*(n + 1)\*(a^2 - b^2)), Int[(a + b\*Csc[c + d\*x])^(n + 1)\*Simp[(a^2 - b^2)\*(n + 1) - a\*b\*(n + 1)\*Csc[c + d\*x] + b^2\*(n + 2)\*Csc[c + d\*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2\*n]

**Rule 3831**

Int[csc[(e\_.) + (f\_.)\*(x\_)]/(csc[(e\_.) + (f\_.)\*(x\_)])\*(b\_.) + (a\_), x\_Symbol] := Dist[1/b, Int[1/(1 + (a\*Sin[e + f\*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

**Rule 3919**

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx &= \frac{1}{2} \operatorname{Subst} \left( \int \frac{1}{(a + b \operatorname{sech}(c + dx))^2} dx, x, x^2 \right) \\ &= \frac{b^2 \tanh(c + dx^2)}{2a(a^2 - b^2)d(a + b \operatorname{sech}(c + dx^2))} - \frac{\operatorname{Subst} \left( \int \frac{-a^2 + b^2 + ab \operatorname{sech}(c + dx)}{a + b \operatorname{sech}(c + dx)} dx, x, x^2 \right)}{2a(a^2 - b^2)} \\ &= \frac{x^2}{2a^2} + \frac{b^2 \tanh(c + dx^2)}{2a(a^2 - b^2)d(a + b \operatorname{sech}(c + dx^2))} - \frac{(b(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{\operatorname{sech}(c + dx)}{a + b \operatorname{sech}(c + dx)} dx, x, x^2 \right)}{2a^2(a^2 - b^2)} \\ &= \frac{x^2}{2a^2} + \frac{b^2 \tanh(c + dx^2)}{2a(a^2 - b^2)d(a + b \operatorname{sech}(c + dx^2))} - \frac{(2a^2 - b^2) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a \cosh(c + dx)}{b}} dx, x, x^2 \right)}{2a^2(a^2 - b^2)} \\ &= \frac{x^2}{2a^2} + \frac{b^2 \tanh(c + dx^2)}{2a(a^2 - b^2)d(a + b \operatorname{sech}(c + dx^2))} + \frac{(i(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, x^2 \right)}{a^2(a^2 - b^2)} \\ &= \frac{x^2}{2a^2} - \frac{b(2a^2 - b^2) \tan^{-1} \left( \frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c + dx^2)\right)}{\sqrt{a+b}} \right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2 \tanh(c + dx^2)}{2a(a^2 - b^2)d(a + b \operatorname{sech}(c + dx^2))} \end{aligned}$$

**Mathematica [A]** time = 0.46, size = 220, normalized size = 1.79

$$\frac{b \left( (a^2 - b^2)^{3/2} (c + dx^2) + ab\sqrt{a^2 - b^2} \sinh(c + dx^2) + (4a^2b - 2b^3) \tan^{-1} \left( \frac{(b-a) \tanh\left(\frac{1}{2}(c + dx^2)\right)}{\sqrt{a^2 - b^2}} \right) \right) + a \cosh(c + dx^2)}{2a^2d(a-b)(a+b)\sqrt{a^2 - b^2} (a \cosh(c + dx^2) + a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sech[c + d*x^2])^2, x]
```

```
[Out] (a*((a^2 - b^2)^(3/2)*(c + d*x^2) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh
[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x^2] + b*((a^2 - b^2)^(3/2)*(
c + d*x^2) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^2)/2])/Sqrt[a
^2 - b^2]) + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x^2]))/(2*a^2*(a - b)*(a + b)*S
qrt[a^2 - b^2]*d*(b + a*Cosh[c + d*x^2]))
```

**fricas [B]** time = 0.46, size = 1314, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{2} \left( (a^5 - 2a^3b^2 + ab^4) dx^2 \cosh(dx^2 + c)^2 + (a^5 - 2a^3b^2 + ab^4) dx^2 \sinh(dx^2 + c)^2 - 2a^3b^2 + 2ab^4 + (a^5 - 2a^3b^2 + ab^4) dx^2 - (2a^3b - ab^3 + (2a^3b - ab^3) \cosh(dx^2 + c)^2 + (2a^3b - ab^3) \sinh(dx^2 + c)^2 + 2(2a^2b^2 - b^4) \cosh(dx^2 + c) + 2(2a^2b^2 - b^4 + (2a^3b - ab^3) \cosh(dx^2 + c)) \sinh(dx^2 + c)) \sqrt{(-a^2 + b^2) \log((a^2 \cosh(dx^2 + c)^2 + a^2 \sinh(dx^2 + c)^2 + 2ab \cosh(dx^2 + c) - a^2 + 2b^2 + 2(a^2 \cosh(dx^2 + c) + ab) \sinh(dx^2 + c) + 2\sqrt{-a^2 + b^2} (a \cosh(dx^2 + c) + a \sinh(dx^2 + c) + b)) / (a \cosh(dx^2 + c)^2 + a \sinh(dx^2 + c)^2 + 2b \cosh(dx^2 + c) + 2(a \cosh(dx^2 + c) + b) \sinh(dx^2 + c) + a)} - 2(a^2b^3 - b^5 - (a^4b - 2a^2b^3 + b^5) dx^2) \cosh(dx^2 + c) - 2(a^2b^3 - b^5 - (a^4b - 2a^2b^3 + b^5) dx^2) \sinh(dx^2 + c) \right) / \left( (a^7 - 2a^5b^2 + a^3b^4) dx^2 \cosh(dx^2 + c)^2 + (a^7 - 2a^5b^2 + a^3b^4) dx^2 \sinh(dx^2 + c)^2 + 2(a^6b - 2a^4b^3 + a^2b^5) dx^2 \cosh(dx^2 + c) + (a^7 - 2a^5b^2 + a^3b^4) dx + 2((a^7 - 2a^5b^2 + a^3b^4) dx^2 \cosh(dx^2 + c) + (a^6b - 2a^4b^3 + a^2b^5) dx) \sinh(dx^2 + c) \right), \frac{1}{2} \left( (a^5 - 2a^3b^2 + ab^4) dx^2 \cosh(dx^2 + c)^2 + (a^5 - 2a^3b^2 + ab^4) dx^2 \sinh(dx^2 + c)^2 - 2a^3b^2 + 2ab^4 + (a^5 - 2a^3b^2 + ab^4) dx^2 + 2(2a^3b - ab^3 + (2a^3b - ab^3) \cosh(dx^2 + c)^2 + (2a^3b - ab^3) \sinh(dx^2 + c)^2 + 2(2a^2b^2 - b^4) \cosh(dx^2 + c) + 2(2a^2b^2 - b^4 + (2a^3b - ab^3) \cosh(dx^2 + c)) \sinh(dx^2 + c)) \sqrt{a^2 - b^2} \arctan\left(\frac{a \cosh(dx^2 + c) + a \sinh(dx^2 + c) + b}{\sqrt{a^2 - b^2}}\right) - 2(a^2b^3 - b^5 - (a^4b - 2a^2b^3 + b^5) dx^2) \cosh(dx^2 + c) - 2(a^2b^3 - b^5 - (a^4b - 2a^2b^3 + b^5) dx^2) \sinh(dx^2 + c) \right) / \left( (a^7 - 2a^5b^2 + a^3b^4) dx^2 \cosh(dx^2 + c)^2 + (a^7 - 2a^5b^2 + a^3b^4) dx^2 \sinh(dx^2 + c)^2 + 2(a^6b - 2a^4b^3 + a^2b^5) dx^2 \cosh(dx^2 + c) + (a^7 - 2a^5b^2 + a^3b^4) dx + 2((a^7 - 2a^5b^2 + a^3b^4) dx^2 \cosh(dx^2 + c) + (a^6b - 2a^4b^3 + a^2b^5) dx) \sinh(dx^2 + c) \right)$$

**giac** [A] time = 0.15, size = 148, normalized size = 1.20

$$\frac{(2a^2b - b^3) \arctan\left(\frac{ae^{(dx^2+c)} + b}{\sqrt{a^2 - b^2}}\right)}{(a^4d - a^2b^2d)\sqrt{a^2 - b^2}} - \frac{b^3e^{(dx^2+c)} + ab^2}{(a^4d - a^2b^2d)\left(ae^{(2dx^2+2c)} + 2be^{(dx^2+c)} + a\right)} + \frac{dx^2 + c}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] 
$$-(2a^2b - b^3) \arctan\left(\frac{a e^{(dx^2 + c)} + b}{\sqrt{a^2 - b^2}}\right) / \left( (a^4d - a^2b^2d) \sqrt{a^2 - b^2} \right) - (b^3 e^{(dx^2 + c)} + ab^2) / \left( (a^4d - a^2b^2d) (a e^{(2dx^2 + 2c)} + 2b e^{(dx^2 + c)} + a) \right) + 1/2 (dx^2 + c) / (a^2d)$$

**maple** [B] time = 0.27, size = 234, normalized size = 1.90

$$\frac{\ln\left(\tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right) - 1\right)}{2da^2} + \frac{b^2 \tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{da(a^2 - b^2)\left(a\left(\tanh^2\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right) - \left(\tanh^2\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)b + a + b\right)} - \frac{2b \arctan\left(\frac{a e^{(dx^2 + c)} + b}{\sqrt{a^2 - b^2}}\right)}{d(a + b)(a - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*sech(d\*x^2+c))^2,x)

```
[Out] -1/2/d/a^2*ln(tanh(1/2*d*x^2+1/2*c)-1)+1/d/a*b^2/(a^2-b^2)*tanh(1/2*d*x^2+1/2*c)/(a*tanh(1/2*d*x^2+1/2*c)^2-tanh(1/2*d*x^2+1/2*c)^2*b+a+b)-2/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x^2+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x^2+1/2*c)/((a+b)*(a-b))^(1/2))+1/2/d/a^2*ln(tanh(1/2*d*x^2+1/2*c)+1)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more details)Is 4*b^2-4*a^2 positive or negative?
```

**mupad** [B] time = 1.76, size = 316, normalized size = 2.57

$$\frac{\frac{b^2}{d(a^2-b^2)} + \frac{b^3 e^{d x^2+c}}{a d(a^2-b^2)} + \frac{x^2}{2 a^2}}{a + 2 b e^{d x^2+c} + a e^{2 d x^2+2 c}} + \frac{b \ln\left(\frac{2 b x e^{d x^2+c} (2 a^2-b^2)}{a^3 (a^2-b^2)} - \frac{2 b x (a+b e^{d x^2+c}) (2 a^2-b^2)}{a^3 (a+b)^{3/2} (b-a)^{3/2}}\right) (2 a^2-b^2)}{2 a^2 d (a+b)^{3/2} (b-a)^{3/2}} - \frac{b \ln\left(\frac{2 b x e^{d x^2+c} (2 a^2-b^2)}{a^3 (a^2-b^2)} - \frac{2 b x (a+b e^{d x^2+c}) (2 a^2-b^2)}{a^3 (a+b)^{3/2} (b-a)^{3/2}}\right) (2 a^2-b^2)}{2 a^2 d (a+b)^{3/2} (b-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/cosh(c + d*x^2))^2,x)
```

```
[Out] (b^2/(d*(a*b^2 - a^3)) + (b^3*exp(c + d*x^2))/(a*d*(a*b^2 - a^3)))/(a + 2*b*exp(c + d*x^2) + a*exp(2*c + 2*d*x^2)) + x^2/(2*a^2) + (b*log((2*b*x*exp(c + d*x^2)*(2*a^2 - b^2))/(a^3*(a^2 - b^2)) - (2*b*x*(a + b*exp(c + d*x^2))*(2*a^2 - b^2))/(a^3*(a + b)^(3/2)*(b - a)^(3/2))))*(2*a^2 - b^2)/(2*a^2*d*(a + b)^(3/2)*(b - a)^(3/2)) - (b*log(((2*b*x*exp(c + d*x^2)*(2*a^2 - b^2))/(a^3*(a^2 - b^2)) + (2*b*x*(a + b*exp(c + d*x^2))*(2*a^2 - b^2))/(a^3*(a + b)^(3/2)*(b - a)^(3/2))))*(2*a^2 - b^2)/(2*a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*sech(d*x**2+c))**2,x)
```

```
[Out] Integral(x/(a + b*sech(c + d*x**2))**2, x)
```

$$3.28 \quad \int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Optimal.** Leaf size=21

$$\operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*sech(d\*x^2+c))^2, x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sech[c + d\*x^2])^2), x]

[Out] Defer[Int][1/(x\*(a + b\*Sech[c + d\*x^2])^2), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx$$

**Mathematica [A]** time = 104.70, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sech[c + d\*x^2])^2), x]

[Out] Integrate[1/(x\*(a + b\*Sech[c + d\*x^2])^2), x]

**fricas [A]** time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{b^2x\operatorname{sech}(dx^2+c)^2+2abx\operatorname{sech}(dx^2+c)+a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(d\*x^2+c))^2, x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*sech(d\*x^2 + c)^2 + 2\*a\*b\*x\*sech(d\*x^2 + c) + a^2\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{sech}(dx^2+c)+a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(d\*x^2+c))^2, x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*x^2 + c) + a)^2\*x), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left( a + b \operatorname{sech}(d x^2 + c) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(1/x/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^3 e^{(dx^2+c)} + ab^2}{(a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)}) x^2 e^{(2dx^2)} + 2(a^4 b d e^c - a^2 b^3 d e^c) x^2 e^{(dx^2)} + (a^5 d - a^3 b^2 d) x^2} + \frac{\log(x)}{a^2} - \int \frac{1}{(a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)}) x^2 e^{(2dx^2)} + 2(a^4 b d e^c - a^2 b^3 d e^c) x^2 e^{(dx^2)} + (a^5 d - a^3 b^2 d) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out]  $-(b^3 e^{(d*x^2 + c)} + a*b^2)/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^2*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^{(d*x^2)} + (a^5*d - a^3*b^2*d)*x^2) + \log(x)/a^2 - \operatorname{integrate}(2*(a*b^2 + (b^3*e^c + (2*a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^{(d*x^2)})/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^3*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^3*e^{(d*x^2)} + (a^5*d - a^3*b^2*d)*x^3), x)$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x \left( a + \frac{b}{\cosh(d x^2 + c)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b/cosh(c + d\*x^2))^2),x)

[Out] int(1/(x\*(a + b/cosh(c + d\*x^2))^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left( a + b \operatorname{sech}(c + dx^2) \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(1/(x\*(a + b\*sech(c + d\*x\*\*2))\*\*2), x)

$$3.29 \quad \int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

**Optimal.** Leaf size=21

$$\operatorname{Int} \left( \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2}, x \right)$$

[Out] Unintegrable(1/x^2/(a+b\*sech(d\*x^2+c))^2,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*Sech[c + d\*x^2])^2),x]

[Out] Defer[Int][1/(x^2\*(a + b\*Sech[c + d\*x^2])^2), x]

Rubi steps

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

**Mathematica [A]** time = 67.40, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*Sech[c + d\*x^2])^2),x]

[Out] Integrate[1/(x^2\*(a + b\*Sech[c + d\*x^2])^2), x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{1}{b^2 x^2 \operatorname{sech}(dx^2 + c)^2 + 2 a b x^2 \operatorname{sech}(dx^2 + c) + a^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*sech(d\*x^2 + c)^2 + 2\*a\*b\*x^2\*sech(d\*x^2 + c) + a^2\*x^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*x^2 + c) + a)^2\*x^2), x)

**maple** [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(d x^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(1/x^2/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^3 d e^{(2c)} - a b^2 d e^{(2c)}) x^2 e^{(2 d x^2)} + a b^2 + (a^3 d - a b^2 d) x^2 + (b^3 e^c + 2(a^2 b d e^c - b^3 d e^c) x^2) e^{(d x^2)}}{(a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)}) x^3 e^{(2 d x^2)} + 2(a^4 b d e^c - a^2 b^3 d e^c) x^3 e^{(d x^2)} + (a^5 d - a^3 b^2 d) x^3} - \int \frac{1}{(a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)}) x^3 e^{(2 d x^2)} + 2(a^4 b d e^c - a^2 b^3 d e^c) x^3 e^{(d x^2)} + (a^5 d - a^3 b^2 d) x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out] -((a^3\*d\*e^(2\*c) - a\*b^2\*d\*e^(2\*c))\*x^2\*e^(2\*d\*x^2) + a\*b^2 + (a^3\*d - a\*b^2\*d)\*x^2 + (b^3\*e^c + 2\*(a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x^2)\*e^(d\*x^2))/((a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*x^3\*e^(2\*d\*x^2) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*x^3\*e^(d\*x^2) + (a^5\*d - a^3\*b^2\*d)\*x^3) - integrate((3\*a\*b^2 + (3\*b^3\*e^c + 2\*(2\*a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x^2)\*e^(d\*x^2))/((a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*x^4\*e^(2\*d\*x^2) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*x^4\*e^(d\*x^2) + (a^5\*d - a^3\*b^2\*d)\*x^4), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \left( a + \frac{b}{\cosh(d x^2 + c)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b/cosh(c + d\*x^2))^2),x)

[Out] int(1/(x^2\*(a + b/cosh(c + d\*x^2))^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d x^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*sech(c + d\*x\*\*2))\*\*2), x)



$$3.30 \quad \int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

**Optimal.** Leaf size=21

$$\operatorname{Int} \left( \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2}, x \right)$$

[Out] Unintegrable(1/x^3/(a+b\*sech(d\*x^2+c))^2,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^3\*(a + b\*Sech[c + d\*x^2])^2),x]

[Out] Defer[Int][1/(x^3\*(a + b\*Sech[c + d\*x^2])^2), x]

Rubi steps

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

**Mathematica [A]** time = 68.24, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^3\*(a + b\*Sech[c + d\*x^2])^2),x]

[Out] Integrate[1/(x^3\*(a + b\*Sech[c + d\*x^2])^2), x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{1}{b^2 x^3 \operatorname{sech}(dx^2 + c)^2 + 2 a b x^3 \operatorname{sech}(dx^2 + c) + a^2 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*sech(d\*x^2+c))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^3\*sech(d\*x^2 + c)^2 + 2\*a\*b\*x^3\*sech(d\*x^2 + c) + a^2\*x^3), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*sech(d\*x^2+c))^2,x, algorithm="giac")

[Out] sage0\*x

**maple** [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(dx^2 + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b\*sech(d\*x^2+c))^2,x)

[Out] int(1/x^3/(a+b\*sech(d\*x^2+c))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^3 d e^{(2c)} - a b^2 d e^{(2c)}) x^2 e^{(2dx^2)} + 2 a b^2 + (a^3 d - a b^2 d) x^2 + 2 (b^3 e^c + (a^2 b d e^c - b^3 d e^c) x^2) e^{(dx^2)}}{2 \left( (a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)}) x^4 e^{(2dx^2)} + 2 (a^4 b d e^c - a^2 b^3 d e^c) x^4 e^{(dx^2)} + (a^5 d - a^3 b^2 d) x^4 \right)} \int \frac{1}{(a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)}) x^4 e^{(2dx^2)} + 2 (a^4 b d e^c - a^2 b^3 d e^c) x^4 e^{(dx^2)} + (a^5 d - a^3 b^2 d) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*sech(d\*x^2+c))^2,x, algorithm="maxima")

[Out]  $-1/2 * ((a^3 * d * e^{(2*c)} - a * b^2 * d * e^{(2*c)}) * x^2 * e^{(2*d*x^2)} + 2 * a * b^2 + (a^3 * d - a * b^2 * d) * x^2 + 2 * (b^3 * e^c + (a^2 * b * d * e^c - b^3 * d * e^c) * x^2) * e^{(d*x^2)}) / ((a^5 * d * e^{(2*c)} - a^3 * b^2 * d * e^{(2*c)}) * x^4 * e^{(2*d*x^2)} + 2 * (a^4 * b * d * e^c - a^2 * b^3 * d * e^c) * x^4 * e^{(d*x^2)} + (a^5 * d - a^3 * b^2 * d) * x^4) - \text{integrate}(2 * (2 * a * b^2 + (2 * b^3 * e^c + (2 * a^2 * b * d * e^c - b^3 * d * e^c) * x^2) * e^{(d*x^2)}) / ((a^5 * d * e^{(2*c)} - a^3 * b^2 * d * e^{(2*c)}) * x^5 * e^{(2*d*x^2)} + 2 * (a^4 * b * d * e^c - a^2 * b^3 * d * e^c) * x^5 * e^{(d*x^2)} + (a^5 * d - a^3 * b^2 * d) * x^5), x)$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^3 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3\*(a + b/cosh(c + d\*x^2))^2),x)

[Out] int(1/(x^3\*(a + b/cosh(c + d\*x^2))^2),x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*sech(d\*x\*\*2+c))\*\*2,x)

[Out] Integral(1/(x\*\*3\*(a + b\*sech(c + d\*x\*\*2))\*\*2), x)

$$3.31 \quad \int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx$$

Optimal. Leaf size=6

$$-\tanh\left(\frac{1}{x}\right)$$

[Out] -tanh(1/x)

Rubi [A] time = 0.02, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {5436, 3767, 8}

$$-\tanh\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[Sech[x^(-1)]^2/x^2,x]

[Out] -Tanh[x^(-1)]

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx &= -\operatorname{Subst}\left(\int \operatorname{sech}^2(x) dx, x, \frac{1}{x}\right) \\ &= -\left(i \operatorname{Subst}\left(\int 1 dx, x, -i \tanh\left(\frac{1}{x}\right)\right)\right) \\ &= -\tanh\left(\frac{1}{x}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 6, normalized size = 1.00

$$-\tanh\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x^(-1)]^2/x^2,x]

[Out] -Tanh[x<sup>(-1)</sup>]

**fricas** [B] time = 0.38, size = 28, normalized size = 4.67

$$\frac{2}{\cosh\left(\frac{1}{x}\right)^2 + 2 \cosh\left(\frac{1}{x}\right) \sinh\left(\frac{1}{x}\right) + \sinh\left(\frac{1}{x}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(1/x)<sup>2</sup>/x<sup>2</sup>, x, algorithm="fricas")

[Out] 2/(cosh(1/x)<sup>2</sup> + 2\*cosh(1/x)\*sinh(1/x) + sinh(1/x)<sup>2</sup> + 1)

**giac** [A] time = 0.11, size = 12, normalized size = 2.00

$$\frac{2}{e^{\frac{2}{x}} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(1/x)<sup>2</sup>/x<sup>2</sup>, x, algorithm="giac")

[Out] 2/(e<sup>(2/x)</sup> + 1)

**maple** [A] time = 0.27, size = 7, normalized size = 1.17

$$-\tanh\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(1/x)<sup>2</sup>/x<sup>2</sup>, x)

[Out] -tanh(1/x)

**maxima** [A] time = 0.30, size = 12, normalized size = 2.00

$$\frac{2}{e^{\frac{2}{x}} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(1/x)<sup>2</sup>/x<sup>2</sup>, x, algorithm="maxima")

[Out] 2/(e<sup>(2/x)</sup> + 1)

**mupad** [B] time = 1.26, size = 12, normalized size = 2.00

$$\frac{2}{e^{2/x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x<sup>2</sup>\*cosh(1/x)<sup>2</sup>), x)

[Out] 2/(exp(2/x) + 1)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(1/x)\*\*2/x\*\*2, x)

[Out] Integral(sech(1/x)\*\*2/x\*\*2, x)

### 3.32 $\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$

**Optimal.** Leaf size=426

$$\frac{ax^4}{4} - \frac{10080ib\operatorname{Li}_8(-ie^{c+d\sqrt{x}})}{d^8} + \frac{10080ib\operatorname{Li}_8(ie^{c+d\sqrt{x}})}{d^8} + \frac{10080ib\sqrt{x}\operatorname{Li}_7(-ie^{c+d\sqrt{x}})}{d^7} - \frac{10080ib\sqrt{x}\operatorname{Li}_7(ie^{c+d\sqrt{x}})}{d^7} - 50$$

```
[Out] 1/4*a*x^4+4*b*x^(7/2)*arctan(exp(c+d*x^(1/2)))/d+1680*I*b*x^(3/2)*polylog(5,
-I*exp(c+d*x^(1/2)))/d^5-10080*I*b*polylog(7,I*exp(c+d*x^(1/2)))*x^(1/2)/d
^7-420*I*b*x^2*polylog(4,-I*exp(c+d*x^(1/2)))/d^4-84*I*b*x^(5/2)*polylog(3,
I*exp(c+d*x^(1/2)))/d^3+14*I*b*x^3*polylog(2,I*exp(c+d*x^(1/2)))/d^2-5040*I
*b*x*polylog(6,-I*exp(c+d*x^(1/2)))/d^6+84*I*b*x^(5/2)*polylog(3,-I*exp(c+d
*x^(1/2)))/d^3-1680*I*b*x^(3/2)*polylog(5,I*exp(c+d*x^(1/2)))/d^5+5040*I*b*
x*polylog(6,I*exp(c+d*x^(1/2)))/d^6-10080*I*b*polylog(8,-I*exp(c+d*x^(1/2))
)/d^8+10080*I*b*polylog(7,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^7-14*I*b*x^3*polyl
og(2,-I*exp(c+d*x^(1/2)))/d^2+10080*I*b*polylog(8,I*exp(c+d*x^(1/2)))/d^8+4
20*I*b*x^2*polylog(4,I*exp(c+d*x^(1/2)))/d^4
```

**Rubi [A]** time = 0.40, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {14, 5436, 4180, 2531, 6609, 2282, 6589}

$$-\frac{14ibx^3\operatorname{PolyLog}(2,-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3\operatorname{PolyLog}(2,ie^{c+d\sqrt{x}})}{d^2} + \frac{84ibx^{5/2}\operatorname{PolyLog}(3,-ie^{c+d\sqrt{x}})}{d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}(3,ie^{c+d\sqrt{x}})}{d^3} - 50$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 + (4*b*x^(7/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((14*I)*b*x^3*PolyL
og[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((14*I)*b*x^3*PolyLog[2, I*E^(c + d*Sq
rt[x])])/d^2 + ((84*I)*b*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 -
((84*I)*b*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((420*I)*b*x^2*Pol
yLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((420*I)*b*x^2*PolyLog[4, I*E^(c + d
*Sqrt[x])])/d^4 + ((1680*I)*b*x^(3/2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d
^5 - ((1680*I)*b*x^(3/2)*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 - ((5040*I)*b
*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((5040*I)*b*x*PolyLog[6, I*E^(
c + d*Sqrt[x])])/d^6 + ((10080*I)*b*Sqrt[x]*PolyLog[7, (-I)*E^(c + d*Sqrt[x
])])/d^7 - ((10080*I)*b*Sqrt[x]*PolyLog[7, I*E^(c + d*Sqrt[x])])/d^7 - ((10
080*I)*b*PolyLog[8, (-I)*E^(c + d*Sqrt[x])])/d^8 + ((10080*I)*b*PolyLog[8,
I*E^(c + d*Sqrt[x])])/d^8
```

#### Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))^*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

#### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```

)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

#### Rule 4180

```

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_
))^m_], x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

#### Rule 5436

```

Int[(x_)^m_)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^n_])^p_], x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

```

#### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

```

#### Rule 6609

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

```

#### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx &= \int (ax^3 + bx^3 \operatorname{sech}(c + d\sqrt{x})) dx \\
&= \frac{ax^4}{4} + b \int x^3 \operatorname{sech}(c + d\sqrt{x}) dx \\
&= \frac{ax^4}{4} + (2b) \operatorname{Subst} \left( \int x^7 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{(14ib) \operatorname{Subst} \left( \int x^6 \log(1 - ie^{c+dx}) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{ax^4}{4} + \frac{4bx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 1.95, size = 415, normalized size = 0.97

$$\frac{ax^4}{4} + \frac{2ib(d^7 x^{7/2} \log(1 - ie^{c+d\sqrt{x}}) - d^7 x^{7/2} \log(1 + ie^{c+d\sqrt{x}}) - 7d^6 x^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}}) + 7d^6 x^3 \operatorname{Li}_2(ie^{c+d\sqrt{x}}) + 42d^5 x^2 \operatorname{Li}_3(-ie^{c+d\sqrt{x}}) - 42d^5 x^2 \operatorname{Li}_3(ie^{c+d\sqrt{x}}) - 210d^4 x^2 \operatorname{Li}_4(-ie^{c+d\sqrt{x}}) + 210d^4 x^2 \operatorname{Li}_4(ie^{c+d\sqrt{x}}) + 840d^3 x^{3/2} \operatorname{Li}_5(-ie^{c+d\sqrt{x}}) - 840d^3 x^{3/2} \operatorname{Li}_5(ie^{c+d\sqrt{x}}) - 2520d^2 x \operatorname{Li}_6(-ie^{c+d\sqrt{x}}) + 2520d^2 x \operatorname{Li}_6(ie^{c+d\sqrt{x}}) + 5040d \operatorname{Li}_7(-ie^{c+d\sqrt{x}}) - 5040d \operatorname{Li}_7(ie^{c+d\sqrt{x}}) - 5040 \operatorname{Li}_8(-ie^{c+d\sqrt{x}}) + 5040 \operatorname{Li}_8(ie^{c+d\sqrt{x}}))}{d^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Sech[c + d\*Sqrt[x]]),x]

[Out] (a\*x^4)/4 + ((2\*I)\*b\*(d^7\*x^(7/2)\*Log[1 - I\*E^(c + d\*Sqrt[x])] - d^7\*x^(7/2)\*Log[1 + I\*E^(c + d\*Sqrt[x])] - 7\*d^6\*x^3\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 7\*d^6\*x^3\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + 42\*d^5\*x^(5/2)\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 42\*d^5\*x^(5/2)\*PolyLog[3, I\*E^(c + d\*Sqrt[x])] - 210\*d^4\*x^2\*PolyLog[4, (-I)\*E^(c + d\*Sqrt[x])] + 210\*d^4\*x^2\*PolyLog[4, I\*E^(c + d\*Sqrt[x])] + 840\*d^3\*x^(3/2)\*PolyLog[5, (-I)\*E^(c + d\*Sqrt[x])] - 840\*d^3\*x^(3/2)\*PolyLog[5, I\*E^(c + d\*Sqrt[x])] - 2520\*d^2\*x\*PolyLog[6, (-I)\*E^(c + d\*Sqrt[x])] + 2520\*d^2\*x\*PolyLog[6, I\*E^(c + d\*Sqrt[x])] + 5040\*d\*Sqrt[x]\*PolyLog[7, (-I)\*E^(c + d\*Sqrt[x])] - 5040\*d\*Sqrt[x]\*PolyLog[7, I\*E^(c + d\*Sqrt[x])] - 5040\*PolyLog[8, (-I)\*E^(c + d\*Sqrt[x])] + 5040\*PolyLog[8, I\*E^(c + d\*Sqrt[x])])/d^8

**fricas [F]** time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}(bx^3 \operatorname{sech}(d\sqrt{x} + c) + ax^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(b\*x^3\*sech(d\*sqrt(x) + c) + a\*x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)\*x^3, x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(x^3\*(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} ax^4 + 2b \int \frac{x^3 e^{(d\sqrt{x}+c)}}{e^{(2d\sqrt{x}+2c)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] 1/4\*a\*x^4 + 2\*b\*integrate(x^3\*e^(d\*sqrt(x) + c)/(e^(2\*d\*sqrt(x) + 2\*c) + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b/cosh(c + d\*x^(1/2))),x)

[Out] int(x^3\*(a + b/cosh(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*sech(c+d\*x\*\*(1/2))),x)

[Out] Integral(x\*\*3\*(a + b\*sech(c + d\*sqrt(x))), x)



### 3.33 $\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$

**Optimal.** Leaf size=310

$$\frac{ax^3}{3} - \frac{240ib\operatorname{Li}_6(-ie^{c+d\sqrt{x}})}{d^6} + \frac{240ib\operatorname{Li}_6(ie^{c+d\sqrt{x}})}{d^6} + \frac{240ib\sqrt{x}\operatorname{Li}_5(-ie^{c+d\sqrt{x}})}{d^5} - \frac{240ib\sqrt{x}\operatorname{Li}_5(ie^{c+d\sqrt{x}})}{d^5} - \frac{120ibx\operatorname{Li}_4(-ie^{c+d\sqrt{x}})}{d^4} + \frac{120ibx\operatorname{Li}_4(ie^{c+d\sqrt{x}})}{d^4}$$

```
[Out] 1/3*a*x^3+4*b*x^(5/2)*arctan(exp(c+d*x^(1/2)))/d-10*I*b*x^2*polylog(2,-I*exp(c+d*x^(1/2)))/d^2+10*I*b*x^2*polylog(2,I*exp(c+d*x^(1/2)))/d^2+40*I*b*x^(3/2)*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-40*I*b*x^(3/2)*polylog(3,I*exp(c+d*x^(1/2)))/d^3-120*I*b*x*polylog(4,-I*exp(c+d*x^(1/2)))/d^4+120*I*b*x*polylog(4,I*exp(c+d*x^(1/2)))/d^4-240*I*b*polylog(6,-I*exp(c+d*x^(1/2)))/d^6+240*I*b*polylog(6,I*exp(c+d*x^(1/2)))/d^6+240*I*b*polylog(5,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^5-240*I*b*polylog(5,I*exp(c+d*x^(1/2)))*x^(1/2)/d^5
```

**Rubi [A]** time = 0.28, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {14, 5436, 4180, 2531, 6609, 2282, 6589}

$$\frac{10ibx^2\operatorname{PolyLog}(2,-ie^{c+d\sqrt{x}})}{d^2} + \frac{10ibx^2\operatorname{PolyLog}(2,ie^{c+d\sqrt{x}})}{d^2} + \frac{40ibx^{3/2}\operatorname{PolyLog}(3,-ie^{c+d\sqrt{x}})}{d^3} - \frac{40ibx^{3/2}\operatorname{PolyLog}(3,ie^{c+d\sqrt{x}})}{d^3} - \frac{120ibx\operatorname{PolyLog}(4,-ie^{c+d\sqrt{x}})}{d^4} + \frac{120ibx\operatorname{PolyLog}(4,ie^{c+d\sqrt{x}})}{d^4} - \frac{240ib\sqrt{x}\operatorname{PolyLog}(6,-ie^{c+d\sqrt{x}})}{d^6} + \frac{240ib\sqrt{x}\operatorname{PolyLog}(6,ie^{c+d\sqrt{x}})}{d^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sech[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 + (4*b*x^(5/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((10*I)*b*x^2*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((10*I)*b*x^2*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 + ((40*I)*b*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((40*I)*b*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((120*I)*b*x*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((120*I)*b*x*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 + ((240*I)*b*Sqrt[x]*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((240*I)*b*Sqrt[x]*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 - ((240*I)*b*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((240*I)*b*PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6
```

#### Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_))^(n_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_)*((F_)((c_)*((a_)+(b_)*(x_)))^(n_))]*((f_)+(g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 4180



**Mathematica [A]** time = 1.81, size = 311, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{2ib \left( d^5 x^{5/2} \log(1 - ie^{c+d\sqrt{x}}) - d^5 x^{5/2} \log(1 + ie^{c+d\sqrt{x}}) - 5d^4 x^2 \text{Li}_2(-ie^{c+d\sqrt{x}}) + 5d^4 x^2 \text{Li}_2(ie^{c+d\sqrt{x}}) + 20 \right)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Sech[c + d\*Sqrt[x]]),x]

[Out] (a\*x^3)/3 + ((2\*I)\*b\*(d^5\*x^(5/2)\*Log[1 - I\*E^(c + d\*Sqrt[x])] - d^5\*x^(5/2)\*Log[1 + I\*E^(c + d\*Sqrt[x])] - 5\*d^4\*x^2\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 5\*d^4\*x^2\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + 20\*d^3\*x^(3/2)\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 20\*d^3\*x^(3/2)\*PolyLog[3, I\*E^(c + d\*Sqrt[x])] - 60\*d^2\*x\*PolyLog[4, (-I)\*E^(c + d\*Sqrt[x])] + 60\*d^2\*x\*PolyLog[4, I\*E^(c + d\*Sqrt[x])] + 120\*d\*Sqrt[x]\*PolyLog[5, (-I)\*E^(c + d\*Sqrt[x])] - 120\*d\*Sqrt[x]\*PolyLog[5, I\*E^(c + d\*Sqrt[x])] - 120\*PolyLog[6, (-I)\*E^(c + d\*Sqrt[x])] + 120\*PolyLog[6, I\*E^(c + d\*Sqrt[x])])/d^6

**fricas [F]** time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}(bx^2 \text{sech}(d\sqrt{x} + c) + ax^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(b\*x^2\*sech(d\*sqrt(x) + c) + a\*x^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \text{sech}(d\sqrt{x} + c) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)\*x^2, x)

**maple [F]** time = 0.55, size = 0, normalized size = 0.00

$$\int x^2 (a + b \text{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(x^2\*(a+b\*sech(c+d\*x^(1/2))),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}ax^3 + 2b \int \frac{x^2 e^{(d\sqrt{x}+c)}}{e^{(2d\sqrt{x}+2c)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 2\*b\*integrate(x^2\*e^(d\*sqrt(x) + c)/(e^(2\*d\*sqrt(x) + 2\*c) + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b/cosh(c + d*x^(1/2))),x)`

[Out] `int(x^2*(a + b/cosh(c + d*x^(1/2))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*sech(c+d*x**(1/2))),x)`

[Out] `Integral(x**2*(a + b*sech(c + d*sqrt(x))), x)`

### 3.34 $\int x \left( a + b \operatorname{sech} \left( c + d \sqrt{x} \right) \right) dx$

**Optimal.** Leaf size=194

$$\frac{ax^2}{2} - \frac{12ib\operatorname{Li}_4(-ie^{c+d\sqrt{x}})}{d^4} + \frac{12ib\operatorname{Li}_4(ie^{c+d\sqrt{x}})}{d^4} + \frac{12ib\sqrt{x}\operatorname{Li}_3(-ie^{c+d\sqrt{x}})}{d^3} - \frac{12ib\sqrt{x}\operatorname{Li}_3(ie^{c+d\sqrt{x}})}{d^3} - \frac{6ibx\operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2}$$

[Out]  $1/2*a*x^2+4*b*x^{(3/2)}*\arctan(\exp(c+d*x^{(1/2)}))/d-6*I*b*x*\operatorname{polylog}(2,-I*\exp(c+d*x^{(1/2)}))/d^2+6*I*b*x*\operatorname{polylog}(2,I*\exp(c+d*x^{(1/2)}))/d^2-12*I*b*\operatorname{polylog}(4,-I*\exp(c+d*x^{(1/2)}))/d^4+12*I*b*\operatorname{polylog}(4,I*\exp(c+d*x^{(1/2)}))/d^4+12*I*b*\operatorname{polylog}(3,-I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^3-12*I*b*\operatorname{polylog}(3,I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^3$

**Rubi [A]** time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {14, 5436, 4180, 2531, 6609, 2282, 6589}

$$\frac{6ibx\operatorname{PolyLog}(2,-ie^{c+d\sqrt{x}})}{d^2} + \frac{6ibx\operatorname{PolyLog}(2,ie^{c+d\sqrt{x}})}{d^2} + \frac{12ib\sqrt{x}\operatorname{PolyLog}(3,-ie^{c+d\sqrt{x}})}{d^3} - \frac{12ib\sqrt{x}\operatorname{PolyLog}(3,ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*Sech[c + d*Sqrt[x]]),x]`

[Out]  $(a*x^2)/2 + (4*b*x^{(3/2)}*\operatorname{ArcTan}[E^{(c + d*\operatorname{Sqrt}[x])}])/d - ((6*I)*b*x*\operatorname{PolyLog}[2, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((6*I)*b*x*\operatorname{PolyLog}[2, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((12*I)*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 - ((12*I)*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 - ((12*I)*b*\operatorname{PolyLog}[4, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^4 + ((12*I)*b*\operatorname{PolyLog}[4, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^4$

#### Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_))^(n_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^{(c_)*((a_)+(b_)*x)}*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

#### Rule 2531

`Int[Log[1 + (e_)*((F_)^((c_)*((a_)+(b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

#### Rule 4180

`Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x]
- Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x]
/; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx &= \int (ax + bx \operatorname{sech}(c + d\sqrt{x})) dx \\
&= \frac{ax^2}{2} + b \int x \operatorname{sech}(c + d\sqrt{x}) dx \\
&= \frac{ax^2}{2} + (2b) \operatorname{Subst}\left(\int x^3 \operatorname{sech}(c + dx) dx, x, \sqrt{x}\right) \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{(6ib) \operatorname{Subst}\left(\int x^2 \log(1 - ie^{c+dx}) dx, x, \sqrt{x}\right)}{d} + \dots \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6ibx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{6ibx \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} + \frac{12ib}{d^2} \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6ibx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{6ibx \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} + \frac{12ib}{d^2} \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6ibx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{6ibx \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} + \frac{12ib}{d^2} \\
&= \frac{ax^2}{2} + \frac{4bx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6ibx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{6ibx \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} + \frac{12ib}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 1.81, size = 207, normalized size = 1.07

$$\frac{ax^2}{2} + \frac{2ib(d^3x^{3/2} \log(1 - ie^{c+d\sqrt{x}}) - d^3x^{3/2} \log(1 + ie^{c+d\sqrt{x}}) - 3d^2x \operatorname{Li}_2(-ie^{c+d\sqrt{x}}) + 3d^2x \operatorname{Li}_2(ie^{c+d\sqrt{x}}) + 6d\sqrt{x} \operatorname{Li}_2(ie^{c+d\sqrt{x}}))}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sech[c + d*Sqrt[x]]), x]
```

```
[Out] (a*x^2)/2 + ((2*I)*b*(d^3*x^(3/2)*Log[1 - I*E^(c + d*Sqrt[x])] - d^3*x^(3/2)*Log[1 + I*E^(c + d*Sqrt[x])] - 3*d^2*x*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] + 3*d^2*x*PolyLog[2, I*E^(c + d*Sqrt[x])] + 6*d*Sqrt[x]*PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - 6*d*Sqrt[x]*PolyLog[3, I*E^(c + d*Sqrt[x])] - 6*PolyLog[4, (-I)*E^(c + d*Sqrt[x])] + 6*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4
```

**fricas** [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}(bx \operatorname{sech}(d\sqrt{x} + c) + ax, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

[Out] `integral(b*x*sech(d*sqrt(x) + c) + a*x, x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

[Out] `integrate((b*sech(d*sqrt(x) + c) + a)*x, x)`

**maple** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*sech(c+d*x^(1/2))),x)`

[Out] `int(x*(a+b*sech(c+d*x^(1/2))),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}ax^2 + 2b \int \frac{xe^{(d\sqrt{x}+c)}}{e^{(2d\sqrt{x}+2c)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

[Out] `1/2*a*x^2 + 2*b*integrate(x*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b/cosh(c + d*x^(1/2))),x)`

[Out] `int(x*(a + b/cosh(c + d*x^(1/2))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*sech(c+d*x**(1/2))),x)`

[Out] `Integral(x*(a + b*sech(c + d*sqrt(x))), x)`

$$3.35 \quad \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x} dx$$

Optimal. Leaf size=24

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+d\sqrt{x})}{x}, x\right) + a\log(x)$$

[Out] a\*ln(x)+b\*Unintegrable(sech(c+d\*x^(1/2))/x,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])/x,x]

[Out] a\*Log[x] + b\*Defer[Int][Sech[c + d\*Sqrt[x]]/x, x]

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x} dx &= \int \left( \frac{a}{x} + \frac{b\operatorname{sech}(c + d\sqrt{x})}{x} \right) dx \\ &= a\log(x) + b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x} dx \end{aligned}$$

Mathematica [A] time = 6.22, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])/x,x]

[Out] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])/x, x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{sech}(d\sqrt{x} + c) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x,x, algorithm="fricas")

[Out] integral((b\*sech(d\*sqrt(x) + c) + a)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{sech}(d\sqrt{x} + c) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)/x, x)

**maple** [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))/x,x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{e^{(d\sqrt{x}+c)}}{xe^{(2d\sqrt{x}+2c)} + x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x,x, algorithm="maxima")

[Out] 2\*b\*integrate(e^(d\*sqrt(x) + c)/(x\*e^(2\*d\*sqrt(x) + 2\*c) + x), x) + a\*log(x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))/x,x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))/x,x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))/x, x)

$$3.36 \quad \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$$

Optimal. Leaf size=26

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+d\sqrt{x})}{x^2}, x\right) - \frac{a}{x}$$

[Out]  $-a/x+b*\operatorname{Unintegrable}(\operatorname{sech}(c+d*x^{(1/2)})/x^2, x)$

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $-(a/x) + b*\operatorname{Defer}[\operatorname{Int}][\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]]/x^2, x]$

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b\operatorname{sech}(c + d\sqrt{x})}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^2} dx \end{aligned}$$

**Mathematica** [A] time = 7.37, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^2, x]$

**fricas** [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{sech}(d\sqrt{x} + c) + a}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\operatorname{sech}(c+d*x^{(1/2)}))/x^2, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b*\operatorname{sech}(d*\operatorname{sqrt}(x) + c) + a)/x^2, x)$

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)/x^2, x)

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))/x^2,x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{e^{(d\sqrt{x}+c)}}{x^2 e^{(2d\sqrt{x}+2c)} + x^2} dx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^2,x, algorithm="maxima")

[Out] 2\*b\*integrate(e^(d\*sqrt(x) + c)/(x^2\*e^(2\*d\*sqrt(x) + 2\*c) + x^2), x) - a/x

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))/x^2,x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))/x\*\*2,x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))/x\*\*2, x)

### 3.37 $\int x^3 \left( a + b \operatorname{sech} \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=677

$$\frac{a^2 x^4}{4} - \frac{20160iab \operatorname{Li}_8(-ie^{c+d\sqrt{x}})}{d^8} + \frac{20160iab \operatorname{Li}_8(ie^{c+d\sqrt{x}})}{d^8} + \frac{20160iab\sqrt{x} \operatorname{Li}_7(-ie^{c+d\sqrt{x}})}{d^7} - \frac{20160iab\sqrt{x} \operatorname{Li}_7(ie^{c+d\sqrt{x}})}{d^7}$$

[Out]  $315/2*b^2*polylog(7, -exp(2*c+2*d*x^(1/2)))/d^8+2*b^2*x^(7/2)/d+8*a*b*x^(7/2)*arctan(exp(c+d*x^(1/2)))/d-20160*I*a*b*polylog(8, -I*exp(c+d*x^(1/2)))/d^8+20160*I*a*b*polylog(8, I*exp(c+d*x^(1/2)))/d^8+28*I*a*b*x^3*polylog(2, I*exp(c+d*x^(1/2)))/d^2+168*I*a*b*x^(5/2)*polylog(3, -I*exp(c+d*x^(1/2)))/d^3+840*I*a*b*x^2*polylog(4, I*exp(c+d*x^(1/2)))/d^4+3360*I*a*b*x^(3/2)*polylog(5, -I*exp(c+d*x^(1/2)))/d^5+10080*I*a*b*x*polylog(6, I*exp(c+d*x^(1/2)))/d^6+20160*I*a*b*polylog(7, -I*exp(c+d*x^(1/2)))*x^(1/2)/d^7-10080*I*a*b*x*polylog(6, -I*exp(c+d*x^(1/2)))/d^6-20160*I*a*b*polylog(7, I*exp(c+d*x^(1/2)))*x^(1/2)/d^7-28*I*a*b*x^3*polylog(2, -I*exp(c+d*x^(1/2)))/d^2-168*I*a*b*x^(5/2)*polylog(3, I*exp(c+d*x^(1/2)))/d^3-840*I*a*b*x^2*polylog(4, -I*exp(c+d*x^(1/2)))/d^4-3360*I*a*b*x^(3/2)*polylog(5, I*exp(c+d*x^(1/2)))/d^5-14*b^2*x^3*ln(1+exp(2*c+2*d*x^(1/2)))/d^2-42*b^2*x^(5/2)*polylog(2, -exp(2*c+2*d*x^(1/2)))/d^3+105*b^2*x^2*polylog(3, -exp(2*c+2*d*x^(1/2)))/d^4-210*b^2*x^(3/2)*polylog(4, -exp(2*c+2*d*x^(1/2)))/d^5+315*b^2*x*polylog(5, -exp(2*c+2*d*x^(1/2)))/d^6-315*b^2*polylog(6, -exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^7+2*b^2*x^(7/2)*tanh(c+d*x^(1/2))/d+1/4*a^2*x^4$

**Rubi [A]** time = 0.82, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5436, 4190, 4180, 2531, 6609, 2282, 6589, 4184, 3718, 2190}

$$-\frac{28iabx^3 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{28iabx^3 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{168iabx^{5/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{168iabx^{5/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out]  $(2*b^2*x^(7/2))/d + (a^2*x^4)/4 + (8*a*b*x^(7/2)*ArcTan[E^(c + d*Sqrt[x])])/d - (14*b^2*x^3*Log[1 + E^(2*(c + d*Sqrt[x]))]/d^2 - ((28*I)*a*b*x^3*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((28*I)*a*b*x^3*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 - (42*b^2*x^(5/2)*PolyLog[2, -E^(2*(c + d*Sqrt[x]))]/d^3 + ((168*I)*a*b*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((168*I)*a*b*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (105*b^2*x^2*PolyLog[3, -E^(2*(c + d*Sqrt[x]))]/d^4 - ((840*I)*a*b*x^2*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((840*I)*a*b*x^2*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (210*b^2*x^(3/2)*PolyLog[4, -E^(2*(c + d*Sqrt[x]))]/d^5 + ((3360*I)*a*b*x^(3/2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((3360*I)*a*b*x^(3/2)*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (315*b^2*x*PolyLog[5, -E^(2*(c + d*Sqrt[x]))]/d^6 - ((10080*I)*a*b*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((10080*I)*a*b*x*PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6 - (315*b^2*Sqrt[x]*PolyLog[6, -E^(2*(c + d*Sqrt[x]))]/d^7 + ((20160*I)*a*b*Sqrt[x]*PolyLog[7, (-I)*E^(c + d*Sqrt[x])])/d^7 - ((20160*I)*a*b*Sqrt[x]*PolyLog[7, I*E^(c + d*Sqrt[x])])/d^7 + (315*b^2*PolyLog[7, -E^(2*(c + d*Sqrt[x]))]/(2*d^8) - ((20160*I)*a*b*PolyLog[8, (-I)*E^(c + d*Sqrt[x])])/d^8 + ((20160*I)*a*b*PolyLog[8, I*E^(c + d*Sqrt[x])])/d^8 + (2*b^2*x^(7/2)*Tanh[c + d*Sqrt[x]])/d$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp

$$\left( \frac{(c + dx)^m \log[1 + (b(F^{g(e+fx)}))^n]}{a} \right) / (bfgn \log[F]), x - \text{Dist}[(d^m)/(bfgn \log[F]), \text{Int}[(c + dx)^{m-1} \log[1 + (b(F^{g(e+fx)}))^n]/a], x], x] /;$$

$$\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \} \&\& \text{IGtQ}[m, 0]$$

#### Rule 2282

$$\text{Int}[u, x\_Symbol] \text{ :> With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$$

$$\text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_*)((a_*) (v_*)^{(n_*)})^{(m_*)} /;$$

$$\text{FreeQ}\{a, m, n\}, x \} \&\& \text{IntegerQ}[m*n] \&\& \text{!MatchQ}[u, E^{((c_*)((a_*) + (b_*)x))} (F_)[v_] /;$$

$$\text{FreeQ}\{a, b, c\}, x \} \&\& \text{InverseFunctionQ}[F[x]]]$$

#### Rule 2531

$$\text{Int}[\log[1 + (e_*)((F_)^{((c_*)((a_*) + (b_*)x_))})^{(n_*)})^{(f_*) + (g_*)} (x_*)^{(m_*)}, x\_Symbol] \text{ :> -Simp}[(f + gx)^m \text{PolyLog}[2, -(e(F^{c(a+bx)}))^n]] / (b*c*n \log[F]), x] + \text{Dist}[(g^m)/(b*c*n \log[F]), \text{Int}[(f + gx)^{m-1} \text{PolyLog}[2, -(e(F^{c(a+bx)}))^n]], x], x] /;$$

$$\text{FreeQ}\{F, a, b, c, e, f, g, n\}, x \} \&\& \text{GtQ}[m, 0]$$

#### Rule 3718

$$\text{Int}[(c_*) + (d_*)x_*)^{(m_*)} \tan[(e_*) + (\text{Complex}[0, fz_])*(f_*)x_]), x\_Symbol] \text{ :> -Simp}[(I*(c + dx)^{(m+1)}) / (d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[(c + dx)^m E^{(2*(-I*e) + f*fz*x))} / (1 + E^{(2*(-I*e) + f*fz*x))}], x], x] /;$$

$$\text{FreeQ}\{c, d, e, f, fz\}, x \} \&\& \text{IGtQ}[m, 0]$$

#### Rule 4180

$$\text{Int}[\text{csc}[(e_*) + \text{Pi}*(k_*) + (\text{Complex}[0, fz_])*(f_*)x_]) * ((c_*) + (d_*)x_*)^{(m_*)}, x\_Symbol] \text{ :> Simp}[( -2*(c + dx)^m \text{ArcTanh}[E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)}]) / (f*fz*I), x] + (-\text{Dist}[(d^m)/(f*fz*I), \text{Int}[(c + dx)^{m-1} \log[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}]], x], x] + \text{Dist}[(d^m)/(f*fz*I), \text{Int}[(c + dx)^{m-1} \log[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}]], x], x] /;$$

$$\text{FreeQ}\{c, d, e, f, fz\}, x \} \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$$

#### Rule 4184

$$\text{Int}[\text{csc}[(e_*) + (f_*)x_])^{2*} ((c_*) + (d_*)x_*)^{(m_*)}, x\_Symbol] \text{ :> -Simp}[(c + dx)^m \text{Cot}[e + fx]] / f, x] + \text{Dist}[(d^m)/f, \text{Int}[(c + dx)^{m-1} \text{Cot}[e + fx]], x], x] /;$$

$$\text{FreeQ}\{c, d, e, f\}, x \} \&\& \text{GtQ}[m, 0]$$

#### Rule 4190

$$\text{Int}[(\text{csc}[(e_*) + (f_*)x_]) * (b_*) + (a_*)^{(n_*)} * ((c_*) + (d_*)x_*)^{(m_*)}, x\_Symbol] \text{ :> Int}[\text{ExpandIntegrand}[(c + dx)^m, (a + b*\text{Csc}[e + fx])^n], x] /;$$

$$\text{FreeQ}\{a, b, c, d, e, f, m\}, x \} \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0]$$

#### Rule 5436

$$\text{Int}[(x_*)^{(m_*)} * ((a_*) + (b_*) \text{Sech}[(c_*) + (d_*)x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \text{ :> Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)} * (a + b*\text{Sech}[c + dx])^p], x, x^n], x] /;$$

$$\text{FreeQ}\{a, b, c, d, m, n, p\}, x \} \&\& \text{IGtQ}[\text{Simplify}[(m+1)/n], 0] \&\& \text{IntegerQ}[p]$$

#### Rule 6589

$$\text{Int}[\text{PolyLog}[n, (c_*)((a_*) + (b_*)x_*)^{(p_*)}] / ((d_*) + (e_*)x_)), x\_Symbol] \text{ :> Simp}[\text{PolyLog}[n + 1, c*(a + bx)^p] / (e^p), x] /;$$

$$\text{FreeQ}\{a, b, c, d, e\}, x \}$$

, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_.)))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^7 (a + b \operatorname{sech}(c + dx))^2 dx, x, \sqrt{x} \right) \\
 &= 2 \operatorname{Subst} \left( \int (a^2 x^7 + 2abx^7 \operatorname{sech}(c + dx) + b^2 x^7 \operatorname{sech}^2(c + dx)) dx, x, \sqrt{x} \right) \\
 &= \frac{a^2 x^4}{4} + (4ab) \operatorname{Subst} \left( \int x^7 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^7 \operatorname{sech}^2(c + dx) dx, x, \sqrt{x} \right) \\
 &= \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} + \frac{2b^2 x^{7/2} \tanh(c + d\sqrt{x})}{d} - \frac{(28iab) \operatorname{Subst} \left( \int x^6 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right)}{d} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{28iabx^3 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 8.99, size = 739, normalized size = 1.09

$$\cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^4 \cosh(c + d\sqrt{x}) + \frac{2b \cosh(c + d\sqrt{x}) \left( \frac{8be^{2c} d^7 x^{7/2}}{e^{2c} + 1} + i \left( 8ad^7 x^{7/2} \log(1 - ie^{c+d\sqrt{x}}) - 8ad^7 x^{7/2} \log(1 + ie^{c+d\sqrt{x}}) \right) \right)}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (Cosh[c + d\*Sqrt[x]]\*(a + b\*Sech[c + d\*Sqrt[x]])^2\*(a^2\*x^4\*Cosh[c + d\*Sqrt[x]] + (2\*b\*Cosh[c + d\*Sqrt[x]]\*((8\*b\*d^7\*E^(2\*c)\*x^(7/2))/(1 + E^(2\*c)) + I\*(8\*a\*d^7\*x^(7/2)\*Log[1 - I\*E^(c + d\*Sqrt[x])] - 8\*a\*d^7\*x^(7/2)\*Log[1 + I\*E^(c + d\*Sqrt[x])]) + (28\*I)\*b\*d^6\*x^3\*Log[1 + E^(2\*(c + d\*Sqrt[x]))] - 56\*a\*d^6\*x^3\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 56\*a\*d^6\*x^3\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + (84\*I)\*b\*d^5\*x^(5/2)\*PolyLog[2, -E^(2\*(c + d\*Sqrt[x]))] + 336\*a\*d^5\*x^(5/2)\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 336\*a\*d^5\*x^(5/2)\*PolyLog[3, I\*E^(c + d\*Sqrt[x])] - (210\*I)\*b\*d^4\*x^2\*PolyLog[3, -E^(2\*(c + d\*Sqrt[x]))] - 1680\*a\*d^4\*x^2\*PolyLog[4, (-I)\*E^(c + d\*Sqrt[x])] + 1680\*a\*d^4\*x^2\*PolyLog[4, I\*E^(c + d\*Sqrt[x])] + (420\*I)\*b\*d^3\*x^(3/2)\*PolyLog[4, -E^(2\*(c + d\*Sqrt[x]))] + 6720\*a\*d^3\*x^(3/2)\*PolyLog[5, (-I)\*E^(c + d\*Sqrt[x])] - 6720\*a\*d^3\*x^(3/2)\*PolyLog[5, I\*E^(c + d\*Sqrt[x])] - (630\*I)\*b\*d^2\*x\*PolyLog[5, -E^(2\*(c + d\*Sqrt[x]))] - 20160\*a\*d^2\*x\*PolyLog[6, (-I)\*E^(c + d\*Sqrt[x])] + 20160\*a\*d^2\*x\*PolyLog[6, I\*E^(c + d\*Sqrt[x])] + (630\*I)\*b\*d\*Sqrt[x]\*PolyLog[6, -E^(2\*(c + d\*Sqrt[x]))] + 40320\*a\*d\*Sqrt[x]\*PolyLog[7, (-I)\*E^(c + d\*Sqrt[x])] - 40320\*a\*d\*Sqrt[x]\*PolyLog[7, I\*E^(c + d\*Sqrt[x])] - (315\*I)\*b\*PolyLog[7, -E^(2\*(c + d\*Sqrt[x]))] - 40320\*a\*PolyLog[8, (-I)\*E^(c + d\*Sqrt[x])] + 40320\*a\*PolyLog[8, I\*E^(c + d\*Sqrt[x])]))/d^8 + (8\*b^2\*x^(7/2)\*Sech[c]\*Sinh[d\*Sqrt[x]]/d)/(4\*(b + a\*Cosh[c + d\*Sqrt[x]])^2)

**fricas** [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^3 \operatorname{sech}(d\sqrt{x} + c)^2 + 2abx^3 \operatorname{sech}(d\sqrt{x} + c) + a^2x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^3\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^3\*sech(d\*sqrt(x) + c) + a^2\*x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2\*x^3, x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x^3\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 dx^4 e^{(2d\sqrt{x}+2c)} + a^2 dx^4 - 16 b^2 x^{\frac{7}{2}}}{4 (d e^{(2d\sqrt{x}+2c)} + d)} + \int \frac{2 (2 ab dx^3 e^{(d\sqrt{x}+c)} + 7 b^2 x^{\frac{5}{2}})}{d e^{(2d\sqrt{x}+2c)} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] 1/4\*(a^2\*d\*x^4\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*x^4 - 16\*b^2\*x^(7/2))/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d) + integrate(2\*(2\*a\*b\*d\*x^3\*e^(d\*sqrt(x) + c) + 7\*b^2\*x^(5/2))/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^3\*(a + b/cosh(c + d\*x^(1/2)))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*3\*(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)



### 3.38 $\int x^2 \left( a + b \operatorname{sech} \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=497

$$\frac{a^2 x^3}{3} - \frac{480iab \operatorname{Li}_6(-ie^{c+d\sqrt{x}})}{d^6} + \frac{480iab \operatorname{Li}_6(ie^{c+d\sqrt{x}})}{d^6} + \frac{480iab\sqrt{x} \operatorname{Li}_5(-ie^{c+d\sqrt{x}})}{d^5} - \frac{480iab\sqrt{x} \operatorname{Li}_5(ie^{c+d\sqrt{x}})}{d^5} - \frac{240ia}{d^5}$$

```
[Out] 2*b^2*x^(5/2)/d+1/3*a^2*x^3+8*a*b*x^(5/2)*arctan(exp(c+d*x^(1/2)))/d-10*b^2*x^2*ln(1+exp(2*c+2*d*x^(1/2)))/d^2-20*I*a*b*x^2*polylog(2,-I*exp(c+d*x^(1/2)))/d^2+240*I*a*b*x*polylog(4,I*exp(c+d*x^(1/2)))/d^4-20*b^2*x^(3/2)*polylog(2,-exp(2*c+2*d*x^(1/2)))/d^3+20*I*a*b*x^2*polylog(2,I*exp(c+d*x^(1/2)))/d^2+80*I*a*b*x^(3/2)*polylog(3,-I*exp(c+d*x^(1/2)))/d^3+30*b^2*x*polylog(3,-exp(2*c+2*d*x^(1/2)))/d^4-80*I*a*b*x^(3/2)*polylog(3,I*exp(c+d*x^(1/2)))/d^3-240*I*a*b*x*polylog(4,-I*exp(c+d*x^(1/2)))/d^4+15*b^2*polylog(5,-exp(2*c+2*d*x^(1/2)))/d^6+480*I*a*b*polylog(6,I*exp(c+d*x^(1/2)))/d^6-480*I*a*b*polylog(5,I*exp(c+d*x^(1/2)))*x^(1/2)/d^5-30*b^2*polylog(4,-exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^5-480*I*a*b*polylog(6,-I*exp(c+d*x^(1/2)))/d^6+480*I*a*b*polylog(5,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^5+2*b^2*x^(5/2)*tanh(c+d*x^(1/2))/d
```

**Rubi [A]** time = 0.60, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5436, 4190, 4180, 2531, 6609, 2282, 6589, 4184, 3718, 2190}

$$\frac{20iabx^2 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{20iabx^2 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{80iabx^{3/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{80iabx^{3/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^(5/2))/d + (a^2*x^3)/3 + (8*a*b*x^(5/2)*ArcTan[E^(c + d*Sqrt[x])])/d - (10*b^2*x^2*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((20*I)*a*b*x^2*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((20*I)*a*b*x^2*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 - (20*b^2*x^(3/2)*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((80*I)*a*b*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((80*I)*a*b*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (30*b^2*x*polylog[3, -E^(2*(c + d*Sqrt[x]))])/d^4 - ((240*I)*a*b*x*polylog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((240*I)*a*b*x*polylog[4, I*E^(c + d*Sqrt[x])])/d^4 - (30*b^2*Sqrt[x]*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((480*I)*a*b*Sqrt[x]*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((480*I)*a*b*Sqrt[x]*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (15*b^2*PolyLog[5, -E^(2*(c + d*Sqrt[x]))])/d^6 - ((480*I)*a*b*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((480*I)*a*b*PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6 + (2*b^2*x^(5/2)*Tanh[c + d*Sqrt[x]])/d
```

**Rule 2190**

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*(F_)^((g_)*(e_) + (f_)*(x_)))^(n_), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n])/a)]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n])/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2282**

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
```

$(F\_)[v\_]$  /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*(a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3718

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] :> -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(1 + E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 4190

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Csc[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*(a\_.) + (b\_.)\*(x\_)))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^5 (a + b \operatorname{sech}(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int (a^2 x^5 + 2abx^5 \operatorname{sech}(c + dx) + b^2 x^5 \operatorname{sech}^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{a^2 x^3}{3} + (4ab) \operatorname{Subst} \left( \int x^5 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^5 \operatorname{sech}^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} + \frac{2b^2 x^{5/2} \tanh(c + d\sqrt{x})}{d} - \frac{(20iab) \operatorname{Subst} \left( \int x^5 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{20iabx^2 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{20iabx^2 \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
&= \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 8.31, size = 573, normalized size = 1.15

$$\cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^3 \cosh(c + d\sqrt{x}) + \frac{3b \cosh(c + d\sqrt{x}) \left( \frac{4be^{2c} d^5 x^{5/2}}{e^{2c} + 1} + i(4ad^5 x^{5/2} \log(1 - ie^{c+d\sqrt{x}})) \right)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (Cosh[c + d\*Sqrt[x]]\*(a + b\*Sech[c + d\*Sqrt[x]])^2\*(a^2\*x^3\*Cosh[c + d\*Sqrt[x]] + (3\*b\*Cosh[c + d\*Sqrt[x]]\*((4\*b\*d^5\*E^(2\*c)\*x^(5/2))/(1 + E^(2\*c)) + I\*(4\*a\*d^5\*x^(5/2)\*Log[1 - I\*E^(c + d\*Sqrt[x])] - 4\*a\*d^5\*x^(5/2)\*Log[1 + I\*E^(c + d\*Sqrt[x])] + (10\*I)\*b\*d^4\*x^2\*Log[1 + E^(2\*(c + d\*Sqrt[x]))] - 20\*a\*d^4\*x^2\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 20\*a\*d^4\*x^2\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + (20\*I)\*b\*d^3\*x^(3/2)\*PolyLog[2, -E^(2\*(c + d\*Sqrt[x]))] + 80\*a\*d^3\*x^(3/2)\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 80\*a\*d^3\*x^(3/2)\*PolyLog[3, I\*E^(c + d\*Sqrt[x])] - (30\*I)\*b\*d^2\*x\*PolyLog[3, -E^(2\*(c + d\*Sqrt[x]))] - 240\*a\*d^2\*x\*PolyLog[4, (-I)\*E^(c + d\*Sqrt[x])] + 240\*a\*d^2\*x\*PolyLog[4, I\*E^(c + d\*Sqrt[x])] + (30\*I)\*b\*d\*Sqrt[x]\*PolyLog[4, -E^(2\*(c + d\*Sqrt[x]))] + 480\*a\*d\*Sqrt[x]\*PolyLog[5, (-I)\*E^(c + d\*Sqrt[x])] - 480\*a\*d\*Sqrt[x]\*PolyLog[5, I\*E^(c + d\*Sqrt[x])] - (15\*I)\*b\*PolyLog[5, -E^(2\*(c + d\*Sqrt[x]))] - 480\*a\*PolyLog[6, (-I)\*E^(c + d\*Sqrt[x])] + 480\*a\*PolyLog[6, I\*E^(c + d\*Sqrt[x])])

+ d\*Sqrt[x])))))/d^6 + (6\*b^2\*x^(5/2)\*Sech[c]\*Sinh[d\*Sqrt[x]]/d)/(3\*(b + a\*Cosh[c + d\*Sqrt[x]])^2)

**fricas** [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(b^2x^2 \operatorname{sech}(d\sqrt{x} + c)^2 + 2abx^2 \operatorname{sech}(d\sqrt{x} + c) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^2\*sech(d\*sqrt(x) + c) + a^2\*x^2, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2\*x^2, x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x^2\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 dx^3 e^{(2d\sqrt{x}+2c)} + a^2 dx^3 - 12 b^2 x^{\frac{5}{2}}}{3 (de^{(2d\sqrt{x}+2c)} + d)} + \int \frac{2 (2 abdx^2 e^{(d\sqrt{x}+c)} + 5 b^2 x^{\frac{3}{2}})}{de^{(2d\sqrt{x}+2c)} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] 1/3\*(a^2\*d\*x^3\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*x^3 - 12\*b^2\*x^(5/2))/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d) + integrate(2\*(2\*a\*b\*d\*x^2\*e^(d\*sqrt(x) + c) + 5\*b^2\*x^(3/2))/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^2\*(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*sech(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**2*(a + b*sech(c + d*sqrt(x)))**2, x)
```

### 3.39 $\int x \left( a + b \operatorname{sech} \left( c + d \sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=319

$$\frac{a^2 x^2}{2} - \frac{24iab \operatorname{Li}_4(-ie^{c+d\sqrt{x}})}{d^4} + \frac{24iab \operatorname{Li}_4(ie^{c+d\sqrt{x}})}{d^4} + \frac{24iab \sqrt{x} \operatorname{Li}_3(-ie^{c+d\sqrt{x}})}{d^3} - \frac{24iab \sqrt{x} \operatorname{Li}_3(ie^{c+d\sqrt{x}})}{d^3} - \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{12iabx \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2}$$

[Out]  $2*b^2*x^{(3/2)}/d+1/2*a^2*x^2+8*a*b*x^{(3/2)}*\arctan(\exp(c+d*x^{(1/2)}))/d-6*b^2*x*\ln(1+\exp(2*c+2*d*x^{(1/2)}))/d^2-12*I*a*b*x*\operatorname{polylog}(2,-I*\exp(c+d*x^{(1/2)}))/d^2+12*I*a*b*x*\operatorname{polylog}(2,I*\exp(c+d*x^{(1/2)}))/d^2+3*b^2*\operatorname{polylog}(3,-\exp(2*c+2*d*x^{(1/2)}))/d^4-24*I*a*b*\operatorname{polylog}(4,-I*\exp(c+d*x^{(1/2)}))/d^4+24*I*a*b*\operatorname{polylog}(4,I*\exp(c+d*x^{(1/2)}))/d^4-6*b^2*\operatorname{polylog}(2,-\exp(2*c+2*d*x^{(1/2)}))*x^{(1/2)}/d^3+24*I*a*b*\operatorname{polylog}(3,-I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^3-24*I*a*b*\operatorname{polylog}(3,I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^3+2*b^2*x^{(3/2)}*\tanh(c+d*x^{(1/2)})/d$

**Rubi [A]** time = 0.43, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {5436, 4190, 4180, 2531, 6609, 2282, 6589, 4184, 3718, 2190}

$$-\frac{12iabx \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{12iabx \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{24iab \sqrt{x} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{24iab \sqrt{x} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])^2, x]$

[Out]  $(2*b^2*x^{(3/2)})/d + (a^2*x^2)/2 + (8*a*b*x^{(3/2)}*\operatorname{ArcTan}[E^{(c + d*\operatorname{Sqrt}[x])}])/d - (6*b^2*x*\operatorname{Log}[1 + E^{(2*(c + d*\operatorname{Sqrt}[x])})])/d^2 - ((12*I)*a*b*x*\operatorname{PolyLog}[2, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((12*I)*a*b*x*\operatorname{PolyLog}[2, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 - (6*b^2*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, -E^{(2*(c + d*\operatorname{Sqrt}[x])})])/d^3 + ((24*I)*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 - ((24*I)*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 + (3*b^2*\operatorname{PolyLog}[3, -E^{(2*(c + d*\operatorname{Sqrt}[x])})])/d^4 - ((24*I)*a*b*\operatorname{PolyLog}[4, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^4 + ((24*I)*a*b*\operatorname{PolyLog}[4, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^4 + (2*b^2*x^{(3/2)}*\operatorname{Tanh}[c + d*\operatorname{Sqrt}[x]])/d$

#### Rule 2190

$\operatorname{Int}[(((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)*((c_)+(d_)*(x_))})^{(m_)}]/((a_)+(b_)*((F_)^{((g_)*(e_)+(f_)*(x_)))})^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x))))^n)/a]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x))))^n)/a], x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$   $\operatorname{FunctionOfExponentialQ}[u, x] \ \&\& \ \operatorname{!MatchQ}[u, (w_)*((a_)*(v_)^{(n_))})^{(m_)} /;$   $\operatorname{FreeQ}\{a, m, n\}, x\} \ \&\& \ \operatorname{IntegerQ}[m*n] \ \&\& \ \operatorname{!MatchQ}[u, E^{((c_)*((a_)+(b_)*x))}*(F_)[v_] /;$   $\operatorname{FreeQ}\{a, b, c\}, x\} \ \&\& \ \operatorname{InverseFunctionQ}[F[x]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1+(e_)*((F_)^{(c_)*((a_)+(b_)*(x_)))})^{(n_)}]*((f_)+(g_)*(x_))^{(m_)}], x\_Symbol] \rightarrow -\operatorname{Simp}[(f+g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a+b*x))))^n)]/(b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f+g*x)^{(m-1)}*\operatorname{PolyLog}[2, -(e*(F^{(c*(a+b*x))))^n)], x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, e, f\}, x\}$

, g, n}, x] && GtQ[m, 0]

#### Rule 3718

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)], x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

#### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-I*e) + f*fz*x)/E^(I*k*Pi)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-I*e) + f*fz*x)/E^(I*k*Pi)], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> -Simp[(c + d*x)^m*Cot[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

#### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.))))^(p_.)], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int x (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^3 (a + b \operatorname{sech}(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int (a^2 x^3 + 2abx^3 \operatorname{sech}(c + dx) + b^2 x^3 \operatorname{sech}^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{a^2 x^2}{2} + (4ab) \operatorname{Subst} \left( \int x^3 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^3 \operatorname{sech}^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{a^2 x^2}{2} + \frac{8abx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} + \frac{2b^2 x^{3/2} \tanh(c + d\sqrt{x})}{d} - \frac{(12iab) \operatorname{Subst} \left( \int x^3 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{2b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} + \frac{8abx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} + \frac{8abx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6b^2 x \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} + \frac{8abx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6b^2 x \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} + \frac{8abx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6b^2 x \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^{3/2}}{d} + \frac{a^2 x^2}{2} + \frac{8abx^{3/2} \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{6b^2 x \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{12iabx \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 8.70, size = 459, normalized size = 1.44

$$\cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^2 \cosh(c + d\sqrt{x}) + \frac{2b \cosh(c + d\sqrt{x}) \left( \frac{4be^{2c} d^3 x^{3/2}}{e^{2c} + 1} + i(-12(ad^2 x - ibd\sqrt{x}) \operatorname{Li}_2(-ie^{c+d\sqrt{x}})) \right)}{d^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (Cosh[c + d\*Sqrt[x]]\*(a + b\*Sech[c + d\*Sqrt[x]])^2\*(a^2\*x^2\*Cosh[c + d\*Sqrt[x]] + (2\*b\*Cosh[c + d\*Sqrt[x]]\*((4\*b\*d^3\*E^(2\*c))\*x^(3/2))/(1 + E^(2\*c)) + I\*((12\*I)\*b\*d^2\*x\*Log[1 - I\*E^(c + d\*Sqrt[x])] + 4\*a\*d^3\*x^(3/2)\*Log[1 - I\*E^(c + d\*Sqrt[x])] + (12\*I)\*b\*d^2\*x\*Log[1 + I\*E^(c + d\*Sqrt[x])] - 4\*a\*d^3\*x^(3/2)\*Log[1 + I\*E^(c + d\*Sqrt[x])] - (6\*I)\*b\*d^2\*x\*Log[1 + E^(2\*(c + d\*Sqrt[x]))] - 12\*((-I)\*b\*d\*Sqrt[x] + a\*d^2\*x)\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 12\*(I\*b\*d\*Sqrt[x] + a\*d^2\*x)\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + 24\*a\*d\*Sqrt[x]\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 24\*a\*d\*Sqrt[x]\*PolyLog[3, I\*E^(c + d\*Sqrt[x])] - (3\*I)\*b\*PolyLog[3, -E^(2\*(c + d\*Sqrt[x]))] - 24\*a\*PolyLog[4, (-I)\*E^(c + d\*Sqrt[x])] + 24\*a\*PolyLog[4, I\*E^(c + d\*Sqrt[x])]))/d^4 + (4\*b^2\*x^(3/2)\*Sech[c]\*Sinh[d\*Sqrt[x]])/d)/(2\*(b + a\*Cosh[c + d\*Sqrt[x]])^2)

**fricas [F]** time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( b^2 x \operatorname{sech}(d\sqrt{x} + c)^2 + 2abx \operatorname{sech}(d\sqrt{x} + c) + a^2 x, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")



[Out] integral(b^2\*x\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x\*sech(d\*sqrt(x) + c) + a^2\*x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2\*x, x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 dx^2 e^{(2d\sqrt{x}+2c)} + a^2 dx^2 - 8b^2 x^{\frac{3}{2}}}{2(d e^{(2d\sqrt{x}+2c)} + d)} + \int \frac{2(2abdx e^{(d\sqrt{x}+c)} + 3b^2\sqrt{x})}{d e^{(2d\sqrt{x}+2c)} + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] 1/2\*(a^2\*d\*x^2\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*x^2 - 8\*b^2\*x^(3/2))/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d) + integrate(2\*(2\*a\*b\*d\*x\*e^(d\*sqrt(x) + c) + 3\*b^2\*sqrt(x))/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x\*(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)

$$3.40 \quad \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x}, x\right)$$

[Out] Unintegrable((a+b\*sech(c+d\*x^(1/2)))^2/x,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])^2/x,x]

[Out] Defer[Int][(a + b\*Sech[c + d\*Sqrt[x]])^2/x, x]

Rubi steps

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx = \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$$

**Mathematica [A]** time = 62.23, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x,x]

[Out] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x, x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \operatorname{sech}(d\sqrt{x} + c)^2 + 2ab \operatorname{sech}(d\sqrt{x} + c) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x,x, algorithm="fricas")

[Out] integral((b^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sech(d\*sqrt(x) + c) + a^2)/x, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{sech}(d\sqrt{x} + c) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2/x, x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))^2/x,x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))^2/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \log(x) - \frac{4b^2\sqrt{x}}{dxe^{(2d\sqrt{x}+2c)} + dx} + \int \frac{2(2abdxe^{(d\sqrt{x}+c)} - b^2\sqrt{x})}{dx^2e^{(2d\sqrt{x}+2c)} + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x,x, algorithm="maxima")

[Out] a^2\*log(x) - 4\*b^2\*sqrt(x)/(d\*x\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x) + integrate(2\*(2\*a\*b\*d\*x\*e^(d\*sqrt(x) + c) - b^2\*sqrt(x))/(d\*x^2\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cosh(c+d\sqrt{x})}\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))^2/x,x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))^2/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*\*2/x,x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))\*\*2/x, x)

$$3.41 \quad \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2}, x\right)$$

[Out] Unintegrable((a+b\*sech(c+d\*x^(1/2)))^2/x^2, x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^2, x]

[Out] Defer[Int][(a + b\*Sech[c + d\*Sqrt[x]])^2/x^2, x]

Rubi steps

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx = \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$$

**Mathematica [A]** time = 28.30, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^2, x]

[Out] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^2, x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \operatorname{sech}(d\sqrt{x} + c)^2 + 2ab \operatorname{sech}(d\sqrt{x} + c) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^2, x, algorithm="fricas")

[Out] integral((b^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sech(d\*sqrt(x) + c) + a^2)/x^2, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2/x^2, x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))^2/x^2,x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))^2/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2 dx e^{(2d\sqrt{x}+2c)} + a^2 dx + 4b^2 \sqrt{x}}{dx^2 e^{(2d\sqrt{x}+2c)} + dx^2} + \int \frac{2(2abdxe^{(d\sqrt{x}+c)} - 3b^2\sqrt{x})}{dx^3 e^{(2d\sqrt{x}+2c)} + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^2,x, algorithm="maxima")

[Out] -(a^2\*d\*x\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*x + 4\*b^2\*sqrt(x))/(d\*x^2\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x^2) + integrate(2\*(2\*a\*b\*d\*x\*e^(d\*sqrt(x) + c) - 3\*b^2\*sqrt(x))/(d\*x^3\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cosh(c+d\sqrt{x})}\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))^2/x^2,x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))^2/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*\*2/x\*\*2,x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))\*\*2/x\*\*2, x)

$$3.42 \quad \int \frac{x^3}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=961

$$\frac{x^4}{4a} - \frac{2b \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} + \frac{2b \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} - \frac{14b \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} + \frac{14b \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} + \dots$$

[Out]  $\frac{1}{4}x^4/a - 2bx^{7/2} \ln(1+a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a\sqrt{-a^2+b^2}) - 2bx^{7/2} \ln(1+a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a\sqrt{-a^2+b^2}) - 14b^2x^3 \operatorname{polylog}(2, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a^2\sqrt{-a^2+b^2}) + 14b^2x^3 \operatorname{polylog}(2, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a^2\sqrt{-a^2+b^2}) + 84b^2x^{5/2} \operatorname{polylog}(3, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a^3\sqrt{-a^2+b^2}) - 84b^2x^{5/2} \operatorname{polylog}(3, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a^3\sqrt{-a^2+b^2}) - 420b^2x^2 \operatorname{polylog}(4, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a^4\sqrt{-a^2+b^2}) + 420b^2x^2 \operatorname{polylog}(4, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a^4\sqrt{-a^2+b^2}) + 1680b^2x^{3/2} \operatorname{polylog}(5, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a^5\sqrt{-a^2+b^2}) - 1680b^2x^{3/2} \operatorname{polylog}(5, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a^5\sqrt{-a^2+b^2}) - 5040b^2x \operatorname{polylog}(6, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a^6\sqrt{-a^2+b^2}) + 5040b^2x \operatorname{polylog}(6, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a^6\sqrt{-a^2+b^2}) - 10080b^2 \operatorname{polylog}(8, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (a^8\sqrt{-a^2+b^2}) + 10080b^2 \operatorname{polylog}(8, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (a^8\sqrt{-a^2+b^2}) + 10080b^2 \operatorname{polylog}(7, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) x^{1/2} / (a^7\sqrt{-a^2+b^2}) - 10080b^2 \operatorname{polylog}(7, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) x^{1/2} / (a^7\sqrt{-a^2+b^2})$

**Rubi [A]** time = 1.33, antiderivative size = 961, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{x^4}{4a} - \frac{2b \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} + \frac{2b \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a\sqrt{b^2-a^2} d} - \frac{14b \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} + \frac{14b \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) x^3}{a\sqrt{b^2-a^2} d^2} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3/(a + b\operatorname{Sech}[c + d\sqrt{x}]), x]$

[Out]  $\frac{x^4}{4a} - (2bx^{7/2} \operatorname{Log}[1 + (aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2})]) / (a\sqrt{-a^2 + b^2}d) + (2bx^{7/2} \operatorname{Log}[1 + (aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2})]) / (a\sqrt{-a^2 + b^2}d) - (14b^2x^3 \operatorname{PolyLog}[2, -((aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a^2\sqrt{-a^2 + b^2}d^2) + (14b^2x^3 \operatorname{PolyLog}[2, -((aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a^2\sqrt{-a^2 + b^2}d^2) + (84b^2x^{5/2} \operatorname{PolyLog}[3, -((aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a^3\sqrt{-a^2 + b^2}d^3) - (84b^2x^{5/2} \operatorname{PolyLog}[3, -((aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a^3\sqrt{-a^2 + b^2}d^3) - (420b^2x^2 \operatorname{PolyLog}[4, -((aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a^4\sqrt{-a^2 + b^2}d^4) + (420b^2x^2 \operatorname{PolyLog}[4, -((aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a^4\sqrt{-a^2 + b^2}d^4) + (1680b^2x^{3/2} \operatorname{PolyLog}[5, -((aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a^5\sqrt{-a^2 + b^2}d^5) - (1680b^2x^{3/2} \operatorname{PolyLog}[5, -((aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a^5\sqrt{-a^2 + b^2}d^5) - (5040b^2x \operatorname{PolyLog}[6, -((aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a^6\sqrt{-a^2 + b^2}d^6) + (5040b^2x \operatorname{PolyLog}[6, -((aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a^6\sqrt{-a^2 + b^2}d^6) + (10080b^2 \operatorname{Sqrt}[x] \operatorname{PolyLog}[7, -((aE^{(c+d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a^7\sqrt{-a^2 + b^2}d^7) - (10080b^2 \operatorname{Sqrt}[x] \operatorname{PolyLog}[7, -((aE^{(c+d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a^7\sqrt{-a^2 + b^2}d^7)$

$$\frac{t[x]}{(b + \sqrt{-a^2 + b^2})} \Big/ \left( \frac{a\sqrt{-a^2 + b^2}d^7 - (10080b \text{PolyLog}[8, -((aE^{(c + d\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))])}{a\sqrt{-a^2 + b^2}d^8} + (10080b \text{PolyLog}[8, -((aE^{(c + d\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))])}{a\sqrt{-a^2 + b^2}d^8} \right)$$

#### Rule 2190

$$\text{Int}[\frac{((F_{}^((g_{})(e_{} + (f_{})(x_{})))^((n_{})*((c_{} + (d_{})(x_{}))^((m_{})))/((a_{} + (b_{})(F_{}^((g_{})(e_{} + (f_{})(x_{})))^((n_{}))), x\_Symbol]} :> \text{Simp}[\frac{((c + d*x)^m \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n]/a)]}{(b*f*g*n \text{Log}[F])}, x] - \text{Dist}[\frac{(d*m)}{(b*f*g*n \text{Log}[F])}, \text{Int}[(c + d*x)^{(m-1)} \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$$

#### Rule 2264

$$\text{Int}[\frac{(F_{}^((u_{})*((f_{} + (g_{})(x_{}))^((m_{})))/((a_{} + (b_{})(F_{}^((u_{} + (c_{})(F_{}^((v_{})))))^((v_{}))), x\_Symbol]} :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[\frac{(2*c)}{q}, \text{Int}[\frac{(f + g*x)^m F^u}{(b - q + 2*c F^u)}, x], x] - \text{Dist}[\frac{(2*c)}{q}, \text{Int}[\frac{(f + g*x)^m F^u}{(b + q + 2*c F^u)}, x], x]] /; \text{FreeQ}\{F, a, b, c, f, g\}, x\} \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 2282

$$\text{Int}[u, x\_Symbol] :> \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_{})*((a_{})(v_{}^((n_{})))^((m_{})) /; \text{FreeQ}\{a, m, n\}, x\} \&\& \text{IntegerQ}[m*n] \&\& \text{!MatchQ}[u, E^{((c_{})(a_{} + (b_{})(x_{})))* (F_{})[v_{}]} /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{InverseFunctionQ}[F[x]]]$$

#### Rule 2531

$$\text{Int}[\text{Log}[1 + (e_{})*((F_{}^((c_{})*((a_{} + (b_{})(x_{})))^((n_{})))*((f_{} + (g_{})(x_{}))^((m_{}))), x\_Symbol]} :> -\text{Simp}[\frac{((f + g*x)^m \text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n])]}{(b*c*n \text{Log}[F])}, x] + \text{Dist}[\frac{(g*m)}{(b*c*n \text{Log}[F])}, \text{Int}[(f + g*x)^{(m-1)} \text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0]$$

#### Rule 3320

$$\text{Int}[\frac{((c_{} + (d_{})(x_{}))^((m_{})))/((a_{} + (b_{})\sin[(e_{} + \text{Pi}*(k_{} + (\text{Complex}[0, fz_{}])*(f_{})(x_{}))])], x\_Symbol]} :> \text{Dist}[2, \text{Int}[\frac{((c + d*x)^m E^{-(I*e + f*fz*x)}}{(E^{(I*Pi*(k - 1/2))}*(b + (2*a E^{-(I*e + f*fz*x)})/E^{(I*Pi*(k - 1/2))} - (b E^{(2*(-I*e + f*fz*x)})/E^{(2*I*k*Pi))})}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, fz\}, x\} \&\& \text{IntegerQ}[2*k] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 4191

$$\text{Int}[(\text{csc}[(e_{} + (f_{})(x_{})]*(b_{} + (a_{}))^((n_{}))*((c_{} + (d_{})(x_{}))^((m_{}))), x\_Symbol]} :> \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, 1/(\text{Sin}[e + f*x]^n/(b + a*\text{Sin}[e + f*x]^n)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 5436

$$\text{Int}[(x_{}^((m_{}))*((a_{} + (b_{})\text{Sech}[(c_{} + (d_{})(x_{}))^((n_{}))])^((p_{}))), x\_Symbol]} :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Sech}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x\} \&\& \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \&\& \text{IntegerQ}[p]$$

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(p_.)]], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x]
&& GtQ[m, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^7}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^7}{a} - \frac{bx^7}{a(b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^4}{4a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^7}{b + a \cosh(c + dx)} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^4}{4a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^7}{a + 2be^{c+dx} + ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^4}{4a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^7}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^7}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{(14b) \operatorname{Subst} \left( \int \frac{e^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} dx, x, \sqrt{x} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}} \\
&= \frac{x^4}{4a} - \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{7/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{14bx^3 \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2}}
\end{aligned}$$

**Mathematica [A]** time = 3.25, size = 939, normalized size = 0.98

$$\left( b + a \cosh(c + d\sqrt{x}) \right) \left( x^4 - \frac{8be^c \left( x^{7/2} \log \left( \frac{e^{2c+d\sqrt{x}} a}{be^c - \sqrt{(b^2 - a^2)e^{2c}} + 1} \right) d^7 - x^{7/2} \log \left( \frac{e^{2c+d\sqrt{x}} a}{e^c b + \sqrt{(b^2 - a^2)e^{2c}} + 1} \right) d^7 + 7x^3 \operatorname{Li}_2 \left( -\frac{ae^{2c+d\sqrt{x}}}{be^c - \sqrt{(b^2 - a^2)e^{2c}}} \right) d^6 - 7x^3 \operatorname{Li}_2 \left( -\frac{ae^{2c+d\sqrt{x}}}{e^c b + \sqrt{(b^2 - a^2)e^{2c}}} \right) d^6 \right)}{a\sqrt{-a^2 + b^2} d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Sech[c + d\*Sqrt[x]]),x]

[Out] ((b + a\*Cosh[c + d\*Sqrt[x]]\*(x^4 - (8\*b\*E^c\*(d^7\*x^(7/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - d^7\*x^(7/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 7\*d^6\*x^3\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 7\*d^6\*x^3\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 42\*d^5\*x^(5/2)\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 42\*d^5\*x^(5/2)\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 210\*d^4\*x^2\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 210\*d^4\*x^2\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 840\*d^3\*x^(3/2)\*PolyLog[5, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 840\*d^3\*x^(3/2)\*PolyLog[5, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 2520\*d^2\*x\*PolyLog[6, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 2520\*d^2\*x\*PolyLog[6, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 5040\*d\*Sqrt[x]\*PolyLog[7, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 5040\*d\*Sqrt[x]\*PolyLog[7, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) + 5040\*PolyLog[8, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))]) - 5040\*PolyLog[8, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))])]/(d^8\*Sqrt[(-a^2 + b^2)\*E^(2\*c)])\*Sech[c + d\*Sqrt[x]]/(4\*a\*(a + b\*Sech[c + d\*Sqrt[x]]))

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^3}{b \operatorname{sech}(d\sqrt{x} + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(x^3/(b\*sech(d\*sqrt(x) + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(x^3/(b\*sech(d\*sqrt(x) + c) + a), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(x^3/(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a + \frac{b}{\cosh(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b/cosh(c + d\*x^(1/2))),x)

[Out] int(x^3/(a + b/cosh(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*sech(c+d\*x\*\*(1/2))),x)

[Out] Integral(x\*\*3/(a + b\*sech(c + d\*sqrt(x))), x)

### 3.43 $\int \frac{x^2}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

**Optimal.** Leaf size=721

$$\frac{240b\operatorname{Li}_6\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} + \frac{240b\operatorname{Li}_6\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} + \frac{240b\sqrt{x}\operatorname{Li}_5\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{240b\sqrt{x}\operatorname{Li}_5\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{120bx\operatorname{Li}_4\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{120bx\operatorname{Li}_4\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}$$

[Out]  $\frac{1}{3}x^3/a - 2bx^{5/2} \ln(1+a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (b-\sqrt{-a^2+b^2}) / a/d / (-a^2+b^2)^{1/2} + 2bx^{5/2} \ln(1+a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (b+\sqrt{-a^2+b^2}) / a/d / (-a^2+b^2)^{1/2} - 10bx^2 \operatorname{polylog}(2, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (b-\sqrt{-a^2+b^2}) / a/d^2 / (-a^2+b^2)^{1/2} + 10bx^2 \operatorname{polylog}(2, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (b+\sqrt{-a^2+b^2}) / a/d^2 / (-a^2+b^2)^{1/2} + 40bx^{3/2} \operatorname{polylog}(3, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (b-\sqrt{-a^2+b^2}) / a/d^3 / (-a^2+b^2)^{1/2} - 40bx^{3/2} \operatorname{polylog}(3, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (b+\sqrt{-a^2+b^2}) / a/d^3 / (-a^2+b^2)^{1/2} - 120bx \operatorname{polylog}(4, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (b-\sqrt{-a^2+b^2}) / a/d^4 / (-a^2+b^2)^{1/2} + 120bx \operatorname{polylog}(4, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (b+\sqrt{-a^2+b^2}) / a/d^4 / (-a^2+b^2)^{1/2} - 240bx \operatorname{polylog}(6, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (b-\sqrt{-a^2+b^2}) / a/d^6 / (-a^2+b^2)^{1/2} + 240bx \operatorname{polylog}(6, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (b+\sqrt{-a^2+b^2}) / a/d^6 / (-a^2+b^2)^{1/2} + 240bx \operatorname{polylog}(5, -a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2})) / (b-\sqrt{-a^2+b^2}) / a/d^5 / (-a^2+b^2)^{1/2} - 240bx \operatorname{polylog}(5, -a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2})) / (b+\sqrt{-a^2+b^2}) / a/d^5 / (-a^2+b^2)^{1/2}$

**Rubi [A]** time = 1.05, antiderivative size = 721, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{10bx^2\operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{10bx^2\operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{40bx^{3/2}\operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{40bx^{3/2}\operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2/(a + b\operatorname{Sech}[c + d\sqrt{x}]), x]$

[Out]  $x^3/(3*a) - (2*b*x^{5/2}*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2})]) / (a*\sqrt{-a^2 + b^2}*d) + (2*b*x^{5/2}*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2})]) / (a*\sqrt{-a^2 + b^2}*d) - (10*b*x^2*\operatorname{PolyLog}[2, -((a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^2) + (10*b*x^2*\operatorname{PolyLog}[2, -((a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^2) + (40*b*x^{3/2}*\operatorname{PolyLog}[3, -((a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^3) - (40*b*x^{3/2}*\operatorname{PolyLog}[3, -((a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^3) - (120*b*x*\operatorname{PolyLog}[4, -((a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^4) + (120*b*x*\operatorname{PolyLog}[4, -((a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^4) + (240*b*\sqrt{x}*\operatorname{PolyLog}[5, -((a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^5) - (240*b*\sqrt{x}*\operatorname{PolyLog}[5, -((a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^5) - (240*b*\operatorname{PolyLog}[6, -((a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^6) + (240*b*\operatorname{PolyLog}[6, -((a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2}))]) / (a*\sqrt{-a^2 + b^2}*d^6)$

**Rule 2190**

$\operatorname{Int}[\frac{(F_*)^{(g_*)} * ((e_*) + (f_*) * (x_*)))^{(n_*)} * ((c_*) + (d_*) * (x_*))^{(m_*)}}{((a_*) + (b_*) * ((F_*)^{(g_*)} * ((e_*) + (f_*) * (x_*)))^{(n_*)})}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d*x)^m * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x))))^n]/a]}{(b*f*g*n*\operatorname{Log}[F])}, x] - \operatorname{Dist}[\frac{(d*m)}{(b*f*g*n*\operatorname{Log}[F])}, \operatorname{Int}[(c + d*x)^{m-1} * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x))))^n]]]$

))<sup>n</sup>)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)<sup>(u\_)</sup>\*((f\_.) + (g\_.)\*(x\_)<sup>(m\_.)</sup>)/((a\_.) + (b\_.)\*(F\_)<sup>(u\_)</sup> + (c\_.)\*  
\*(F\_)<sup>(v\_)</sup>), x\_Symbol] := With[{q = Rt[b<sup>2</sup> - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[  
((f + g\*x)<sup>m</sup>\*F<sup>u</sup>)/(b - q + 2\*c\*F<sup>u</sup>), x], x] - Dist[(2\*c)/q, Int[  
((f + g\*x)<sup>m</sup>\*F<sup>u</sup>)/(b + q + 2\*c\*F<sup>u</sup>), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,  
2\*u] && LinearQ[u, x] && NeQ[b<sup>2</sup> - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]  
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi  
onOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)<sup>(n\_)</sup>)<sup>(m\_)</sup> /; FreeQ[  
{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E<sup>((c\_.)\*(a\_.) + (b\_.)\*x)</sup>\*  
(F\_)<sup>[v\_]</sup> /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)<sup>((c\_.)\*(a\_.) + (b\_.)\*(x\_)))<sup>(n\_.)</sup>]\*((f\_.) + (g\_.)  
\*(x\_)<sup>(m\_.)</sup>), x\_Symbol] := -Simp[((f + g\*x)<sup>m</sup>\*PolyLog[2, -(e\*(F<sup>(c\*(a + b\*x))</sup>  
)<sup>n</sup>)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)<sup>(m -  
1)</sup>\*PolyLog[2, -(e\*(F<sup>(c\*(a + b\*x))</sup>)<sup>n</sup>], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]</sup>

#### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_)<sup>(m\_.)</sup>)/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Comple  
x[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)<sup>m</sup>\*E<sup>-(I\*e) +  
f\*fz\*x)</sup>]/(E<sup>(I\*Pi\*(k - 1/2))</sup>\*(b + (2\*a\*E<sup>-(I\*e) + f\*fz\*x)</sup>)/E<sup>(I\*Pi\*(k - 1/  
2)) - (b\*E<sup>(2\*(-I\*e) + f\*fz\*x)</sup>)/E<sup>(2\*I\*k\*Pi)</sup>), x], x] /; FreeQ[{a, b, c,  
d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a<sup>2</sup> - b<sup>2</sup>, 0] && IGtQ[m, 0]</sup>

#### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))<sup>(n\_.)</sup>\*((c\_.) + (d\_.)\*(x\_)<sup>(m\_.)</sup>  
, x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)<sup>m</sup>, 1/(Sin[e + f\*x]<sup>n</sup>/(b + a\*Si  
n[e + f\*x]<sup>n</sup>), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt  
Q[m, 0]

#### Rule 5436

Int[(x\_)<sup>(m\_.)</sup>\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)<sup>(n\_)])<sup>(p\_.)</sup>, x\_Symbo  
l] := Dist[1/n, Subst[Int[x<sup>(Simplify[(m + 1)/n] - 1)</sup>\*(a + b\*Sech[c + d\*x])  
<sup>p</sup>, x], x, x<sup>n</sup>], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m  
+ 1)/n], 0] && IntegerQ[p]</sup>

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)<sup>(p\_.)</sup>]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)<sup>p</sup>]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_)<sup>(m\_.)</sup>\*PolyLog[n\_, (d\_.)\*((F\_)<sup>((c\_.)\*(a\_.) + (b\_.  
)\*(x\_)))<sup>(p\_.)</sup>], x\_Symbol] := Simp[((e + f\*x)<sup>m</sup>\*PolyLog[n + 1, d\*(F<sup>(c\*(a  
+ b\*x))</sup>)<sup>p</sup>]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)<sup>^</sup></sup>

$(m - 1) * \text{PolyLog}[n + 1, d * (F^{(c * (a + b * x)))^p], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^5}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right) \\
 &= 2 \operatorname{Subst} \left( \int \left( \frac{x^5}{a} - \frac{bx^5}{a(b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
 &= \frac{x^3}{3a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^5}{b + a \cosh(c + dx)} dx, x, \sqrt{x} \right)}{a} \\
 &= \frac{x^3}{3a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^5}{a + 2be^{c+dx} + ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
 &= \frac{x^3}{3a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^5}{2b - 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^5}{2b + 2\sqrt{-a^2 + b^2} + 2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2 + b^2}} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{(10b) \operatorname{Subst} \left( \int x^4}{\sqrt{-a^2 + b^2}} \right)}{\sqrt{-a^2 + b^2}} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} \\
 &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} + \frac{2bx^{5/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d} - \frac{10bx^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a\sqrt{-a^2 + b^2} d}
 \end{aligned}$$

**Mathematica [A]** time = 2.18, size = 744, normalized size = 1.03

$$d^6 x^3 \sqrt{e^{2c} (b^2 - a^2)} - 6be^c d^5 x^{5/2} \log \left( \frac{ae^{2c+d\sqrt{x}}}{be^c - \sqrt{e^{2c} (b^2 - a^2)}} + 1 \right) + 6be^c d^5 x^{5/2} \log \left( \frac{ae^{2c+d\sqrt{x}}}{\sqrt{e^{2c} (b^2 - a^2)} + be^c} + 1 \right) - 30be^c d^4 x^2 \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Sech[c + d\*sqrt[x]]),x]

```
[Out] (d^6*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^3 - 6*b*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c) + d*Sqrt[x])]/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 6*b*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - 30*b*d^4*E^c*x^2*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 30*b*d^4*E^c*x^2*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 120*b*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 120*b*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 360*b*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 360*b*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 720*b*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 720*b*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 720*b*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 720*b*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))]/(3*a*d^6*Sqrt[(-a^2 + b^2)*E^(2*c)])
```

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{b \operatorname{sech}(d\sqrt{x} + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*sech(d*sqrt(x) + c) + a), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*sech(d*sqrt(x) + c) + a), x)
```

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*sech(c+d*x^(1/2))),x)
```

```
[Out] int(x^2/(a+b*sech(c+d*x^(1/2))),x)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a + \frac{b}{\cosh(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b/cosh(c + d\*x^(1/2))), x)

[Out] int(x^2/(a + b/cosh(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*sech(c+d\*x\*\*(1/2))), x)

[Out] Integral(x\*\*2/(a + b\*sech(c + d\*sqrt(x))), x)



$$3.44 \quad \int \frac{x}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=481

$$\frac{12b\operatorname{Li}_4\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{12b\operatorname{Li}_4\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{12b\sqrt{x}\operatorname{Li}_3\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{12b\sqrt{x}\operatorname{Li}_3\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{6bx\operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{6bx\operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}$$

[Out]  $1/2*x^2/a-2*b*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)}+2*b*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)}-6*b*x*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d^2/(-a^2+b^2)^{(1/2)}+6*b*x*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d^2/(-a^2+b^2)^{(1/2)}-12*b*polylog(4,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d^4/(-a^2+b^2)^{(1/2)}+12*b*polylog(4,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d^4/(-a^2+b^2)^{(1/2)}+12*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a/d^3/(-a^2+b^2)^{(1/2)}-12*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a/d^3/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 0.86, antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6bx\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{6bx\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sech[c + d\*Sqrt[x]]), x]

[Out]  $x^2/(2*a) - (2*b*x^{(3/2)}*\log[1 + (a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^{(3/2)}*\log[1 + (a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (6*b*x*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (6*b*x*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (12*b*Sqrt[x]*PolyLog[3, -((a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*Sqrt[x]*PolyLog[3, -((a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*PolyLog[4, -((a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (12*b*PolyLog[4, -((a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4)$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2264**

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*((F\_)^(v\_))), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

**Rule 2282**

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3320

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^3}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^3}{a} - \frac{bx^3}{a(b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^2}{2a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^3}{b+a \cosh(c+dx)} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^2}{2a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^3}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{x^2}{2a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2+b^2}} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^3}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2+b^2}} \\
&= \frac{x^2}{2a} - \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{(6b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^3}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2+b^2}} \\
&= \frac{x^2}{2a} - \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{x^2}{2a} - \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{x^2}{2a} - \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{x^2}{2a} - \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{x^2}{2a} - \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}}
\end{aligned}$$

**Mathematica [A]** time = 21.46, size = 508, normalized size = 1.06

$$d^4 x^2 \sqrt{e^{2c} (b^2 - a^2)} - 4be^c d^3 x^{3/2} \log \left( \frac{ae^{2c+d\sqrt{x}}}{be^c - \sqrt{e^{2c}(b^2 - a^2)}} + 1 \right) + 4be^c d^3 x^{3/2} \log \left( \frac{ae^{2c+d\sqrt{x}}}{\sqrt{e^{2c}(b^2 - a^2)} + be^c} + 1 \right) - 12be^c d^2 x \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Sech[c + d\*Sqrt[x]]), x]

[Out] (d^4\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^2 - 4\*b\*d^3\*E^c\*x^(3/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + 4\*b\*d^3\*E^c\*x^(3/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 12\*b\*d^2\*E^c\*x\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])))] + 12\*b\*d^2\*E^c\*x\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])))] + 24\*b\*d\*E^c\*Sqrt[x]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])))] - 24\*b\*d\*E^c\*Sqrt[x]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])))] - 24\*b\*E^c\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])))] + 24\*b\*E^c\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])))])/(2\*a\*d^4\*Sqrt[(-a^2 + b^2)\*E^(2\*c)])

**fricas** [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{b \operatorname{sech}(d\sqrt{x} + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(x/(b\*sech(d\*sqrt(x) + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(x/(b\*sech(d\*sqrt(x) + c) + a), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(x/(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a + \frac{b}{\cosh(c + d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b/cosh(c + d\*x^(1/2))),x)

[Out] int(x/(a + b/cosh(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(c+d\*x\*\*(1/2))),x)

[Out] Integral(x/(a + b\*sech(c + d\*sqrt(x))), x)

$$3.45 \quad \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

**Optimal.** Leaf size=23

$$\operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*sech(c+d\*x^(1/2))), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

[Out] Defer[Int][1/(x\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

**Mathematica [A]** time = 5.57, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

[Out] Integrate[1/(x\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

**fricas [A]** time = 0.45, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bx \operatorname{sech}(d\sqrt{x} + c) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(1/(b\*x\*sech(d\*sqrt(x) + c) + a\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(c+d\*x^(1/2))), x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*sqrt(x) + c) + a)\*x), x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a+b*sech(c+d*x^(1/2))),x)`

[Out] `int(1/x/(a+b*sech(c+d*x^(1/2))),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-2b \int \frac{e^{(d\sqrt{x}+c)}}{a^2xe^{(2d\sqrt{x}+2c)} + 2abxe^{(d\sqrt{x}+c)} + a^2x} dx + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

[Out] `-2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x*e^(2*d*sqrt(x) + 2*c) + 2*a*b*x*e^(d*sqrt(x) + c) + a^2*x), x) + log(x)/a`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b/cosh(c + d*x^(1/2))))),x)`

[Out] `int(1/(x*(a + b/cosh(c + d*x^(1/2))))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*sech(c+d*x**(1/2))),x)`

[Out] `Integral(1/(x*(a + b*sech(c + d*sqrt(x))))), x)`

$$3.46 \quad \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$$

Optimal. Leaf size=26

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+d\sqrt{x})}{x^2}, x\right) - \frac{a}{x}$$

[Out]  $-a/x + b*\operatorname{Unintegrable}(\operatorname{sech}(c+d*x^{(1/2)})/x^2, x)$

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $-(a/x) + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]]/x^2, x]]$

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx &= \int \left( \frac{a}{x^2} + \frac{b\operatorname{sech}(c + d\sqrt{x})}{x^2} \right) dx \\ &= -\frac{a}{x} + b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^2} dx \end{aligned}$$

Mathematica [A] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^2, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^2, x]$

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{sech}(d\sqrt{x} + c) + a}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\operatorname{sech}(c+d*x^{(1/2)}))/x^2, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b*\operatorname{sech}(d*\operatorname{sqrt}(x) + c) + a)/x^2, x)$

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)/x^2, x)

**maple** [A] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))/x^2,x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{e^{(d\sqrt{x}+c)}}{x^2 e^{(2d\sqrt{x}+2c)} + x^2} dx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^2,x, algorithm="maxima")

[Out] 2\*b\*integrate(e^(d\*sqrt(x) + c)/(x^2\*e^(2\*d\*sqrt(x) + 2\*c) + x^2), x) - a/x

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))/x^2,x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))/x\*\*2,x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))/x\*\*2, x)



$$3.47 \quad \int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=2851

result too large to display

```
[Out] 10080*b^2*polylog(7,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)
/d^8+10080*b^2*polylog(7,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2
-b^2)/d^8+10080*b^3*polylog(8,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2
/(-a^2+b^2)^(3/2)/d^8-10080*b^3*polylog(8,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)
^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8-20160*b*polylog(8,-a*exp(c+d*x^(1/2))/(b-
(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)+20160*b*polylog(8,-a*exp(c+d*x^
(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)+2*b^2*x^(7/2)/a^2/(a^
2-b^2)/d+840*b*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/
d^4/(-a^2+b^2)^(1/2)+3360*b*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+
b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-3360*b*x^(3/2)*polylog(5,-a*exp(c+d*x
^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-10080*b*x*polylog(6,
-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+10080*b*
x*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1
/2)-10080*b^2*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a
^2/(a^2-b^2)/d^7-10080*b^2*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2
)))*x^(1/2)/a^2/(a^2-b^2)/d^7-10080*b^3*polylog(7,-a*exp(c+d*x^(1/2))/(b-(-
a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7+10080*b^3*polylog(7,-a*ex
p(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7+20160
*b*polylog(7,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^7/(-a^
2+b^2)^(1/2)-20160*b*polylog(7,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^
(1/2)/a^2/d^7/(-a^2+b^2)^(1/2)-14*b^2*x^3*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+
b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+2*b^3*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-14*b^2*x^3*ln(1+a*exp(c+d*x^(1/2))/(
b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-2*b^3*x^(7/2)*ln(1+a*exp(c+d*x^(1/2)
))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-84*b^2*x^(5/2)*polylog(2,-a*
exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+14*b^3*x^3*polylog
(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-84*b^
2*x^(5/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)
/d^3-14*b^3*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(3/2)/d^2+420*b^2*x^2*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(
1/2)))/a^2/(a^2-b^2)/d^4-84*b^3*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+420*b^2*x^2*polylog(3,-a*exp(c+d*
x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+84*b^3*x^(5/2)*polylog(3,-
a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-1680*b^2*
x^(3/2)*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d
^5+420*b^3*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^
2+b^2)^(3/2)/d^4-1680*b^2*x^(3/2)*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^
2)^(1/2)))/a^2/(a^2-b^2)/d^5-420*b^3*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(
-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+5040*b^2*x*polylog(5,-a*exp(c+d*
x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-1680*b^3*x^(3/2)*polylog(5
,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+5040*b^
2*x*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^6+1
680*b^3*x^(3/2)*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a
^2+b^2)^(3/2)/d^5+5040*b^3*x*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(3/2)/d^6-5040*b^3*x*polylog(6,-a*exp(c+d*x^(1/2))/(b+
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6-4*b*x^(7/2)*ln(1+a*exp(c+d*x^(1
/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+4*b*x^(7/2)*ln(1+a*exp(c+
d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-28*b*x^3*polylog(2,
-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+28*b*x^3
*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/
2)+168*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^
3/(-a^2+b^2)^(1/2)-168*b*x^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2
)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-840*b*x^2*polylog(4,-a*exp(c+d*x^(1/2)))/
```

$(b - (-a^2 + b^2)^{1/2}) / a^2 / d^4 / (-a^2 + b^2)^{1/2} + 1/4 * x^4 / a^2 + 2 * b^2 * x^{7/2} * \sinh(c + d * x^{1/2}) / a / (a^2 - b^2) / d / (b + a * \cosh(c + d * x^{1/2}))$

**Rubi [A]** time = 3.78, antiderivative size = 2851, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2531, 6609, 2282, 6589, 5562}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out]  $(2 * b^2 * x^{7/2}) / (a^2 * (a^2 - b^2) * d) + x^4 / (4 * a^2) - (14 * b^2 * x^3 * \text{Log}[1 + (a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^2) + (2 * b^3 * x^{7/2} * \text{Log}[1 + (a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d) - (4 * b * x^{7/2} * \text{Log}[1 + (a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d) - (14 * b^2 * x^3 * \text{Log}[1 + (a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^2) - (2 * b^3 * x^{7/2} * \text{Log}[1 + (a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d) + (4 * b * x^{7/2} * \text{Log}[1 + (a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d) - (84 * b^2 * x^{5/2} * \text{PolyLog}[2, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^3) + (14 * b^3 * x^3 * \text{PolyLog}[2, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^2) - (28 * b * x^3 * \text{PolyLog}[2, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^2) - (84 * b^2 * x^{5/2} * \text{PolyLog}[2, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^3) - (14 * b^3 * x^3 * \text{PolyLog}[2, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^2) + (28 * b * x^3 * \text{PolyLog}[2, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^2) + (420 * b^2 * x^2 * \text{PolyLog}[3, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^4) - (84 * b^3 * x^{5/2} * \text{PolyLog}[3, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^3) + (168 * b * x^{5/2} * \text{PolyLog}[3, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^3) + (420 * b^2 * x^2 * \text{PolyLog}[3, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^4) + (84 * b^3 * x^{5/2} * \text{PolyLog}[3, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^3) - (168 * b * x^{5/2} * \text{PolyLog}[3, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^3) - (1680 * b^2 * x^{3/2} * \text{PolyLog}[4, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^5) + (420 * b^3 * x^2 * \text{PolyLog}[4, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^4) - (840 * b * x^2 * \text{PolyLog}[4, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^4) - (1680 * b^2 * x^{3/2} * \text{PolyLog}[4, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^5) - (420 * b^3 * x^2 * \text{PolyLog}[4, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^4) + (840 * b * x^2 * \text{PolyLog}[4, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^4) + (5040 * b^2 * x * \text{PolyLog}[5, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^6) - (1680 * b^3 * x^{3/2} * \text{PolyLog}[5, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^5) + (3360 * b * x^{3/2} * \text{PolyLog}[5, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^5) + (5040 * b^2 * x * \text{PolyLog}[5, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^6) + (1680 * b^3 * x^{3/2} * \text{PolyLog}[5, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^5) - (3360 * b * x^{3/2} * \text{PolyLog}[5, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^5) - (10080 * b^2 * \text{Sqrt}[x] * \text{PolyLog}[6, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^7) + (5040 * b^3 * x * \text{PolyLog}[6, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^6) - (10080 * b * x * \text{PolyLog}[6, -(a * E^c + d * \text{Sqrt}[x]) / (b - \text{Sqrt}[-a^2 + b^2])]) / (a^2 * \text{Sqrt}[-a^2 + b^2] * d^6) - (10080 * b^2 * \text{Sqrt}[x] * \text{PolyLog}[6, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (a^2 - b^2) * d^7) - (5040 * b^3 * x * \text{PolyLog}[6, -(a * E^c + d * \text{Sqrt}[x]) / (b + \text{Sqrt}[-a^2 + b^2])]) / (a^2 * (-a^2 + b^2)^{3/2} * d^6)$

$$\begin{aligned} & )^{(3/2)*d^6} + (10080*b*x*PolyLog[6, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^6) + (10080*b^2*PolyLog[7, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(a^2 - b^2)*d^8) - (10080*b^3*Sqrt[x]*PolyLog[7, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^{(3/2)*d^7} + (20160*b*Sqrt[x]*PolyLog[7, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^7) + (10080*b^2*PolyLog[7, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(a^2 - b^2)*d^8) + (10080*b^3*Sqrt[x]*PolyLog[7, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^{(3/2)*d^7} - (20160*b*Sqrt[x]*PolyLog[7, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^7) + (10080*b^3*PolyLog[8, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^{(3/2)*d^8} - (20160*b*PolyLog[8, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^8) - (10080*b^3*PolyLog[8, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^{(3/2)*d^8} + (20160*b*PolyLog[8, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^8) + (2*b^2*x^{(7/2)*Sinh[c + d*Sqrt[x]]/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*Sqrt[x]])) \end{aligned}$$

#### Rule 2190

$$\begin{aligned} & \text{Int}[\text{((F\_)}^{\text{(g\_)}*(\text{e\_}) + (\text{f\_})*(\text{x\_}))}^{\text{(n\_)}*(\text{(c\_)} + (\text{d\_})*(\text{x\_}))}^{\text{(m\_)}}/ \\ & \text{((a\_)} + (\text{b\_})*(\text{F\_})^{\text{(g\_)}*(\text{e\_}) + (\text{f\_})*(\text{x\_}))}^{\text{(n\_)}}, \text{x\_Symbol}] \text{:> Simp} \\ & [((\text{c} + \text{d*x})^{\text{m}}*\text{Log}[1 + (\text{b}*(\text{F}^{\text{(g*(e + f*x))}^{\text{n}})/\text{a})]/(\text{b*f*g*n*Log}[F]), \text{x}] - \text{Dist} \\ & [(\text{d*m})/(\text{b*f*g*n*Log}[F]), \text{Int}[(\text{c} + \text{d*x})^{\text{(m - 1)}*Log}[1 + (\text{b}*(\text{F}^{\text{(g*(e + f*x))}^{\text{n}})/\text{a})], \text{x}], \text{x}] \text{/; FreeQ}\{\text{F}, \text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{g}, \text{n}\}, \text{x}\} \&\& \text{IGtQ}\{\text{m}, 0\} \end{aligned}$$

#### Rule 2264

$$\begin{aligned} & \text{Int}[\text{((F\_)}^{\text{(u\_)}*(\text{f\_}) + (\text{g\_})*(\text{x\_}))}^{\text{(m\_)}}/(\text{(a\_)} + (\text{b\_})*(\text{F\_})^{\text{(u\_)} + (\text{c\_})} \\ & *(\text{F\_})^{\text{(v\_)}}, \text{x\_Symbol}] \text{:> With}\{\text{q} = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/\text{q}, \text{Int} \\ & [((\text{f} + \text{g*x})^{\text{m}}*\text{F}^{\text{u}})/(\text{b} - \text{q} + 2*c*\text{F}^{\text{u}}), \text{x}], \text{x}] - \text{Dist}[(2*c)/\text{q}, \text{Int} \\ & [((\text{f} + \text{g*x})^{\text{m}}*\text{F}^{\text{u}})/(\text{b} + \text{q} + 2*c*\text{F}^{\text{u}}), \text{x}], \text{x}]] \text{/; FreeQ}\{\text{F}, \text{a}, \text{b}, \text{c}, \text{f}, \text{g}\}, \text{x}\} \&\& \text{EqQ}\{\text{v}, \\ & 2*u\} \&\& \text{LinearQ}\{\text{u}, \text{x}\} \&\& \text{NeQ}\{b^2 - 4*a*c, 0\} \&\& \text{IGtQ}\{\text{m}, 0\} \end{aligned}$$

#### Rule 2282

$$\begin{aligned} & \text{Int}[\text{u\_}, \text{x\_Symbol}] \text{:> With}\{\text{v} = \text{FunctionOfExponential}[\text{u}, \text{x}]\}, \text{Dist}[\text{v}/\text{D}[\text{v}, \text{x}] \\ & , \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[\text{u}, \text{x}]/\text{x}, \text{x}], \text{x}, \text{v}], \text{x}]] \text{/; FunctionOfExponentialQ}\{\text{u}, \text{x}\} \&\& \\ & \text{!MatchQ}\{\text{u}, (\text{w\_})*(\text{a\_})*(\text{v\_})^{\text{(n\_)}}^{\text{(m\_)}}\} \text{/; FreeQ}\{\text{a}, \text{m}, \text{n}\}, \text{x}\} \&\& \text{IntegerQ}\{\text{m*n}\} \&\& \text{!MatchQ}\{\text{u}, \text{E}^{\text{((c\_)}*(\text{a\_}) + (\text{b\_})*\text{x}))} \\ & (\text{F\_})[\text{v\_}]\} \text{/; FreeQ}\{\text{a}, \text{b}, \text{c}\}, \text{x}\} \&\& \text{InverseFunctionQ}\{\text{F}[\text{x}]\} \end{aligned}$$

#### Rule 2531

$$\begin{aligned} & \text{Int}[\text{Log}[1 + (\text{e\_})*(\text{F\_})^{\text{(c\_)}*(\text{a\_}) + (\text{b\_})*(\text{x\_}))}^{\text{(n\_)}}]*(\text{(f\_)} + (\text{g\_}) \\ & *(\text{x\_}))^{\text{(m\_)}}, \text{x\_Symbol}] \text{:> -Simp}[\text{((f} + \text{g*x})^{\text{m}}*\text{PolyLog}[2, -(\text{e}*(\text{F}^{\text{(c*(a + b*x))}^{\text{n}})) \\ & )^{\text{n}}]/(\text{b*c*n*Log}[F]), \text{x}] + \text{Dist}[(\text{g*m})/(\text{b*c*n*Log}[F]), \text{Int}[(\text{f} + \text{g*x})^{\text{(m - 1)}* \\ & \text{PolyLog}[2, -(\text{e}*(\text{F}^{\text{(c*(a + b*x))}^{\text{n}}))], \text{x}], \text{x}]] \text{/; FreeQ}\{\text{F}, \text{a}, \text{b}, \text{c}, \text{e}, \text{f}, \\ & \text{g}, \text{n}\}, \text{x}\} \&\& \text{GtQ}\{\text{m}, 0\} \end{aligned}$$

#### Rule 3320

$$\begin{aligned} & \text{Int}[\text{((c\_)} + (\text{d\_})*(\text{x\_}))^{\text{(m\_)}}/(\text{(a\_)} + (\text{b\_})*\text{sin}[(\text{e\_}) + \text{Pi}*(\text{k\_}) + (\text{Complex}[0, \text{fz\_}]]*(\text{f\_})*(\text{x\_}))], \text{x\_Symbol}] \text{:> Dist}[2, \text{Int} \\ & [((\text{c} + \text{d*x})^{\text{m}}*\text{E}^{\text{-((I*e) + f*fz*x)}})/(\text{E}^{\text{(I*Pi*(k - 1/2))}*(\text{b} + (2*a*\text{E}^{\text{-((I*e) + f*fz*x)}})/\text{E}^{\text{(I*Pi*(k - 1/2))}} \\ & - (\text{b}*\text{E}^{\text{(2*(-(I*e) + f*fz*x))}})/\text{E}^{\text{(2*I*k*Pi)}}), \text{x}], \text{x}]] \text{/; FreeQ}\{\text{a}, \text{b}, \text{c}, \\ & \text{d}, \text{e}, \text{f}, \text{fz}\}, \text{x}\} \&\& \text{IntegerQ}\{2*k\} \&\& \text{NeQ}\{a^2 - b^2, 0\} \&\& \text{IGtQ}\{\text{m}, 0\} \end{aligned}$$

#### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*cos[e + f*x])/(f*(a^2 - b^2)*(a + b*sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*cos[e + f*x])/(a
+ b*sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

#### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

#### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

#### Rule 5562

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_
.)*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rubi steps



**Mathematica** [A] time = 18.74, size = 3033, normalized size = 1.06

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out]  $(x^4(b + a\cosh[c + d\sqrt{x}])^2\operatorname{sech}[c + d\sqrt{x}]^2)/(4a^2(a + b\operatorname{sech}[c + d\sqrt{x}])^2) + (2bE^c(b + a\cosh[c + d\sqrt{x}])^2(2bE^c x^{7/2} + ((1 + E^{2c}) * (-7bd^6\sqrt{(-a^2 + b^2)E^{2c}}) * x^3 \operatorname{Log}[1 + (aE^{2c} + d\sqrt{x})]/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})]) - 2a^2d^7E^c x^{7/2} \operatorname{Log}[1 + (aE^{2c} + d\sqrt{x})]/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})]) + b^2d^7E^c x^{7/2} \operatorname{Log}[1 + (aE^{2c} + d\sqrt{x})]/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})]) - 7bd^6\sqrt{(-a^2 + b^2)E^{2c}} * x^3 \operatorname{Log}[1 + (aE^{2c} + d\sqrt{x})]/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})]) + 2a^2d^7E^c x^{7/2} \operatorname{Log}[1 + (aE^{2c} + d\sqrt{x})]/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})]) - b^2d^7E^c x^{7/2} \operatorname{Log}[1 + (aE^{2c} + d\sqrt{x})]/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})]) + 7d^5(-6b\sqrt{(-a^2 + b^2)E^{2c}} - 2a^2dE^c\sqrt{x} + b^2dE^c\sqrt{x}) * x^{5/2} \operatorname{PolyLog}[2, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 7d^5(6b\sqrt{(-a^2 + b^2)E^{2c}} - 2a^2dE^c\sqrt{x} + b^2dE^c\sqrt{x}) * x^{5/2} \operatorname{PolyLog}[2, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 210bd^4\sqrt{(-a^2 + b^2)E^{2c}} * x^2 \operatorname{PolyLog}[3, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 84a^2d^5E^c x^{5/2} \operatorname{PolyLog}[3, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 42b^2d^5E^c x^{5/2} \operatorname{PolyLog}[3, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 210bd^4\sqrt{(-a^2 + b^2)E^{2c}} * x^2 \operatorname{PolyLog}[3, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 84a^2d^5E^c x^{5/2} \operatorname{PolyLog}[3, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 42b^2d^5E^c x^{5/2} \operatorname{PolyLog}[3, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 840bd^3\sqrt{(-a^2 + b^2)E^{2c}} * x^{3/2} \operatorname{PolyLog}[4, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 420a^2d^4E^c x^2 \operatorname{PolyLog}[4, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 210b^2d^4E^c x^2 \operatorname{PolyLog}[4, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 840bd^3\sqrt{(-a^2 + b^2)E^{2c}} * x^{3/2} \operatorname{PolyLog}[4, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 420a^2d^4E^c x^2 \operatorname{PolyLog}[4, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 210b^2d^4E^c x^2 \operatorname{PolyLog}[4, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 2520bd^2\sqrt{(-a^2 + b^2)E^{2c}} * x \operatorname{PolyLog}[5, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 1680a^2d^3E^c x^{3/2} \operatorname{PolyLog}[5, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 840b^2d^3E^c x^{3/2} \operatorname{PolyLog}[5, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 2520bd^2\sqrt{(-a^2 + b^2)E^{2c}} * x \operatorname{PolyLog}[5, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 1680a^2d^3E^c x^{3/2} \operatorname{PolyLog}[5, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 840b^2d^3E^c x^{3/2} \operatorname{PolyLog}[5, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 5040bd\sqrt{(-a^2 + b^2)E^{2c}} * \sqrt{x} \operatorname{PolyLog}[6, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 5040a^2d^2E^c x \operatorname{PolyLog}[6, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 2520b^2d^2E^c x \operatorname{PolyLog}[6, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 5040bd\sqrt{(-a^2 + b^2)E^{2c}} * \sqrt{x} \operatorname{PolyLog}[6, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 5040a^2d^2E^c x \operatorname{PolyLog}[6, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 2520b^2d^2E^c x \operatorname{PolyLog}[6, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 5040b\sqrt{(-a^2 + b^2)E^{2c}} * \operatorname{PolyLog}[7, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] + 10080a^2dE^c\sqrt{x} \operatorname{PolyLog}[7, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 5040b^2dE^c\sqrt{x} \operatorname{PolyLog}[7, -((aE^{2c} + d\sqrt{x})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}})))] - 5040b^2dE^c\sqrt{x} \operatorname{PolyLog}[7, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] + 10080a^2dE^c\sqrt{x} \operatorname{PolyLog}[7, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))] - 5040b^2dE^c\sqrt{x} \operatorname{PolyLog}[7, -((aE^{2c} + d\sqrt{x})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}})))]$

$y \log[7, -((aE^{2c + d\sqrt{x}})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}}))] + 5040b\sqrt{(-a^2 + b^2)E^{2c}} \text{PolyLog}[7, -((aE^{2c + d\sqrt{x}})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}}))] - 10080a^2dE^c\sqrt{x} \text{PolyLog}[7, -((aE^{2c + d\sqrt{x}})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}}))] + 5040b^2dE^c\sqrt{x} \text{PolyLog}[7, -((aE^{2c + d\sqrt{x}})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}}))] - 10080a^2E^c \text{PolyLog}[8, -((aE^{2c + d\sqrt{x}})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}}))] + 5040b^2E^c \text{PolyLog}[8, -((aE^{2c + d\sqrt{x}})/(bE^c - \sqrt{(-a^2 + b^2)E^{2c}}))] + 10080a^2E^c \text{PolyLog}[8, -((aE^{2c + d\sqrt{x}})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}}))] - 5040b^2E^c \text{PolyLog}[8, -((aE^{2c + d\sqrt{x}})/(bE^c + \sqrt{(-a^2 + b^2)E^{2c}}))] / (d^7E^c\sqrt{(-a^2 + b^2)E^{2c}}) \text{sech}[c + d\sqrt{x}]^2 / (a^2(a^2 - b^2)d*(1 + E^{2c})*(a + b\text{sech}[c + d\sqrt{x}])^2 + (2*(b + a\text{Cosh}[c + d\sqrt{x}])\text{sech}[c]\text{sech}[c + d\sqrt{x}]^2*(b^3x^{7/2}\text{Sinh}[c] - a*b^2x^{7/2}\text{Sinh}[d\sqrt{x}])) / (a^2*(-a + b)*(a + b)*d*(a + b\text{sech}[c + d\sqrt{x}])^2)$

**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^3}{b^2 \text{sech}(d\sqrt{x} + c)^2 + 2ab \text{sech}(d\sqrt{x} + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x^3/(b^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sech(d\*sqrt(x) + c) + a^2), x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.57Not invertible Error: Bad Argument Value

**maple** [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \text{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x^3/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* h

elp (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a + \frac{b}{\cosh(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^3/(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*3/(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)



$$3.48 \quad \int \frac{x^2}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=2123

result too large to display

```
[Out] 240*b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d
^6+240*b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2
)/d^6+240*b^3*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d^6-240*b^3*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2))
)/a^2/(-a^2+b^2)^(3/2)/d^6-480*b*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)
^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+480*b*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-
a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+2*b^2*x^(5/2)/a^2/(a^2-b^2)/d+240
*b*x*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)
^(1/2)-240*b^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/
a^2/(a^2-b^2)/d^5-240*b^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)
))*x^(1/2)/a^2/(a^2-b^2)/d^5-240*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2
+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^5+240*b^3*polylog(5,-a*exp(c+d
*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^5+480*b*poly
log(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^5/(-a^2+b^2)^(
1/2)-480*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2
/d^5/(-a^2+b^2)^(1/2)-10*b^2*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2
)))/a^2/(a^2-b^2)/d^2+2*b^3*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d-10*b^2*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b
^2)^(1/2)))/a^2/(a^2-b^2)/d^2-2*b^3*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-40*b^2*x^(3/2)*polylog(2,-a*exp(c+d*x
^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+10*b^3*x^2*polylog(2,-a*exp
(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-40*b^2*x^(3/2)
*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-10*b
^3*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(
3/2)/d^2+120*b^2*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/
(a^2-b^2)/d^4-40*b^3*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+1/3*x^3/a^2+2*b^2*x^(5/2)*sinh(c+d*x^(1/2))/
a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))+120*b^2*x*polylog(3,-a*exp(c+d*x^(1/2)
))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+40*b^3*x^(3/2)*polylog(3,-a*exp(
c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+120*b^3*x*polyl
og(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-120
*b^3*x*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(
3/2)/d^4-4*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-
a^2+b^2)^(1/2)+4*b*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^
2/d/(-a^2+b^2)^(1/2)-20*b*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(
1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+20*b*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-
a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+80*b*x^(3/2)*polylog(3,-a*exp(c+
d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-80*b*x^(3/2)*poly
log(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-24
0*b*x*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)
^(1/2)
```

**Rubi [A]** time = 2.98, antiderivative size = 2123, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2531, 6609, 2282, 6589, 5562}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^(5/2))/(a^2*(a^2 - b^2)*d) + x^3/(3*a^2) - (10*b^2*x^2*Log[1 + (a*
E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*
```

$$\begin{aligned}
& x^{(5/2)} \cdot \text{Log}\left[1 + \frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]}\right] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d) - (4 \cdot b \cdot x^{(5/2)} \cdot \text{Log}\left[1 + \frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]}\right]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d) - (10 \cdot b^2 \cdot x^2 \cdot \text{Log}\left[1 + \frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]}\right]) / (b + \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot (a^2 - b^2) \cdot d^2) - (2 \cdot b^3 \cdot x^{(5/2)} \cdot \text{Log}\left[1 + \frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]}\right]) / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d) \\
& + (4 \cdot b \cdot x^{(5/2)} \cdot \text{Log}\left[1 + \frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]}\right]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d) - (40 \cdot b^2 \cdot x^{(3/2)} \cdot \text{PolyLog}[2, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^3) + (10 \cdot b^3 \cdot x^2 \cdot \text{PolyLog}[2, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^2) \\
& - (20 \cdot b \cdot x^2 \cdot \text{PolyLog}[2, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^2) - (40 \cdot b^2 \cdot x^{(3/2)} \cdot \text{PolyLog}[2, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^3) - (10 \cdot b^3 \cdot x^2 \cdot \text{PolyLog}[2, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^2) \\
& + (20 \cdot b \cdot x^2 \cdot \text{PolyLog}[2, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^2) + (120 \cdot b^2 \cdot x \cdot \text{PolyLog}[3, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (b - \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot (a^2 - b^2) \cdot d^4) - (40 \cdot b^3 \cdot x^{(3/2)} \cdot \text{PolyLog}[3, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^3) \\
& + (80 \cdot b \cdot x^{(3/2)} \cdot \text{PolyLog}[3, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^3) + (120 \cdot b^2 \cdot x \cdot \text{PolyLog}[3, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^4) + (40 \cdot b^3 \cdot x^{(3/2)} \cdot \text{PolyLog}[3, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^3) \\
& - (80 \cdot b \cdot x^{(3/2)} \cdot \text{PolyLog}[3, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^3) - (240 \cdot b^2 \cdot \text{Sqrt}[x] \cdot \text{PolyLog}[4, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^5) + (120 \cdot b^3 \cdot x \cdot \text{PolyLog}[4, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^4) \\
& - (240 \cdot b \cdot x \cdot \text{PolyLog}[4, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (b - \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^4) - (240 \cdot b^2 \cdot \text{Sqrt}[x] \cdot \text{PolyLog}[4, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^5) \\
& - (120 \cdot b^3 \cdot x \cdot \text{PolyLog}[4, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (b + \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^4) + (240 \cdot b \cdot x \cdot \text{PolyLog}[4, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (b + \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^4) + (240 \cdot b^2 \cdot \text{PolyLog}[5, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^6) \\
& - (240 \cdot b^3 \cdot \text{Sqrt}[x] \cdot \text{PolyLog}[5, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^5) + (480 \cdot b \cdot \text{Sqrt}[x] \cdot \text{PolyLog}[5, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (b - \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^5) + (240 \cdot b^2 \cdot \text{PolyLog}[5, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (a^2 - b^2) \cdot d^6) \\
& + (240 \cdot b^3 \cdot \text{Sqrt}[x] \cdot \text{PolyLog}[5, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (b + \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^5) - (480 \cdot b \cdot \text{Sqrt}[x] \cdot \text{PolyLog}[5, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (b + \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^5) + (240 \cdot b^3 \cdot \text{PolyLog}[6, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^6) \\
& - (480 \cdot b \cdot \text{PolyLog}[6, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b - \text{Sqrt}[-a^2 + b^2]})] / (b - \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^6) - (240 \cdot b^3 \cdot \text{PolyLog}[6, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (a^2 \cdot (-a^2 + b^2)^{(3/2)} \cdot d^6) + (480 \cdot b \cdot \text{PolyLog}[6, -\frac{a \cdot E^{(c + d \cdot \text{Sqrt}[x])}}{b + \text{Sqrt}[-a^2 + b^2]})] / (b + \text{Sqrt}[-a^2 + b^2]) / (a^2 \cdot \text{Sqrt}[-a^2 + b^2] \cdot d^6) \\
& + (2 \cdot b^2 \cdot x^{(5/2)} \cdot \text{Sinh}[c + d \cdot \text{Sqrt}[x]]) / (a \cdot (a^2 - b^2) \cdot d \cdot (b + a \cdot \text{Cosh}[c + d \cdot \text{Sqrt}[x]]))
\end{aligned}$$

#### Rule 2190

$$\begin{aligned}
& \text{Int}\left[\frac{((F_)^{((g_.) \cdot ((e_.) + (f_.) \cdot (x_)))})^{(n_.)} \cdot ((c_.) + (d_.) \cdot (x_))^{(m_.)}}{((a_.) + (b_.) \cdot ((F_)^{((g_.) \cdot ((e_.) + (f_.) \cdot (x_)))})^{(n_.)})}, x\_Symbol] \rightarrow \text{Simp} \\
& \left[\frac{(c + d \cdot x)^m \cdot \text{Log}\left[1 + \frac{b \cdot (F^{(g \cdot (e + f \cdot x)))^n}{a}}{b \cdot f \cdot g \cdot n \cdot \text{Log}[F]}\right]}{(b \cdot f \cdot g \cdot n \cdot \text{Log}[F])}, x\right] - \text{Dist} \\
& \left[\frac{(d \cdot m)}{b \cdot f \cdot g \cdot n \cdot \text{Log}[F]}, \text{Int}\left[\frac{(c + d \cdot x)^{(m-1)} \cdot \text{Log}\left[1 + \frac{b \cdot (F^{(g \cdot (e + f \cdot x)))^n}{a}}{b \cdot f \cdot g \cdot n \cdot \text{Log}[F]}\right]}{a}, x\right], x\right] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]
\end{aligned}$$

#### Rule 2264

$$\begin{aligned}
& \text{Int}\left[\frac{(F_)^{(u_)} \cdot ((f_.) + (g_.) \cdot (x_))^{(m_.)}}{((a_.) + (b_.) \cdot (F_)^{(u_)} + (c_.) \cdot (F_)^{(v_)}), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4 \cdot a \cdot c, 2]\}, \text{Dist}\left[\frac{(2 \cdot c)}{q}, \text{Int}\left[\frac{(f + g \cdot x)^m \cdot F^u}{(b - q + 2 \cdot c \cdot F^u)}, x\right], x\right] - \text{Dist}\left[\frac{(2 \cdot c)}{q}, \text{Int}\left[\frac{(f + g \cdot x)^m \cdot F^u}{(b + q + 2 \cdot c \cdot F^u)}, x\right], x\right]
\end{aligned}$$

$m \cdot F^u / (b + q + 2 \cdot c \cdot F^u), x], x]] /;$  FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_))))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :=> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m-1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 3320

Int[((c\_) + (d\_)\*(x\_))^(m\_)/((a\_) + (b\_)\*sin[(e\_) + Pi\*(k\_) + (Complex[0, fz\_] \* (f\_)\*(x\_))]), x\_Symbol] :=> Dist[2, Int[((c + d\*x)^m \* E^(-(I\*e) + f\*fz\*x))/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-(I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/2)) - (b\*E^(2\*(-(I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 3324

Int[((c\_) + (d\_)\*(x\_))^(m\_)/((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)]^2, x\_Symbol] :=> Simp[(b\*(c + d\*x)^m \* Cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*Sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*Sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m-1)\*Cos[e + f\*x])/(a + b\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 4191

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(b\_) + (a\_))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :=> Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 5436

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*Sech[(c\_) + (d\_)\*(x\_)]^(n\_))^(p\_), x\_Symbol] :=> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 5562

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*Sinh[(c\_) + (d\_)\*(x\_)]/(Cosh[(c\_) + (d\_)\*(x\_)]\*(b\_) + (a\_)), x\_Symbol] :=> -Simp[(e + f\*x)^(m+1)/(b\*f\*(m+1)), x] + (Int[((e + f\*x)^m \* E^(c + d\*x))/(a - Rt[a^2 - b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m \* E^(c + d\*x))/(a + Rt[a^2 - b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol]
:> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x]
&& GtQ[m, 0]
```

### Rubi steps



```
[Out] (x^3*(b + a*Cosh[c + d*Sqrt[x]])^2*Sech[c + d*Sqrt[x]]^2)/(3*a^2*(a + b*Sech[c + d*Sqrt[x]]^2) + (2*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])^2*(2*b*E^c*x^(5/2) + ((1 + E^(2*c))*(-5*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - 5*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 5*d^3*(-4*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 5*d^3*(4*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 60*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 40*a^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 20*b^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 60*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 40*a^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 20*b^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 120*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 120*a^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 60*b^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 120*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 120*a^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 60*b^2*d^2*E^c*x*PolyLog[4, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 120*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 240*a^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 120*b^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 120*b*Sqrt[(-a^2 + b^2)*E^(2*c)]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 240*a^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 120*b^2*d*E^c*Sqrt[x]*PolyLog[5, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 240*a^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 120*b^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 240*a^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 120*b^2*E^c*PolyLog[6, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])])]/(d^5*E^c*Sqrt[(-a^2 + b^2)*E^(2*c)])*Sech[c + d*Sqrt[x]]^2)/(a^2*(a^2 - b^2)*d*(1 + E^(2*c))*(a + b*Sech[c + d*Sqrt[x]]^2) + (2*(b + a*Cosh[c + d*Sqrt[x]])*Sech[c]*Sech[c + d*Sqrt[x]]^2*(b^3*x^(5/2)*Sinh[c] - a*b^2*x^(5/2)*Sinh[d*Sqrt[x]]))/(a^2*(-a + b)*(a + b)*d*(a + b*Sech[c + d*Sqrt[x]]^2)
```

**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^2}{b^2 \operatorname{sech}(d\sqrt{x} + c)^2 + 2ab \operatorname{sech}(d\sqrt{x} + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2),
```

x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] Exception raised: AttributeError >> type

**maple** [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + b \operatorname{sech}(c + d\sqrt{x})\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see 'assume?' for more details)Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^2/(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + b \operatorname{sech}(c + d\sqrt{x})\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*2/(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)

$$3.49 \quad \int \frac{x}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=1395

$$\frac{2x^{3/2} \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{2x^{3/2} \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{6x \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{6x \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{12\sqrt{x} \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{12\sqrt{x} \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2}$$

[Out]  $12*b^2*\operatorname{polylog}(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^4+12*b^2*\operatorname{polylog}(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^4+12*b^3*\operatorname{polylog}(4,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^4-12*b^3*\operatorname{polylog}(4,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^4-24*b*\operatorname{polylog}(4,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/d^4/(-a^2+b^2)^{(1/2)}+24*b*\operatorname{polylog}(4,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/d^4/(-a^2+b^2)^{(1/2)}+2*b^2*x^{(3/2)}/a^2/(a^2-b^2)/d+1/2*x^2/a^2+2*b^2*x^{(3/2)}*\sinh(c+d*x^{(1/2)})/a/(a^2-b^2)/d/(b+a*\cosh(c+d*x^{(1/2)}))-6*b^2*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^2+2*b^3*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d-6*b^2*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^2-2*b^3*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d+6*b^3*x*\operatorname{polylog}(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^2-6*b^3*x*\operatorname{polylog}(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^2-4*b*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/d/(-a^2+b^2)^{(1/2)}+4*b*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/d/(-a^2+b^2)^{(1/2)}-12*b*x*\operatorname{polylog}(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/d^2/(-a^2+b^2)^{(1/2)}+12*b*x*\operatorname{polylog}(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/d^2/(-a^2+b^2)^{(1/2)}-12*b^2*\operatorname{polylog}(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(a^2-b^2)/d^3-12*b^2*\operatorname{polylog}(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(a^2-b^2)/d^3-12*b^3*\operatorname{polylog}(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^3+12*b^3*\operatorname{polylog}(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^3+24*b*\operatorname{polylog}(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/d^3/(-a^2+b^2)^{(1/2)}-24*b*\operatorname{polylog}(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/d^3/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 2.31, antiderivative size = 1395, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2531, 6609, 2282, 6589, 5562}

$$\frac{2x^{3/2} \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{2x^{3/2} \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{6x \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{6x \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Sech[c + d\*Sqrt[x]])^2, x]

[Out]  $(2*b^2*x^{(3/2)})/(a^2*(a^2 - b^2)*d) + x^2/(2*a^2) - (6*b^2*x*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2})])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*x^{(3/2)}*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2})])/(a^2*(-a^2 + b^2)^{(3/2)*d}) - (4*b*x^{(3/2)}*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2})])/(a^2*\sqrt{-a^2 + b^2}*d) - (6*b^2*x*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2})])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x^{(3/2)}*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2})])/(a^2*(-a^2 + b^2)^{(3/2)*d}) + (4*b*x^{(3/2)}*\operatorname{Log}[1 + (a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2})])/(a^2*\sqrt{-a^2 + b^2}*d) - (12*b^2*\sqrt{x}*\operatorname{PolyLog}[2, -(a*E^{(c + d*\sqrt{x})})/(b - \sqrt{-a^2 + b^2})])/(a^2*(a^2 - b^2)*d) + (12*b^2*\sqrt{x}*\operatorname{PolyLog}[2, -(a*E^{(c + d*\sqrt{x})})/(b + \sqrt{-a^2 + b^2})])/(a^2*(a^2 - b^2)*d)$



$$\begin{aligned} & t[-a^2 + b^2]))]/(a^2*(a^2 - b^2)*d^3) + (6*b^3*x*PolyLog[2, -((a*E^(c + d \\ & *Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (12*b*x \\ & *PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^ \\ & 2 + b^2]*d^2) - (12*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqr \\ & t[-a^2 + b^2]))]/(a^2*(a^2 - b^2)*d^3) - (6*b^3*x*PolyLog[2, -((a*E^(c + d \\ & *Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (12*b*x \\ & *PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^ \\ & 2 + b^2]*d^2) + (12*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + \\ & b^2]))]/(a^2*(a^2 - b^2)*d^4) - (12*b^3*Sqrt[x]*PolyLog[3, -((a*E^(c + d* \\ & Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^3) + (24*b*Sq \\ & rt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqr \\ & t[-a^2 + b^2]*d^3) + (12*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[- \\ & a^2 + b^2]))]/(a^2*(a^2 - b^2)*d^4) + (12*b^3*Sqrt[x]*PolyLog[3, -((a*E^(c \\ & + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^3) - (24 \\ & *b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^ \\ & 2*Sqrt[-a^2 + b^2]*d^3) + (12*b^3*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - S \\ & qrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (24*b*PolyLog[4, -((a*E^ \\ & (c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^4) - (12 \\ & *b^3*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^ \\ & 2 + b^2)^(3/2)*d^4) + (24*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a \\ & ^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^4) + (2*b^2*x^(3/2)*Sinh[c + d*Sqrt[x \\ & ]])/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*Sqrt[x]])) \end{aligned}$$
Rule 2190

$$\begin{aligned} & \text{Int}[\text{((F\_)}^{\text{(g\_)}* \text{(e\_)} + \text{(f\_)}* \text{(x\_)}))^{\text{(n\_)}* \text{(c\_)} + \text{(d\_)}* \text{(x\_)}^{\text{(m\_)}}/ \\ & \text{((a\_)} + \text{(b\_)}* \text{(F\_)}^{\text{(g\_)}* \text{(e\_)} + \text{(f\_)}* \text{(x\_)}))^{\text{(n\_)}}, \text{x\_Symbol}] \text{:>} \text{Simp} \\ & [ \text{((c + d*x)}^{\text{m}}* \text{Log}[1 + \text{(b*(F}^{\text{g*(e + f*x))}}^{\text{n}}/\text{a})]/\text{(b*f*g*n*Log[F])}, \text{x}] - \text{Di} \\ & \text{st}[\text{(d*m)}/\text{(b*f*g*n*Log[F])}, \text{Int}[\text{(c + d*x)}^{\text{(m - 1)}}* \text{Log}[1 + \text{(b*(F}^{\text{g*(e + f*x))}}^{\text{n}} \\ & )]/\text{a}], \text{x}], \text{x}] \text{/; FreeQ}\{\text{F}, \text{a}, \text{b}, \text{c}, \text{d}, \text{e}, \text{f}, \text{g}, \text{n}\}, \text{x}\} \&\& \text{IGtQ}\{\text{m}, 0\} \end{aligned}$$
Rule 2264

$$\begin{aligned} & \text{Int}[\text{((F\_)}^{\text{(u\_)}* \text{(f\_)} + \text{(g\_)}* \text{(x\_)}^{\text{(m\_)}})/\text{((a\_)} + \text{(b\_)}* \text{(F\_)}^{\text{(u\_)} + \text{(c\_)} \\ & * \text{(F\_)}^{\text{(v\_)}}, \text{x\_Symbol}] \text{:>} \text{With}\{\text{q} = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[\text{(2*c)}/\text{q}, \text{Int} \\ & [ \text{((f + g*x)}^{\text{m}}* \text{F}^{\text{u}})/\text{(b - q + 2*c*F}^{\text{u}}), \text{x}], \text{x}] - \text{Dist}[\text{(2*c)}/\text{q}, \text{Int}[\text{((f + g*x)}^{\text{m}} \\ & * \text{F}^{\text{u}})/\text{(b + q + 2*c*F}^{\text{u}}), \text{x}], \text{x}]] \text{/; FreeQ}\{\text{F}, \text{a}, \text{b}, \text{c}, \text{f}, \text{g}\}, \text{x}\} \&\& \text{EqQ}\{\text{v}, \\ & 2*u\} \&\& \text{LinearQ}\{\text{u}, \text{x}\} \&\& \text{NeQ}\{b^2 - 4*a*c, 0\} \&\& \text{IGtQ}\{\text{m}, 0\} \end{aligned}$$
Rule 2282

$$\begin{aligned} & \text{Int}[\text{u\_}, \text{x\_Symbol}] \text{:>} \text{With}\{\text{v} = \text{FunctionOfExponential}[\text{u}, \text{x}]\}, \text{Dist}[\text{v}/\text{D}[\text{v}, \text{x}] \\ & , \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[\text{u}, \text{x}]/\text{x}, \text{x}], \text{x}, \text{v}], \text{x}]] \text{/; Functi} \\ & \text{onOfExponentialQ}\{\text{u}, \text{x}\} \&\& \text{!MatchQ}\{\text{u}, \text{(w\_)}* \text{(a\_)}* \text{(v\_)}^{\text{(n\_)}}, \text{m}\} \text{/; FreeQ} \\ & \{\text{a}, \text{m}, \text{n}\}, \text{x}\} \&\& \text{IntegerQ}\{\text{m*n}\} \&\& \text{!MatchQ}\{\text{u}, \text{E}^{\text{(c\_)}* \text{(a\_)} + \text{(b\_)}* \text{x}}\} \\ & \text{(F\_)}[\text{v\_}] \text{/; FreeQ}\{\text{a}, \text{b}, \text{c}\}, \text{x}\} \&\& \text{InverseFunctionQ}\{\text{F}[\text{x}]\} \end{aligned}$$
Rule 2531

$$\begin{aligned} & \text{Int}[\text{Log}[1 + \text{(e\_)}* \text{(F\_)}^{\text{(c\_)}* \text{(a\_)} + \text{(b\_)}* \text{(x\_)}))^{\text{(n\_)}}* \text{(f\_)} + \text{(g\_)} \\ & * \text{(x\_)}^{\text{(m\_)}}, \text{x\_Symbol}] \text{:>} -\text{Simp}[\text{((f + g*x)}^{\text{m}}* \text{PolyLog}[2, -\text{(e*(F}^{\text{c*(a + b*x)}}^{\text{n}} \\ & ))^{\text{n}}]/\text{(b*c*n*Log[F])}, \text{x}] + \text{Dist}[\text{(g*m)}/\text{(b*c*n*Log[F])}, \text{Int}[\text{(f + g*x)}^{\text{(m - \\ & 1)}}* \text{PolyLog}[2, -\text{(e*(F}^{\text{c*(a + b*x)}}^{\text{n}})]^{\text{n}}], \text{x}], \text{x}]] \text{/; FreeQ}\{\text{F}, \text{a}, \text{b}, \text{c}, \text{e}, \text{f} \\ & , \text{g}, \text{n}\}, \text{x}\} \&\& \text{GtQ}\{\text{m}, 0\} \end{aligned}$$
Rule 3320

$$\begin{aligned} & \text{Int}[\text{((c\_)} + \text{(d\_)}* \text{(x\_)}^{\text{(m\_)}})/\text{((a\_)} + \text{(b\_)}* \text{sin}[\text{(e\_)} + \text{Pi*(k\_)} + \text{(Comple} \\ & \text{x}[0, \text{fz\_}]* \text{(f\_)}* \text{(x\_)}]), \text{x\_Symbol}] \text{:>} \text{Dist}[2, \text{Int}[\text{(c + d*x)}^{\text{m}}* \text{E}^{\text{-(I*e)} + \\ & \text{f*fz*x}})/\text{(E}^{\text{I*Pi*(k - 1/2)}}* \text{(b + (2*a*E}^{\text{-(I*e)} + \text{f*fz*x}})/\text{E}^{\text{I*Pi*(k - 1/} \end{aligned}$$

2)) - (b\*E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 3324

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^2, x\_Symbol] :> Simp[(b\*(c + d\*x)^m\*cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*sin[e + f\*x]), x], x] - Dist[(b\*d\*m)/(f\*(a^2 - b^2)), Int[((c + d\*x)^(m - 1)\*cos[e + f\*x])/(a + b\*sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, 1/(sin[e + f\*x]^n/(b + a\*sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*sech[(c\_.) + (d\_.)\*(x\_)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### Rule 5562

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*sinh[(c\_.) + (d\_.)\*(x\_)]/(cosh[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.)), x\_Symbol] :> -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 - b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 - b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{x^3}{(a + b \operatorname{sech}(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^3}{a^2} + \frac{b^2 x^3}{a^2 (b + a \cosh(c + dx))^2} - \frac{2bx^3}{a^2 (b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{x^2}{2a^2} - \frac{(4b) \operatorname{Subst} \left( \int \frac{x^3}{b+a \cosh(c+dx)} dx, x, \sqrt{x} \right)}{a^2} + \frac{(2b^2) \operatorname{Subst} \left( \int \frac{x^3}{(b+a \cosh(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{x^2}{2a^2} + \frac{2b^2 x^{3/2} \sinh(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cosh(c + d\sqrt{x}))} - \frac{(8b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^3}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} + \frac{2b^2 x^{3/2} \sinh(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cosh(c + d\sqrt{x}))} - \frac{(4b^3) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^3}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} - \frac{4bx^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^{3/2}}{a^2(a^2 - b^2)d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 17.01, size = 1393, normalized size = 1.00

$$(b + a \cosh(c + d\sqrt{x})) \operatorname{sech}^2(c + d\sqrt{x}) \left( \frac{4x^{3/2} \operatorname{sech}(c)(a \sinh(d\sqrt{x}) - b \sinh(c))b^2}{(a-b)(a+b)d} + \frac{4e^c(b+a \cosh(c+d\sqrt{x}))}{2be^c x^{3/2} + \frac{e^{-c}(1+e^{2c})}{b - \sqrt{-a^2 + b^2}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] ((b + a\*Cosh[c + d\*Sqrt[x]])\*Sech[c + d\*Sqrt[x]]^2\*(x^2\*(b + a\*Cosh[c + d\*Sqrt[x]]) + (4\*b\*E^c\*(b + a\*Cosh[c + d\*Sqrt[x]])\*(2\*b\*E^c\*x^(3/2) + ((1 + E^(2\*c))\*(-3\*b\*d^2\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 2\*a^2\*d^3\*E^c\*x^(3/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + b^2\*d^3\*E^c\*x^(3/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 3\*b\*d^2\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + 2\*a^2\*d^3\*E^c\*x^(3/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - b^2\*d^3\*E^c\*x^(3/2)\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + 3\*d\*(-2\*b\*Sqrt[(-a^2 + b^2)\*E^(2\*c)] - 2\*a^2\*d\*E^c\*Sqrt[x] + b^2\*d\*E^c\*Sqrt[x])\*Sqrt[x]\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 3\*d\*(2\*b\*Sqrt[(-a^2 + b^2)\*E^(2\*c)] - 2\*a^2\*d\*E^c\*Sqrt[x] + b^2\*d\*E^c\*Sqrt[x])\*Sqrt[x]\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 6\*b\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 12\*a^2\*d\*E^c\*Sqrt[x]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 6\*b^2\*d\*E^c\*Sqrt[x]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 6\*b\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 12\*a^2\*d\*E^c\*Sqrt[x]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 6\*b^2\*d\*E^c\*Sqrt[x]\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 12\*a^2\*E^c\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 6\*b^2\*E^c\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 12\*a^2\*E^c\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 6\*b^2\*E^c\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])])])/(d^3\*E^c\*Sqrt[(-a^2 + b^2)\*E^(2\*c)])/(a^2 - b^2)\*d\*(1 + E^(2\*c)) + (4\*b^2\*x^(3/2)\*Sech[c]\*(-b\*Sinh[c]) + a\*Sinh[d\*Sqrt[x]])/((a - b)\*(a + b)\*d))/(2\*a^2\*(a + b\*Sech[c + d\*Sqrt[x]])^2)

**fricas** [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x}{b^2 \operatorname{sech}(d\sqrt{x} + c)^2 + 2ab \operatorname{sech}(d\sqrt{x} + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x/(b^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sech(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x/(b\*sech(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*sech(c+d*x^(1/2)))^2,x)`

[Out] `int(x/(a+b*sech(c+d*x^(1/2)))^2,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a + \frac{b}{\cosh(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b/cosh(c + d*x^(1/2)))^2,x)`

[Out] `int(x/(a + b/cosh(c + d*x^(1/2)))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + b \operatorname{sech}(c + d\sqrt{x})\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*sech(c+d*x**(1/2)))**2,x)`

[Out] `Integral(x/(a + b*sech(c + d*sqrt(x)))**2, x)`

$$3.50 \quad \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

[Out] Defer[Int][1/(x\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Mathematica [A]** time = 120.97, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

[Out] Integrate[1/(x\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

**fricas [A]** time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{b^2x\operatorname{sech}(d\sqrt{x}+c)^2+2abx\operatorname{sech}(d\sqrt{x}+c)+a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x\*sech(d\*sqrt(x) + c) + a^2\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{sech}(d\sqrt{x}+c)+a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*sqrt(x) + c) + a)^2\*x), x)

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(1/x/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4 \left( b^3 \sqrt{x} e^{(d\sqrt{x}+c)} + ab^2 \sqrt{x} \right)}{\left( a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)} \right) x e^{(2d\sqrt{x})} + 2 \left( a^4 b d e^c - a^2 b^3 d e^c \right) x e^{(d\sqrt{x})} + \left( a^5 d - a^3 b^2 d \right) x} + \frac{\log(x)}{a^2} - \int \frac{1}{\left( a^5 d e^{(2c)} - a^3 b^2 d e^{(2c)} \right) x e^{(2d\sqrt{x})} + 2 \left( a^4 b d e^c - a^2 b^3 d e^c \right) x e^{(d\sqrt{x})} + \left( a^5 d - a^3 b^2 d \right) x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] -4\*(b^3\*sqrt(x)\*e^(d\*sqrt(x) + c) + a\*b^2\*sqrt(x))/((a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*x\*e^(2\*d\*sqrt(x)) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*x\*e^(d\*sqrt(x)) + (a^5\*d - a^3\*b^2\*d)\*x) + log(x)/a^2 - integrate(2\*(a\*b^2\*sqrt(x) + (b^3\*sqrt(x)\*e^c + (2\*a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x)\*e^(d\*sqrt(x)))/((a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*x^2\*e^(2\*d\*sqrt(x)) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*x^2\*e^(d\*sqrt(x)) + (a^5\*d - a^3\*b^2\*d)\*x^2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

[Out] int(1/(x\*(a + b/cosh(c + d\*x^(1/2))))^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(1/(x\*(a + b\*sech(c + d\*sqrt(x)))\*\*2), x)

$$3.51 \quad \int \frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2}, x\right)$$

[Out] Unintegrable(1/x^2/(a+b\*sech(c+d\*x^(1/2)))^2, x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

[Out] Defer[Int][1/(x^2\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

Rubi steps

$$\int \frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Mathematica [A] time = 71.53, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

[Out] Integrate[1/(x^2\*(a + b\*Sech[c + d\*Sqrt[x]]))^2], x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{b^2x^2\operatorname{sech}(d\sqrt{x}+c)^2+2abx^2\operatorname{sech}(d\sqrt{x}+c)+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*x^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^2\*sech(d\*sqrt(x) + c) + a^2\*x^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")



[Out] sage0\*x

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(1/x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4ab^2\sqrt{x} + (a^3de^{2c} - ab^2de^{2c})xe^{2d\sqrt{x}} + (a^3d - ab^2d)x + 2(2b^3\sqrt{x}e^c + (a^2bde^c - b^3de^c)x)e^{d\sqrt{x}}}{(a^5de^{2c} - a^3b^2de^{2c})x^2e^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)x^2e^{d\sqrt{x}} + (a^5d - a^3b^2d)x^2} \int \frac{1}{(a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out]  $-(4*a*b^2*\sqrt{x} + (a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*x*e^{(2*d*\sqrt{x})}) + (a^3*d - a*b^2*d)*x + 2*(2*b^3*\sqrt{x}*e^c + (a^2*b*d*e^c - b^3*d*e^c)*x)*e^{(d*\sqrt{x})} / ((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^2*e^{(2*d*\sqrt{x})} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^{(d*\sqrt{x})} + (a^5*d - a^3*b^2*d)*x^2) - \int \frac{1}{(a^5de^{2c} - a^3b^2de^{2c})x^2e^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)x^2e^{d\sqrt{x}} + (a^5d - a^3b^2d)x^2} dx$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

[Out] int(1/(x^2\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*sech(c + d\*sqrt(x)))\*\*2), x)

### 3.52 $\int x^{3/2} \left( a + b \operatorname{sech} \left( c + d\sqrt{x} \right) \right) dx$

**Optimal.** Leaf size=254

$$\frac{2}{5}ax^{5/2} + \frac{48ib\operatorname{Li}_5(-ie^{c+d\sqrt{x}})}{d^5} - \frac{48ib\operatorname{Li}_5(ie^{c+d\sqrt{x}})}{d^5} - \frac{48ib\sqrt{x}\operatorname{Li}_4(-ie^{c+d\sqrt{x}})}{d^4} + \frac{48ib\sqrt{x}\operatorname{Li}_4(ie^{c+d\sqrt{x}})}{d^4} + \frac{24ibx\operatorname{Li}_3(-ie^{c+d\sqrt{x}})}{d^3}$$

[Out]  $\frac{2}{5}a*x^{5/2} + 4*b*x^2*\arctan(\exp(c+d*x^{1/2}))/d - 8*I*b*x^{3/2}*polylog(2, -I*\exp(c+d*x^{1/2}))/d^2 + 8*I*b*x^{3/2}*polylog(2, I*\exp(c+d*x^{1/2}))/d^2 + 24*I*b*x*polylog(3, -I*\exp(c+d*x^{1/2}))/d^3 - 24*I*b*x*polylog(3, I*\exp(c+d*x^{1/2}))/d^3 + 48*I*b*polylog(5, -I*\exp(c+d*x^{1/2}))/d^5 - 48*I*b*polylog(5, I*\exp(c+d*x^{1/2}))/d^5 - 48*I*b*polylog(4, -I*\exp(c+d*x^{1/2}))*x^{1/2}/d^4 + 48*I*b*polylog(4, I*\exp(c+d*x^{1/2}))*x^{1/2}/d^4$

**Rubi [A]** time = 0.22, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {14, 5436, 4180, 2531, 6609, 2282, 6589}

$$-\frac{8ibx^{3/2}\operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2}\operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{24ibx\operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{24ibx\operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^{3/2}*(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]]), x]$

[Out]  $(2*a*x^{5/2})/5 + (4*b*x^2*\operatorname{ArcTan}[E^{(c + d*\operatorname{Sqrt}[x])}])/d - ((8*I)*b*x^{3/2}*\operatorname{PolyLog}[2, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((8*I)*b*x^{3/2}*\operatorname{PolyLog}[2, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((24*I)*b*x*\operatorname{PolyLog}[3, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 - ((24*I)*b*x*\operatorname{PolyLog}[3, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 - ((48*I)*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[4, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^4 + ((48*I)*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[4, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^4 + ((48*I)*b*\operatorname{PolyLog}[5, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^5 - ((48*I)*b*\operatorname{PolyLog}[5, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^5$

#### Rule 14

$\operatorname{Int}[(u_)*((c_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /;$   $\operatorname{FreeQ}\{c, m\}, x \&\& \operatorname{SumQ}[u] \&\& !\operatorname{LinearQ}[u, x] \&\& !\operatorname{MatchQ}[u, (a_)+(b_)*(v_)] /;$   $\operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{InverseFunctionQ}[v]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$   $\operatorname{FunctionOfExponentialQ}[u, x] \&\& !\operatorname{MatchQ}[u, (w_)*((a_)*(v_))^{(n_)}]^{(m_)} /;$   $\operatorname{FreeQ}\{a, m, n\}, x \&\& \operatorname{IntegerQ}[m*n] \&\& !\operatorname{MatchQ}[u, E^{((c_)*((a_)+(b_)*x))}*(F_)] [v_] /;$   $\operatorname{FreeQ}\{a, b, c\}, x \&\& \operatorname{InverseFunctionQ}[F[x]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_)*((F_))^{((c_)*((a_)+(b_)*(x_)))^{(n_)}]}*((f_)+(g_)*(x_))^{(m_)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)]/(b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^{(m-1)}*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n)], x], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, e, f, g, n\}, x \&\& \operatorname{GtQ}[m, 0]$

#### Rule 4180

$\operatorname{Int}[\operatorname{csc}[(e_)+(Pi)*(k_)+(Complex[0, fz_])*(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)})/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1$

- E<sup>-</sup>(-I\*e) + f\*fz\*x)/E<sup>(I\*k\*Pi)]</sup>, x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)<sup>(m - 1)</sup>\*Log[1 + E<sup>-</sup>(-I\*e) + f\*fz\*x)/E<sup>(I\*k\*Pi)]</sup>, x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 5436

Int[(x\_)<sup>(m\_)</sup>\*((a\_) + (b\_) \* Sech[(c\_) + (d\_) \* (x\_)<sup>(n\_)</sup>])<sup>(p\_)</sup>, x\_Symbol] :> Dist[1/n, Subst[Int[x<sup>(Simplify[(m + 1)/n] - 1)</sup> \* (a + b \* Sech[c + d \* x])<sup>p</sup>, x], x, x<sup>n</sup>], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rule 6589

Int[PolyLog[n\_, (c\_) \* ((a\_) + (b\_) \* (x\_)<sup>(p\_)</sup>)] / ((d\_) + (e\_) \* (x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c \* (a + b \* x)<sup>p</sup>] / (e \* p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b \* d, a \* e]

#### Rule 6609

Int[((e\_) + (f\_) \* (x\_)<sup>(m\_)</sup> \* PolyLog[n\_, (d\_) \* ((F\_)<sup>(c\_)</sup> \* ((a\_) + (b\_) \* (x\_))])<sup>(p\_)</sup>, x\_Symbol] :> Simp[((e + f \* x)<sup>m</sup> \* PolyLog[n + 1, d \* (F<sup>c</sup> \* (a + b \* x))<sup>p</sup>] / (b \* c \* p \* Log[F]), x] - Dist[(f \* m) / (b \* c \* p \* Log[F]), Int[(e + f \* x)<sup>(m - 1)</sup> \* PolyLog[n + 1, d \* (F<sup>c</sup> \* (a + b \* x))<sup>p</sup>], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
 \int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x})) dx &= \int (ax^{3/2} + bx^{3/2} \operatorname{sech}(c + d\sqrt{x})) dx \\
 &= \frac{2}{5} ax^{5/2} + b \int x^{3/2} \operatorname{sech}(c + d\sqrt{x}) dx \\
 &= \frac{2}{5} ax^{5/2} + (2b) \operatorname{Subst}\left(\int x^4 \operatorname{sech}(c + dx) dx, x, \sqrt{x}\right) \\
 &= \frac{2}{5} ax^{5/2} + \frac{4bx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{(8ib) \operatorname{Subst}\left(\int x^3 \log(1 - ie^{c+dx}) dx, x, \sqrt{x}\right)}{d} \\
 &= \frac{2}{5} ax^{5/2} + \frac{4bx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8ibx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2}{5} ax^{5/2} + \frac{4bx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8ibx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2}{5} ax^{5/2} + \frac{4bx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8ibx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2}{5} ax^{5/2} + \frac{4bx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8ibx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\
 &= \frac{2}{5} ax^{5/2} + \frac{4bx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8ibx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 2.05, size = 288, normalized size = 1.13

$$\frac{2(ad^5 x^{5/2} + 5ibd^4 x^2 \log(1 - ie^{c+d\sqrt{x}}) - 5ibd^4 x^2 \log(1 + ie^{c+d\sqrt{x}}) - 20ibd^3 x^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}}) + 20ibd^3 x^{3/2} \operatorname{Li}_2(ie^{c+d\sqrt{x}}))}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(3/2)</sup> \* (a + b \* Sech[c + d \* Sqrt[x]]), x]

[Out]  $(2*(a*d^5*x^{(5/2)} + (5*I)*b*d^4*x^2*\text{Log}[1 - I*E^{(c + d*\text{Sqrt}[x])}] - (5*I)*b*d^4*x^2*\text{Log}[1 + I*E^{(c + d*\text{Sqrt}[x])}] - (20*I)*b*d^3*x^{(3/2)}*\text{PolyLog}[2, (-I)*E^{(c + d*\text{Sqrt}[x])}] + (20*I)*b*d^3*x^{(3/2)}*\text{PolyLog}[2, I*E^{(c + d*\text{Sqrt}[x])}] + (60*I)*b*d^2*x*\text{PolyLog}[3, (-I)*E^{(c + d*\text{Sqrt}[x])}] - (60*I)*b*d^2*x*\text{PolyLog}[3, I*E^{(c + d*\text{Sqrt}[x])}] - (120*I)*b*d*\text{Sqrt}[x]*\text{PolyLog}[4, (-I)*E^{(c + d*\text{Sqrt}[x])}] + (120*I)*b*d*\text{Sqrt}[x]*\text{PolyLog}[4, I*E^{(c + d*\text{Sqrt}[x])}] + (120*I)*b*\text{PolyLog}[5, (-I)*E^{(c + d*\text{Sqrt}[x])}] - (120*I)*b*\text{PolyLog}[5, I*E^{(c + d*\text{Sqrt}[x])}]))/(5*d^5)$

**fricas** [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(bx^{\frac{3}{2}} \operatorname{sech}(d\sqrt{x} + c) + ax^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

[Out] `integral(b*x^(3/2)*sech(d*sqrt(x) + c) + a*x^(3/2), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

[Out] `integrate((b*sech(d*sqrt(x) + c) + a)*x^(3/2), x)`

**maple** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x)`

[Out] `int(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5}ax^{\frac{5}{2}} + 2b \int \frac{x^{\frac{3}{2}}e^{(d\sqrt{x}+c)}}{e^{(2d\sqrt{x}+2c)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

[Out] `2/5*a*x^(5/2) + 2*b*integrate(x^(3/2)*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)*(a + b/cosh(c + d*x^(1/2))),x)`

[Out] `int(x^(3/2)*(a + b/cosh(c + d*x^(1/2))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)*(a+b*sech(c+d*x**(1/2))), x)`

[Out] `Integral(x**(3/2)*(a + b*sech(c + d*sqrt(x))), x)`

### 3.53 $\int \sqrt{x} \left( a + b \operatorname{sech} \left( c + d\sqrt{x} \right) \right) dx$

**Optimal.** Leaf size=140

$$\frac{2}{3}ax^{3/2} + \frac{4ib\operatorname{Li}_3(-ie^{c+d\sqrt{x}})}{d^3} - \frac{4ib\operatorname{Li}_3(ie^{c+d\sqrt{x}})}{d^3} - \frac{4ib\sqrt{x}\operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x}\operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} + \frac{4bx \tan^{-1}(e^{c+d\sqrt{x}})}{d}$$

[Out]  $\frac{2}{3}a*x^{3/2} + 4*b*x*\arctan(\exp(c+d*x^{1/2}))/d + 4*I*b*\operatorname{polylog}(3, -I*\exp(c+d*x^{1/2}))/d^3 - 4*I*b*\operatorname{polylog}(3, I*\exp(c+d*x^{1/2}))/d^3 - 4*I*b*\operatorname{polylog}(2, -I*\exp(c+d*x^{1/2}))*x^{1/2}/d^2 + 4*I*b*\operatorname{polylog}(2, I*\exp(c+d*x^{1/2}))*x^{1/2}/d^2$

**Rubi [A]** time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$ , Rules used = {14, 5436, 4180, 2531, 2282, 6589}

$$-\frac{4ib\sqrt{x}\operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x}\operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{4ib\operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]]),x]`

[Out]  $(2*a*x^{3/2})/3 + (4*b*x*\operatorname{ArcTan}[E^{(c + d*\operatorname{Sqrt}[x])}])/d - ((4*I)*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((4*I)*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^2 + ((4*I)*b*\operatorname{PolyLog}[3, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3 - ((4*I)*b*\operatorname{PolyLog}[3, I*E^{(c + d*\operatorname{Sqrt}[x])}])/d^3$

#### Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

#### Rule 2531

`Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]`

#### Rule 4180

`Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/ (f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]`

#### Rule 5436

```
Int[(x_)^(m_)*((a_) + (b_)*Sech[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x})) dx &= \int (a\sqrt{x} + b\sqrt{x} \operatorname{sech}(c + d\sqrt{x})) dx \\ &= \frac{2}{3}ax^{3/2} + b \int \sqrt{x} \operatorname{sech}(c + d\sqrt{x}) dx \\ &= \frac{2}{3}ax^{3/2} + (2b) \operatorname{Subst}\left(\int x^2 \operatorname{sech}(c + dx) dx, x, \sqrt{x}\right) \\ &= \frac{2}{3}ax^{3/2} + \frac{4bx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{(4ib) \operatorname{Subst}\left(\int x \log(1 - ie^{c+dx}) dx, x, \sqrt{x}\right)}{d} \\ &= \frac{2}{3}ax^{3/2} + \frac{4bx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{4ib\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\ &= \frac{2}{3}ax^{3/2} + \frac{4bx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{4ib\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \\ &= \frac{2}{3}ax^{3/2} + \frac{4bx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{4ib\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x} \operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} \end{aligned}$$

**Mathematica [A]** time = 7.66, size = 197, normalized size = 1.41

$$\frac{2(ad^3x^{3/2} - 6bd^2x \tan^{-1}(\cosh(c + d\sqrt{x}) - \sinh(c + d\sqrt{x})) - 6ibd\sqrt{x} \operatorname{Li}_2(-i(\cosh(c + d\sqrt{x}) - \sinh(c + d\sqrt{x}))))}{(3d^3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]]), x]
```

```
[Out] (2*(a*d^3*x^(3/2) - 6*b*d^2*x*ArcTan[Cosh[c + d*Sqrt[x]] - Sinh[c + d*Sqrt[x]]] - (6*I)*b*d*Sqrt[x]*PolyLog[2, (-I)*(Cosh[c + d*Sqrt[x]] - Sinh[c + d*Sqrt[x]])] + (6*I)*b*d*Sqrt[x]*PolyLog[2, I*(Cosh[c + d*Sqrt[x]] - Sinh[c + d*Sqrt[x]])] - (6*I)*b*PolyLog[3, (-I)*(Cosh[c + d*Sqrt[x]] - Sinh[c + d*Sqrt[x]])] + (6*I)*b*PolyLog[3, I*(Cosh[c + d*Sqrt[x]] - Sinh[c + d*Sqrt[x]])]))/(3*d^3)
```

**fricas [F]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}(b\sqrt{x} \operatorname{sech}(d\sqrt{x} + c) + a\sqrt{x}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2), x, algorithm="fricas")
```

```
[Out] integral(b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)\sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))\*x^(1/2),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)\*sqrt(x), x)

**maple** [F] time = 0.53, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}(c + d\sqrt{x})) \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))\*x^(1/2),x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))\*x^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3} ax^{\frac{3}{2}} + 2b \int \frac{\sqrt{x} e^{(d\sqrt{x}+c)}}{e^{(2d\sqrt{x}+2c)} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))\*x^(1/2),x, algorithm="maxima")

[Out] 2/3\*a\*x^(3/2) + 2\*b\*integrate(sqrt(x)\*e^(d\*sqrt(x) + c)/(e^(2\*d\*sqrt(x) + 2\*c) + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*(a + b/cosh(c + d\*x^(1/2))),x)

[Out] int(x^(1/2)\*(a + b/cosh(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*x\*\*(1/2),x)

[Out] Integral(sqrt(x)\*(a + b\*sech(c + d\*sqrt(x))), x)



$$3.54 \quad \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} dx$$

Optimal. Leaf size=26

$$2a\sqrt{x} + \frac{2b \tan^{-1}(\sinh(c+d\sqrt{x}))}{d}$$

[Out] 2\*b\*arctan(sinh(c+d\*x^(1/2)))/d+2\*a\*x^(1/2)

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {14, 5436, 3770}

$$2a\sqrt{x} + \frac{2b \tan^{-1}(\sinh(c+d\sqrt{x}))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])/Sqrt[x], x]

[Out] 2\*a\*Sqrt[x] + (2\*b\*ArcTan[Sinh[c + d\*Sqrt[x]])/d

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 3770

Int[csc[(c\_)+(d\_)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 5436

Int[(x\_)^(m\_)\*((a\_)+(b\_)\*Sech[(c\_)+(d\_)\*(x\_)^(n\_)])^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} dx &= \int \left( \frac{a}{\sqrt{x}} + \frac{b\operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} \right) dx \\ &= 2a\sqrt{x} + b \int \frac{\operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} dx \\ &= 2a\sqrt{x} + (2b) \operatorname{Subst} \left( \int \operatorname{sech}(c+dx) dx, x, \sqrt{x} \right) \\ &= 2a\sqrt{x} + \frac{2b \tan^{-1}(\sinh(c+d\sqrt{x}))}{d} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 30, normalized size = 1.15

$$\frac{2(a(c+d\sqrt{x}) + b \tan^{-1}(\sinh(c+d\sqrt{x})))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])/Sqrt[x], x]

[Out] (2\*(a\*(c + d\*Sqrt[x]) + b\*ArcTan[Sinh[c + d\*Sqrt[x]]]))/d

**fricas** [A] time = 0.52, size = 33, normalized size = 1.27

$$\frac{2(ad\sqrt{x} + 2b \arctan(\cosh(d\sqrt{x} + c) + \sinh(d\sqrt{x} + c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(1/2), x, algorithm="fricas")

[Out] 2\*(a\*d\*sqrt(x) + 2\*b\*arctan(cosh(d\*sqrt(x) + c) + sinh(d\*sqrt(x) + c)))/d

**giac** [A] time = 0.14, size = 29, normalized size = 1.12

$$\frac{2(d\sqrt{x} + c)a}{d} + \frac{4b \arctan(e^{(d\sqrt{x}+c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(1/2), x, algorithm="giac")

[Out] 2\*(d\*sqrt(x) + c)\*a/d + 4\*b\*arctan(e^(d\*sqrt(x) + c))/d

**maple** [A] time = 0.13, size = 23, normalized size = 0.88

$$\frac{2b \arctan(\sinh(c + d\sqrt{x}))}{d} + 2a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))/x^(1/2), x)

[Out] 2\*b\*arctan(sinh(c+d\*x^(1/2)))/d+2\*a\*x^(1/2)

**maxima** [A] time = 0.34, size = 22, normalized size = 0.85

$$2a\sqrt{x} + \frac{2b \arctan(\sinh(d\sqrt{x} + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(1/2), x, algorithm="maxima")

[Out] 2\*a\*sqrt(x) + 2\*b\*arctan(sinh(d\*sqrt(x) + c))/d

**mupad** [B] time = 1.34, size = 43, normalized size = 1.65

$$2a\sqrt{x} + \frac{4 \operatorname{atan}\left(\frac{b e^{d\sqrt{x}} e^c \sqrt{d^2}}{d \sqrt{b^2}}\right) \sqrt{b^2}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))/x^(1/2), x)

[Out] 2\*a\*x^(1/2) + (4\*atan((b\*exp(d\*x^(1/2))\*exp(c)\*(d^2)^(1/2))/(d\*(b^2)^(1/2)))\*(b^2)^(1/2))/(d^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))/x\*\*(1/2), x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))/sqrt(x), x)

$$3.55 \quad \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{3/2}} dx$$

Optimal. Leaf size=30

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+d\sqrt{x})}{x^{3/2}}, x\right) - \frac{2a}{\sqrt{x}}$$

[Out]  $-2*a/x^{(1/2)}+b*\operatorname{Unintegrable}(\operatorname{sech}(c+d*x^{(1/2)})/x^{(3/2)}, x)$

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^{(3/2)}, x]$

[Out]  $(-2*a)/\operatorname{Sqrt}[x] + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]]/x^{(3/2)}, x]$

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx &= \int \left( \frac{a}{x^{3/2}} + \frac{b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} \right) dx \\ &= -\frac{2a}{\sqrt{x}} + b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx \end{aligned}$$

**Mathematica** [A] time = 8.92, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^{(3/2)}, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^{(3/2)}, x]$

**fricas** [A] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\sqrt{x} \operatorname{sech}(d\sqrt{x} + c) + a\sqrt{x}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\operatorname{sech}(c+d*x^{(1/2)}))/x^{(3/2)}, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b*\operatorname{sqrt}(x))*\operatorname{sech}(d*\operatorname{sqrt}(x) + c) + a*\operatorname{sqrt}(x))/x^2, x)$

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(3/2),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)/x^(3/2), x)

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}\left(\frac{c + d\sqrt{x}}{x^{\frac{3}{2}}}\right)}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))/x^(3/2),x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))/x^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{e^{(d\sqrt{x}+c)}}{x^{\frac{3}{2}}e^{(2d\sqrt{x}+2c)} + x^{\frac{3}{2}}} dx - \frac{2a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(3/2),x, algorithm="maxima")

[Out] 2\*b\*integrate(e^(d\*sqrt(x) + c)/(x^(3/2)\*e^(2\*d\*sqrt(x) + 2\*c) + x^(3/2)), x) - 2\*a/sqrt(x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))/x^(3/2),x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))/x^(3/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}\left(\frac{c + d\sqrt{x}}{x^{\frac{3}{2}}}\right)}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))/x\*\*(3/2),x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))/x\*\*(3/2), x)

$$3.56 \quad \int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{5/2}} dx$$

Optimal. Leaf size=32

$$b\operatorname{Int}\left(\frac{\operatorname{sech}(c+d\sqrt{x})}{x^{5/2}}, x\right) - \frac{2a}{3x^{3/2}}$$

[Out]  $-2/3*a/x^{(3/2)}+b*\operatorname{Unintegrable}(\operatorname{sech}(c+d*x^{(1/2)})/x^{(5/2)}, x)$

**Rubi** [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^{(5/2)}, x]$

[Out]  $(-2*a)/(3*x^{(3/2)}) + b*\operatorname{Defer}[\operatorname{Int}[\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]]/x^{(5/2)}, x]$

Rubi steps

$$\begin{aligned} \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx &= \int \left( \frac{a}{x^{5/2}} + \frac{b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} \right) dx \\ &= -\frac{2a}{3x^{3/2}} + b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx \end{aligned}$$

**Mathematica** [A] time = 9.55, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^{(5/2)}, x]$

[Out]  $\operatorname{Integrate}[(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])/x^{(5/2)}, x]$

**fricas** [A] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\sqrt{x} \operatorname{sech}(d\sqrt{x} + c) + a\sqrt{x}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((a+b*\operatorname{sech}(c+d*x^{(1/2)}))/x^{(5/2)}, x, \operatorname{algorithm}="fricas")$

[Out]  $\operatorname{integral}((b*\operatorname{sqrt}(x)*\operatorname{sech}(d*\operatorname{sqrt}(x) + c) + a*\operatorname{sqrt}(x))/x^3, x)$

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(5/2),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)/x^(5/2), x)

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}\left(\frac{c + d\sqrt{x}}{x^{\frac{5}{2}}}\right)}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))/x^(5/2),x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))/x^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{e^{(d\sqrt{x}+c)}}{x^{\frac{5}{2}}e^{(2d\sqrt{x}+2c)} + x^{\frac{5}{2}}} dx - \frac{2a}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))/x^(5/2),x, algorithm="maxima")

[Out] 2\*b\*integrate(e^(d\*sqrt(x) + c)/(x^(5/2)\*e^(2\*d\*sqrt(x) + 2\*c) + x^(5/2)), x) - 2/3\*a/x^(3/2)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))/x^(5/2),x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))/x^(5/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{sech}\left(\frac{c + d\sqrt{x}}{x^{\frac{5}{2}}}\right)}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))/x\*\*(5/2),x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))/x\*\*(5/2), x)

### 3.57 $\int x^{3/2} \left( a + b \operatorname{sech} \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=407

$$\frac{2}{5}a^2x^{5/2} + \frac{96iab\operatorname{Li}_5(-ie^{c+d\sqrt{x}})}{d^5} - \frac{96iab\operatorname{Li}_5(ie^{c+d\sqrt{x}})}{d^5} - \frac{96iab\sqrt{x}\operatorname{Li}_4(-ie^{c+d\sqrt{x}})}{d^4} + \frac{96iab\sqrt{x}\operatorname{Li}_4(ie^{c+d\sqrt{x}})}{d^4} + \frac{48iabx\operatorname{Li}_3(-ie^{c+d\sqrt{x}})}{d^3} - \frac{48iabx\operatorname{Li}_3(ie^{c+d\sqrt{x}})}{d^3} - \frac{48iabx\sqrt{x}\operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{48iabx\sqrt{x}\operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} - \frac{48iabx\operatorname{Li}_1(-ie^{c+d\sqrt{x}})}{d} + \frac{48iabx\operatorname{Li}_1(ie^{c+d\sqrt{x}})}{d}$$

```
[Out] 2*b^2*x^2/d+2/5*a^2*x^(5/2)+8*a*b*x^2*arctan(exp(c+d*x^(1/2)))/d-8*b^2*x^(3/2)*ln(1+exp(2*c+2*d*x^(1/2)))/d^2+16*I*a*b*x^(3/2)*polylog(2,I*exp(c+d*x^(1/2)))/d^2-16*I*a*b*x^(3/2)*polylog(2,-I*exp(c+d*x^(1/2)))/d^2-12*b^2*x*polylog(2,-exp(2*c+2*d*x^(1/2)))/d^3-96*I*a*b*polylog(4,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^4-96*I*a*b*polylog(5,I*exp(c+d*x^(1/2)))/d^5-6*b^2*polylog(4,-exp(2*c+2*d*x^(1/2)))/d^5-48*I*a*b*x*polylog(3,I*exp(c+d*x^(1/2)))/d^3+96*I*a*b*polylog(5,-I*exp(c+d*x^(1/2)))/d^5+12*b^2*polylog(3,-exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^4+96*I*a*b*polylog(4,I*exp(c+d*x^(1/2)))*x^(1/2)/d^4+48*I*a*b*x*polylog(3,-I*exp(c+d*x^(1/2)))/d^3+2*b^2*x^2*tanh(c+d*x^(1/2))/d
```

**Rubi [A]** time = 0.53, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$ , Rules used = {5436, 4190, 4180, 2531, 6609, 2282, 6589, 4184, 3718, 2190}

$$-\frac{16iabx^{3/2}\operatorname{PolyLog}(2,-ie^{c+d\sqrt{x}})}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}(2,ie^{c+d\sqrt{x}})}{d^2} + \frac{48iabx\operatorname{PolyLog}(3,-ie^{c+d\sqrt{x}})}{d^3} - \frac{48iabx\operatorname{PolyLog}(3,ie^{c+d\sqrt{x}})}{d^3} - \frac{48iabx\sqrt{x}\operatorname{PolyLog}(2,-ie^{c+d\sqrt{x}})}{d^2} + \frac{48iabx\sqrt{x}\operatorname{PolyLog}(2,ie^{c+d\sqrt{x}})}{d^2} - \frac{48iabx\operatorname{Li}_1(-ie^{c+d\sqrt{x}})}{d} + \frac{48iabx\operatorname{Li}_1(ie^{c+d\sqrt{x}})}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*b^2*x^2)/d + (2*a^2*x^(5/2))/5 + (8*a*b*x^2*ArcTan[E^(c + d*Sqrt[x])])/d - (8*b^2*x^(3/2)*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((16*I)*a*b*x^(3/2)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((16*I)*a*b*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 - (12*b^2*x*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((48*I)*a*b*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((48*I)*a*b*x*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (12*b^2*Sqrt[x]*PolyLog[3, -E^(2*(c + d*Sqrt[x]))])/d^4 - ((96*I)*a*b*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((96*I)*a*b*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (6*b^2*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((96*I)*a*b*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((96*I)*a*b*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (2*b^2*x^2*Tanh[c + d*Sqrt[x]])/d
```

**Rule 2190**

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2282**

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

**Rule 2531**



```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 3718

```
Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

#### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

#### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^4 (a + b \operatorname{sech}(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int (a^2 x^4 + 2abx^4 \operatorname{sech}(c + dx) + b^2 x^4 \operatorname{sech}^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{2}{5} a^2 x^{5/2} + (4ab) \operatorname{Subst} \left( \int x^4 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^4 \operatorname{sech}^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} + \frac{2b^2 x^2 \tanh(c + d\sqrt{x})}{d} - \frac{(16iab) \operatorname{Subst} \left( \int x^4 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8b^2 x^{3/2} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8b^2 x^{3/2} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8b^2 x^{3/2} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8b^2 x^{3/2} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} + \frac{8abx^2 \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8b^2 x^{3/2} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{16iabx^{3/2} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 6.34, size = 487, normalized size = 1.20

$$2 \cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^{5/2} \cosh(c + d\sqrt{x}) + \frac{5b \cosh(c + d\sqrt{x}) \left( \frac{2be^{2c} d^4 x^2}{e^{2c} + 1} + i(2ad^4 x^2 \log(1 - ie^{c+d\sqrt{x}})) - 2a \right)}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (2\*Cosh[c + d\*Sqrt[x]]\*(a + b\*Sech[c + d\*Sqrt[x]])^2\*(a^2\*x^(5/2)\*Cosh[c + d\*Sqrt[x]] + (5\*b\*Cosh[c + d\*Sqrt[x]]\*((2\*b\*d^4\*E^(2\*c)\*x^2)/(1 + E^(2\*c)) + I\*(2\*a\*d^4\*x^2\*Log[1 - I\*E^(c + d\*Sqrt[x]]) - 2\*a\*d^4\*x^2\*Log[1 + I\*E^(c + d\*Sqrt[x]])] + (4\*I)\*b\*d^3\*x^(3/2)\*Log[1 + E^(2\*(c + d\*Sqrt[x]))] - 8\*a\*d^3\*x^(3/2)\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 8\*a\*d^3\*x^(3/2)\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + (6\*I)\*b\*d^2\*x\*PolyLog[2, -E^(2\*(c + d\*Sqrt[x]))] + 24\*a\*d^2\*x\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 24\*a\*d^2\*x\*PolyLog[3, I\*E^(c + d\*Sqrt[x])] - (6\*I)\*b\*d\*Sqrt[x]\*PolyLog[3, -E^(2\*(c + d\*Sqrt[x]))] - 48\*a\*d\*Sqrt[x]\*PolyLog[4, (-I)\*E^(c + d\*Sqrt[x])] + 48\*a\*d\*Sqrt[x]\*PolyLog[4, I\*E^(c + d\*Sqrt[x])] + (3\*I)\*b\*PolyLog[4, -E^(2\*(c + d\*Sqrt[x]))] + 48\*a\*PolyLog[5, (-I)\*E^(c + d\*Sqrt[x])] - 48\*a\*PolyLog[5, I\*E^(c + d\*Sqrt[x])]))/d^5 + (5\*b^2\*x^2\*Sech[c]\*Sinh[d\*Sqrt[x]])/d)/(5\*(b + a\*Cosh[c + d\*Sqrt[x]])^2)

**fricas [F]** time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( b^2 x^{\frac{3}{2}} \operatorname{sech}(d\sqrt{x} + c)^2 + 2abx^{\frac{3}{2}} \operatorname{sech}(d\sqrt{x} + c) + a^2 x^{\frac{3}{2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^(3/2)\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^(3/2)\*sech(d\*sqrt(x) + c) + a^2\*x^(3/2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2\*x^(3/2), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x^(3/2)\*(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( a^2 d x^{\frac{5}{2}} e^{(2d\sqrt{x}+2c)} + a^2 d x^{\frac{5}{2}} - 10 b^2 x^2 \right)}{5 \left( d e^{(2d\sqrt{x}+2c)} + d \right)} + \int \frac{4 \left( a b d x^{\frac{5}{2}} e^{(d\sqrt{x}+c)} + 2 b^2 x^2 \right)}{d x e^{(2d\sqrt{x}+2c)} + d x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] 2/5\*(a^2\*d\*x^(5/2)\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*x^(5/2) - 10\*b^2\*x^2)/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d) + integrate(4\*(a\*b\*d\*x^(5/2)\*e^(d\*sqrt(x) + c) + 2\*b^2\*x^2)/(d\*x\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^(3/2)\*(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*(3/2)\*(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)

### 3.58 $\int \sqrt{x} \left( a + b \operatorname{sech} \left( c + d\sqrt{x} \right) \right)^2 dx$

**Optimal.** Leaf size=229

$$\frac{2}{3}a^2x^{3/2} + \frac{8iab\operatorname{Li}_3(-ie^{c+d\sqrt{x}})}{d^3} - \frac{8iab\operatorname{Li}_3(ie^{c+d\sqrt{x}})}{d^3} - \frac{8iab\sqrt{x}\operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8iab\sqrt{x}\operatorname{Li}_2(ie^{c+d\sqrt{x}})}{d^2} + \frac{8abx \tan^{-1}(e^{c+d\sqrt{x}})}{d}$$

[Out]  $2*b^2*x/d + 2/3*a^2*x^(3/2) + 8*a*b*x*\arctan(\exp(c+d*x^(1/2)))/d - 2*b^2*\operatorname{polylog}(2, -\exp(2*c+2*d*x^(1/2)))/d^3 + 8*I*a*b*\operatorname{polylog}(3, -I*\exp(c+d*x^(1/2)))/d^3 - 8*I*a*b*\operatorname{polylog}(3, I*\exp(c+d*x^(1/2)))/d^3 - 4*b^2*\ln(1+\exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^2 - 8*I*a*b*\operatorname{polylog}(2, -I*\exp(c+d*x^(1/2)))*x^(1/2)/d^2 + 8*I*a*b*\operatorname{polylog}(2, I*\exp(c+d*x^(1/2)))*x^(1/2)/d^2 + 2*b^2*x*\tanh(c+d*x^(1/2))/d$

**Rubi [A]** time = 0.32, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5436, 4190, 4180, 2531, 2282, 6589, 4184, 3718, 2190, 2279, 2391}

$$-\frac{8iab\sqrt{x}\operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{8iab\sqrt{x}\operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{8iab\operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{8iab\operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2, x]`

[Out]  $(2*b^2*x)/d + (2*a^2*x^(3/2))/3 + (8*a*b*x*\operatorname{ArcTan}[E^(c + d*\operatorname{Sqrt}[x])])/d - (4*b^2*\operatorname{Sqrt}[x]*\operatorname{Log}[1 + E^(2*(c + d*\operatorname{Sqrt}[x]))])/d^2 - ((8*I)*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, (-I)*E^(c + d*\operatorname{Sqrt}[x])])/d^2 + ((8*I)*a*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, I*E^(c + d*\operatorname{Sqrt}[x])])/d^2 - (2*b^2*\operatorname{PolyLog}[2, -E^(2*(c + d*\operatorname{Sqrt}[x]))])/d^3 + ((8*I)*a*b*\operatorname{PolyLog}[3, (-I)*E^(c + d*\operatorname{Sqrt}[x])])/d^3 - ((8*I)*a*b*\operatorname{PolyLog}[3, I*E^(c + d*\operatorname{Sqrt}[x])])/d^3 + (2*b^2*x*\operatorname{Tanh}[c + d*\operatorname{Sqrt}[x]])/d$

#### Rule 2190

`Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]`

#### Rule 2279

`Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]`

#### Rule 2282

`Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]`

#### Rule 2391

`Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]`

Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 3718

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(1 + E^(2\*(-I\*e) + f\*fz\*x))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-I\*e) + f\*fz\*x]/E^(I\*k\*Pi))]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-I\*e) + f\*fz\*x]/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-I\*e) + f\*fz\*x]/E^(I\*k\*Pi)], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 4190

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Csc[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= 2 \operatorname{Subst} \left( \int x^2 (a + b \operatorname{sech}(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int (a^2 x^2 + 2abx^2 \operatorname{sech}(c + dx) + b^2 x^2 \operatorname{sech}^2(c + dx)) dx, x, \sqrt{x} \right) \\
&= \frac{2}{3} a^2 x^{3/2} + (4ab) \operatorname{Subst} \left( \int x^2 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int x^2 \operatorname{sech}^2(c + dx) dx, x, \sqrt{x} \right) \\
&= \frac{2}{3} a^2 x^{3/2} + \frac{8abx \tan^{-1}(e^{c+d\sqrt{x}})}{d} + \frac{2b^2 x \tanh(c + d\sqrt{x})}{d} - \frac{(8iab) \operatorname{Subst} \left( \int x^2 \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right)}{d} \\
&= \frac{2b^2 x}{d} + \frac{2}{3} a^2 x^{3/2} + \frac{8abx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{8iab\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} + \frac{8iab\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x}{d} + \frac{2}{3} a^2 x^{3/2} + \frac{8abx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{4b^2 \sqrt{x} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{8iab\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x}{d} + \frac{2}{3} a^2 x^{3/2} + \frac{8abx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{4b^2 \sqrt{x} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{8iab\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2} \\
&= \frac{2b^2 x}{d} + \frac{2}{3} a^2 x^{3/2} + \frac{8abx \tan^{-1}(e^{c+d\sqrt{x}})}{d} - \frac{4b^2 \sqrt{x} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{8iab\sqrt{x} \operatorname{Li}_2(-ie^{c+d\sqrt{x}})}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 5.90, size = 309, normalized size = 1.35

$$2 \cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^{3/2} \cosh(c + d\sqrt{x}) + \frac{3b \cosh(c + d\sqrt{x}) \left( 2ia \left( d^2 x \log(1 - ie^{c+d\sqrt{x}}) \right) - d^2 x \log(1 + ie^{c+d\sqrt{x}}) \right)}{d^2} \right)$$

3(a + b sech(c + d√x))

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (2\*Cosh[c + d\*Sqrt[x]]\*(a + b\*Sech[c + d\*Sqrt[x]])^2\*(a^2\*x^(3/2)\*Cosh[c + d\*Sqrt[x]] + (3\*b\*Cosh[c + d\*Sqrt[x]]\*((2\*b\*d^2\*E^(2\*c)\*x)/(1 + E^(2\*c)) - b\*(2\*d\*Sqrt[x]\*Log[1 + E^(2\*(c + d\*Sqrt[x]))] + PolyLog[2, -E^(2\*(c + d\*Sqrt[x]))]) + (2\*I)\*a\*(d^2\*x\*Log[1 - I\*E^(c + d\*Sqrt[x])] - d^2\*x\*Log[1 + I\*E^(c + d\*Sqrt[x])] - 2\*d\*Sqrt[x]\*PolyLog[2, (-I)\*E^(c + d\*Sqrt[x])] + 2\*d\*Sqrt[x]\*PolyLog[2, I\*E^(c + d\*Sqrt[x])] + 2\*PolyLog[3, (-I)\*E^(c + d\*Sqrt[x])] - 2\*PolyLog[3, I\*E^(c + d\*Sqrt[x])])))/d^3 + (3\*b^2\*x\*Sech[c]\*Sinh[d\*Sqrt[x]]/d)/(3\*(b + a\*Cosh[c + d\*Sqrt[x]])^2)

**fricas [F]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( b^2 \sqrt{x} \operatorname{sech}(d\sqrt{x} + c)^2 + 2ab\sqrt{x} \operatorname{sech}(d\sqrt{x} + c) + a^2 \sqrt{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2\*x^(1/2),x, algorithm="fricas")

[Out] integral(b^2\*sqrt(x)\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sqrt(x)\*sech(d\*sqrt(x) + c) + a^2\*sqrt(x), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2\*x^(1/2),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2\*sqrt(x), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))^2\*x^(1/2),x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))^2\*x^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( a^2 dx^{\frac{3}{2}} e^{(2d\sqrt{x}+2c)} + a^2 dx^{\frac{3}{2}} - 6b^2 x \right)}{3 \left( de^{(2d\sqrt{x}+2c)} + d \right)} + \int \frac{4 \left( abdx^{\frac{3}{2}} e^{(d\sqrt{x}+c)} + b^2 x \right)}{dxe^{(2d\sqrt{x}+2c)} + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2\*x^(1/2),x, algorithm="maxima")

[Out] 2/3\*(a^2\*d\*x^(3/2)\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*x^(3/2) - 6\*b^2\*x)/(d\*e^(2\*d\*sqrt(x) + 2\*c) + d) + integrate(4\*(a\*b\*d\*x^(3/2)\*e^(d\*sqrt(x) + c) + b^2\*x)/(d\*x\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)\*(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^(1/2)\*(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*\*2\*x\*\*(1/2),x)

[Out] Integral(sqrt(x)\*(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)

$$3.59 \quad \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$$

Optimal. Leaf size=47

$$2a^2\sqrt{x} + \frac{4ab \tan^{-1}(\sinh(c+d\sqrt{x}))}{d} + \frac{2b^2 \tanh(c+d\sqrt{x})}{d}$$

[Out]  $4*a*b*\arctan(\sinh(c+d*x^{(1/2)}))/d+2*a^2*x^{(1/2)}+2*b^2*\tanh(c+d*x^{(1/2)})/d$

**Rubi [A]** time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {5436, 3773, 3770, 3767, 8}

$$2a^2\sqrt{x} + \frac{4ab \tan^{-1}(\sinh(c+d\sqrt{x}))}{d} + \frac{2b^2 \tanh(c+d\sqrt{x})}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])^2/Sqrt[x], x]

[Out]  $2*a^2*\text{Sqrt}[x] + (4*a*b*\text{ArcTan}[\text{Sinh}[c + d*\text{Sqrt}[x]])]/d + (2*b^2*\text{Tanh}[c + d*\text{Sqrt}[x]])/d$

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3773

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.))^2, x\_Symbol] := Simp[a^2\*x, x] + (Dist[2\*a\*b, Int[Csc[c + d\*x], x], x] + Dist[b^2, Int[Csc[c + d\*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]

Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps



$$\begin{aligned}
\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx &= 2 \operatorname{Subst} \left( \int (a + b \operatorname{sech}(c + dx))^2 dx, x, \sqrt{x} \right) \\
&= 2a^2\sqrt{x} + (4ab) \operatorname{Subst} \left( \int \operatorname{sech}(c + dx) dx, x, \sqrt{x} \right) + (2b^2) \operatorname{Subst} \left( \int \operatorname{sech}^2(c + dx) dx, x, \sqrt{x} \right) \\
&= 2a^2\sqrt{x} + \frac{4ab \tan^{-1}(\sinh(c + d\sqrt{x}))}{d} + \frac{(2b^2) \operatorname{Subst}(\int 1 dx, x, -i \tanh(c + d\sqrt{x}))}{d} \\
&= 2a^2\sqrt{x} + \frac{4ab \tan^{-1}(\sinh(c + d\sqrt{x}))}{d} + \frac{2b^2 \tanh(c + d\sqrt{x})}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 48, normalized size = 1.02

$$\frac{2 \left( a \left( a \left( c + d\sqrt{x} \right) + 2b \tan^{-1} \left( \sinh \left( c + d\sqrt{x} \right) \right) \right) + b^2 \tanh \left( c + d\sqrt{x} \right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/Sqrt[x], x]

[Out] (2\*(a\*(a\*(c + d\*Sqrt[x]) + 2\*b\*ArcTan[Sinh[c + d\*Sqrt[x]]])) + b^2\*Tanh[c + d\*Sqrt[x]])/d

**fricas [B]** time = 0.45, size = 194, normalized size = 4.13

$$\frac{2 \left( a^2 d \sqrt{x} \cosh(d\sqrt{x} + c)^2 + 2 a^2 d \sqrt{x} \cosh(d\sqrt{x} + c) \sinh(d\sqrt{x} + c) + a^2 d \sqrt{x} \sinh(d\sqrt{x} + c)^2 + a^2 d \sqrt{x} - d \cosh(d\sqrt{x} + c)^2 + 2 d \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(1/2), x, algorithm="fricas")

[Out] 2\*(a^2\*d\*sqrt(x)\*cosh(d\*sqrt(x) + c)^2 + 2\*a^2\*d\*sqrt(x)\*cosh(d\*sqrt(x) + c)\*sinh(d\*sqrt(x) + c) + a^2\*d\*sqrt(x)\*sinh(d\*sqrt(x) + c)^2 + a^2\*d\*sqrt(x) - 2\*b^2 + 4\*(a\*b\*cosh(d\*sqrt(x) + c)^2 + 2\*a\*b\*cosh(d\*sqrt(x) + c)\*sinh(d\*sqrt(x) + c) + a\*b\*sinh(d\*sqrt(x) + c)^2 + a\*b)\*arctan(cosh(d\*sqrt(x) + c) + sinh(d\*sqrt(x) + c)))/(d\*cosh(d\*sqrt(x) + c)^2 + 2\*d\*cosh(d\*sqrt(x) + c)\*sinh(d\*sqrt(x) + c) + d\*sinh(d\*sqrt(x) + c)^2 + d)

**giac [A]** time = 0.15, size = 55, normalized size = 1.17

$$\frac{2(d\sqrt{x} + c)a^2}{d} + \frac{8ab \arctan(e^{(d\sqrt{x}+c)})}{d} - \frac{4b^2}{d(e^{(2d\sqrt{x}+2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(1/2), x, algorithm="giac")

[Out] 2\*(d\*sqrt(x) + c)\*a^2/d + 8\*a\*b\*arctan(e^(d\*sqrt(x) + c))/d - 4\*b^2/(d\*(e^(2\*d\*sqrt(x) + 2\*c) + 1))

**maple [A]** time = 0.41, size = 51, normalized size = 1.09

$$2a^2\sqrt{x} + \frac{2b^2 \tanh(c + d\sqrt{x})}{d} + \frac{8ab \arctan(e^{c+d\sqrt{x}})}{d} + \frac{2a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))^2/x^(1/2),x)

[Out] 2\*a^2\*x^(1/2)+2\*b^2\*tanh(c+d\*x^(1/2))/d+8/d\*a\*b\*arctan(exp(c+d\*x^(1/2)))+2/d\*a^2\*c

**maxima** [A] time = 0.30, size = 48, normalized size = 1.02

$$2 a^2 \sqrt{x} + \frac{4 a b \arctan\left(\sinh\left(d \sqrt{x} + c\right)\right)}{d} + \frac{4 b^2}{d\left(e^{(-2 d \sqrt{x}-2 c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")

[Out] 2\*a^2\*sqrt(x) + 4\*a\*b\*arctan(sinh(d\*sqrt(x) + c))/d + 4\*b^2/(d\*(e^(-2\*d\*sqrt(x) - 2\*c) + 1))

**mupad** [B] time = 1.32, size = 77, normalized size = 1.64

$$2 a^2 \sqrt{x} + \frac{8 \operatorname{atan}\left(\frac{a b e^{d \sqrt{x}} e^c \sqrt{d^2}}{d \sqrt{a^2 b^2}}\right) \sqrt{a^2 b^2}}{\sqrt{d^2}} - \frac{4 b^2}{d\left(e^{2 c+2 d \sqrt{x}} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))^2/x^(1/2),x)

[Out] 2\*a^2\*x^(1/2) + (8\*atan((a\*b\*exp(d\*x^(1/2))\*exp(c)\*(d^2)^(1/2))/(d\*(a^2\*b^2)^(1/2)))\*(a^2\*b^2)^(1/2))/(d^2)^(1/2) - (4\*b^2)/(d\*(exp(2\*c + 2\*d\*x^(1/2)) + 1))

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d \sqrt{x}))^2}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*\*2/x\*\*(1/2),x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))\*\*2/sqrt(x), x)

$$3.60 \quad \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}}, x\right)$$

[Out] Unintegrable((a+b\*sech(c+d\*x^(1/2)))^2/x^(3/2), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(3/2), x]

[Out] Defer[Int][(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(3/2), x]

Rubi steps

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Mathematica [A] time = 27.87, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(3/2), x]

[Out] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(3/2), x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\sqrt{x}\operatorname{sech}(d\sqrt{x}+c)^2+2ab\sqrt{x}\operatorname{sech}(d\sqrt{x}+c)+a^2\sqrt{x}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(3/2), x, algorithm="fricas")

[Out] integral((b^2\*sqrt(x)\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sqrt(x)\*sech(d\*sqrt(x) + c) + a^2\*sqrt(x))/x^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{sech}(d\sqrt{x}+c)+a)^2}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(3/2),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2/x^(3/2), x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))^2/x^(3/2),x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))^2/x^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2(a^2 d \sqrt{x} e^{(2d\sqrt{x}+2c)} + a^2 d \sqrt{x} + 2b^2)}{dx e^{(2d\sqrt{x}+2c)} + dx} + \int \frac{4(ab dx e^{(d\sqrt{x}+c)} - b^2 \sqrt{x})}{dx^{\frac{5}{2}} e^{(2d\sqrt{x}+2c)} + dx^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")

[Out] -2\*(a^2\*d\*sqrt(x)\*e^(2\*d\*sqrt(x) + 2\*c) + a^2\*d\*sqrt(x) + 2\*b^2)/(d\*x\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x) + integrate(4\*(a\*b\*d\*x\*e^(d\*sqrt(x) + c) - b^2\*sqrt(x))/(d\*x^(5/2)\*e^(2\*d\*sqrt(x) + 2\*c) + d\*x^(5/2)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2}{x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))^2/x^(3/2),x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))^2/x^(3/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*\*2/x\*\*(3/2),x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))\*\*2/x\*\*(3/2), x)

$$3.61 \quad \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}}, x\right)$$

[Out] Unintegrable((a+b\*sech(c+d\*x^(1/2)))^2/x^(5/2), x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(5/2), x]

[Out] Defer[Int][(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(5/2), x]

Rubi steps

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Mathematica [A] time = 27.88, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(5/2), x]

[Out] Integrate[(a + b\*Sech[c + d\*Sqrt[x]])^2/x^(5/2), x]

fricas [A] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2\sqrt{x}\operatorname{sech}(d\sqrt{x}+c)^2+2ab\sqrt{x}\operatorname{sech}(d\sqrt{x}+c)+a^2\sqrt{x}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(5/2), x, algorithm="fricas")

[Out] integral((b^2\*sqrt(x)\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sqrt(x)\*sech(d\*sqrt(x) + c) + a^2\*sqrt(x))/x^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{sech}(d\sqrt{x}+c)+a)^2}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(5/2),x, algorithm="giac")

[Out] integrate((b\*sech(d\*sqrt(x) + c) + a)^2/x^(5/2), x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sech(c+d\*x^(1/2)))^2/x^(5/2),x)

[Out] int((a+b\*sech(c+d\*x^(1/2)))^2/x^(5/2),x)

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + \frac{b}{\cosh(c+d\sqrt{x})}\right)^2}{x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^(1/2)))^2/x^(5/2),x)

[Out] int((a + b/cosh(c + d\*x^(1/2)))^2/x^(5/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sech(c+d\*x\*\*(1/2)))\*\*2/x\*\*(5/2),x)

[Out] Integral((a + b\*sech(c + d\*sqrt(x)))\*\*2/x\*\*(5/2), x)

$$3.62 \quad \int \frac{x^{3/2}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=601

$$\frac{48b\operatorname{Li}_5\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{48b\operatorname{Li}_5\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{48b\sqrt{x}\operatorname{Li}_4\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{48b\sqrt{x}\operatorname{Li}_4\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{24bx\operatorname{Li}_3\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{24bx\operatorname{Li}_3\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}$$

```
[Out] 2/5*x^(5/2)/a-2*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+2*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)-8*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+8*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+24*b*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-24*b*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)+48*b*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-48*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-48*b*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2)+48*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2)
```

**Rubi [A]** time = 0.97, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{8bx^{3/2}\operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{8bx^{3/2}\operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{24bx\operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{24bx\operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]]), x]
```

```
[Out] (2*x^(5/2))/(5*a) - (2*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (2*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (8*b*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (8*b*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (24*b*x*polylog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (24*b*x*polylog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (48*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (48*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (48*b*polylog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) - (48*b*polylog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5)
```

**Rule 2190**

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2264**

```
Int[(((F_)^(u_.)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
```

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 3320

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^4}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^4}{a} - \frac{bx^4}{a(b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{2x^{5/2}}{5a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^4}{b+a \cosh(c+dx)} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{2x^{5/2}}{5a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^4}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{2x^{5/2}}{5a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^4}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2+b^2}} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx}}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{(8b) \operatorname{Subst} \left( \int \frac{e^{c+dx}}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}} \\
&= \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}}
\end{aligned}$$

**Mathematica [A]** time = 2.37, size = 626, normalized size = 1.04

$$2 \left( d^5 x^{5/2} \sqrt{e^{2c} (b^2 - a^2)} - 5 b e^c d^4 x^2 \log \left( \frac{a e^{2c+d\sqrt{x}}}{b e^c - \sqrt{e^{2c} (b^2 - a^2)}} + 1 \right) + 5 b e^c d^4 x^2 \log \left( \frac{a e^{2c+d\sqrt{x}}}{\sqrt{e^{2c} (b^2 - a^2)} + b e^c} + 1 \right) - 20 b e^c d^3 x^{3/2} \operatorname{Li}_2 \left( -\frac{a}{b - \sqrt{e^{2c} (b^2 - a^2)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a + b\*Sech[c + d\*Sqrt[x]]), x]

[Out] (2\*(d^5\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^(5/2) - 5\*b\*d^4\*E^c\*x^2\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + 5\*b\*d^4\*E^c\*x^2\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 20\*b\*d^3\*E^c\*x^(3/2)\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))] + 20\*b\*d^3\*E^c\*x^(3/2)\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))] + 60\*b\*d^2\*E^c\*x\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))] - 60\*b\*d^2\*E^c\*x\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])])

)] - 120\*b\*d\*E^c\*Sqrt[x]\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))] + 120\*b\*d\*E^c\*Sqrt[x]\*PolyLog[4, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)]))] + 120\*b\*E^c\*PolyLog[5, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)]))] - 120\*b\*E^c\*PolyLog[5, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])))]/(5\*a\*d^5\*Sqrt[(-a^2 + b^2)\*E^(2\*c)])

**fricas** [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^{\frac{3}{2}}}{b \operatorname{sech}(d\sqrt{x} + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(x^(3/2)/(b\*sech(d\*sqrt(x) + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(x^(3/2)/(b\*sech(d\*sqrt(x) + c) + a), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{a + \frac{b}{\cosh(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2))), x)`

[Out] `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2))), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)/(a+b*sech(c+d*x**(1/2))), x)`

[Out] `Integral(x**(3/2)/(a + b*sech(c + d*sqrt(x))), x)`

$$3.63 \quad \int \frac{\sqrt{x}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$$

**Optimal.** Leaf size=361

$$\frac{4b\operatorname{Li}_3\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{4b\operatorname{Li}_3\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{4b\sqrt{x}\operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{4b\sqrt{x}\operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{2bx\log\left(\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}+1\right)}{ad\sqrt{b^2-a^2}}$$

[Out]  $2/3*x^{(3/2)}/a-2*b*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)}+2*b*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)}+4*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d^3/(-a^2+b^2)^{(1/2)}-4*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d^3/(-a^2+b^2)^{(1/2)}-4*b*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a/d^2/(-a^2+b^2)^{(1/2)}+4*b*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a/d^2/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 0.78, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {5436, 4191, 3320, 2264, 2190, 2531, 2282, 6589}

$$-\frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{4b\sqrt{x}\operatorname{PolyLog}\left(2,-\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{4b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{4b\operatorname{PolyLog}\left(3,-\frac{ae^{c+d\sqrt{x}}}{\sqrt{b^2-a^2}+b}\right)}{ad^3\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*Sech[c + d\*Sqrt[x]]),x]

[Out]  $(2*x^{(3/2)})/(3*a) - (2*b*x*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x])})]/(b - \operatorname{Sqrt}[-a^2 + b^2])]/(a*\operatorname{Sqrt}[-a^2 + b^2]*d) + (2*b*x*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x])})]/(b + \operatorname{Sqrt}[-a^2 + b^2])]/(a*\operatorname{Sqrt}[-a^2 + b^2]*d) - (4*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b - \operatorname{Sqrt}[-a^2 + b^2]))]/(a*\operatorname{Sqrt}[-a^2 + b^2]*d^2) + (4*b*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[2, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2]))]/(a*\operatorname{Sqrt}[-a^2 + b^2]*d^2) + (4*b*\operatorname{PolyLog}[3, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b - \operatorname{Sqrt}[-a^2 + b^2]))]/(a*\operatorname{Sqrt}[-a^2 + b^2]*d^3) - (4*b*\operatorname{PolyLog}[3, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2]))]/(a*\operatorname{Sqrt}[-a^2 + b^2]*d^3)$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2264**

Int[((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

**Rule 2282**

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))

$(F\_)[v\_]$  /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.))]\*((f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_] \* (f\_.)\*(x\_))], x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-(I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/2)) - (b\*E^(2\*(-(I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Sin[e + f\*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= 2 \operatorname{Subst} \left( \int \frac{x^2}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{2x^{3/2}}{3a} - \frac{(2b) \operatorname{Subst} \left( \int \frac{x^2}{b+a \cosh(c+dx)} dx, x, \sqrt{x} \right)}{3a} \\
&= \frac{2x^{3/2}}{3a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{3a} \\
&= \frac{2x^{3/2}}{3a} - \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{3a} + \frac{(4b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, \sqrt{x} \right)}{3a} \\
&= \frac{2x^{3/2}}{3a} - \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{(4b) \operatorname{Subst} \left( \int x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right) dx, x, \sqrt{x} \right)}{3a} \\
&= \frac{2x^{3/2}}{3a} - \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{4b\sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d^2} \\
&= \frac{2x^{3/2}}{3a} - \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{4b\sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d^2} \\
&= \frac{2x^{3/2}}{3a} - \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d} - \frac{4b\sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right)}{a\sqrt{-a^2+b^2}d^2}
\end{aligned}$$

**Mathematica [A]** time = 8.03, size = 390, normalized size = 1.08

$$\frac{2 \left( d^3 x^{3/2} \sqrt{e^{2c} (b^2 - a^2)} - 3be^c d^2 x \log \left( \frac{ae^{2c+d\sqrt{x}}}{be^c - \sqrt{e^{2c} (b^2 - a^2)}} + 1 \right) + 3be^c d^2 x \log \left( \frac{ae^{2c+d\sqrt{x}}}{\sqrt{e^{2c} (b^2 - a^2)} + be^c} + 1 \right) - 6be^c d \sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}} \right) - 6be^c d \sqrt{x} \operatorname{Li}_2 \left( -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}} \right) \right)}{3ad^3 \sqrt{e^{2c} (b^2 - a^2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b\*Sech[c + d\*Sqrt[x]]), x]

[Out] (2\*(d^3\*Sqrt[(-a^2 + b^2)\*E^(2\*c)]\*x^(3/2) - 3\*b\*d^2\*E^c\*x\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])] + 3\*b\*d^2\*E^c\*x\*Log[1 + (a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])] - 6\*b\*d\*E^c\*Sqrt[x]\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 6\*b\*d\*E^c\*Sqrt[x]\*PolyLog[2, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) + 6\*b\*E^c\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c - Sqrt[(-a^2 + b^2)\*E^(2\*c)])]) - 6\*b\*E^c\*PolyLog[3, -((a\*E^(2\*c + d\*Sqrt[x]))/(b\*E^c + Sqrt[(-a^2 + b^2)\*E^(2\*c)])])])/(3\*a\*d^3\*Sqrt[(-a^2 + b^2)\*E^(2\*c)])

**fricas [F]** time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{x}}{b \operatorname{sech}(d\sqrt{x} + c) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="fricas")

[Out] integral(sqrt(x)/(b\*sech(d\*sqrt(x) + c) + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="giac")

[Out] integrate(sqrt(x)/(b\*sech(d\*sqrt(x) + c) + a), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(x^(1/2)/(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see 'assume?' for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x}}{a + \frac{b}{\cosh(c+d\sqrt{x})}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a + b/cosh(c + d\*x^(1/2))),x)

[Out] int(x^(1/2)/(a + b/cosh(c + d\*x^(1/2))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(a+b\*sech(c+d\*x\*\*(1/2))),x)

[Out] Integral(sqrt(x)/(a + b\*sech(c + d\*sqrt(x))), x)

$$3.64 \quad \int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Optimal. Leaf size=68

$$\frac{2\sqrt{x}}{a} - \frac{4b \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

[Out]  $-4*b*\arctan((a-b)^{(1/2)}*\tanh(1/2*c+1/2*d*x^{(1/2)})/(a+b)^{(1/2)})/a/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}+2*x^{(1/2)}/a$

**Rubi [A]** time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5436, 3783, 2659, 208}

$$\frac{2\sqrt{x}}{a} - \frac{4b \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*Sech[c + d\*Sqrt[x]])),x]

[Out]  $(2*\text{Sqrt}[x])/a - (4*b*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tanh}[(c + d*\text{Sqrt}[x])/2])/\text{Sqrt}[a + b]])/(a*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d)$

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3783

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.) + (a\_)^(-1), x\_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a\*Sin[c + d\*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

Rubi steps



$$\begin{aligned}
\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx &= 2 \operatorname{Subst} \left( \int \frac{1}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right) \\
&= \frac{2\sqrt{x}}{a} - \frac{2 \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a \cosh(c+dx)}{b}} dx, x, \sqrt{x} \right)}{a} \\
&= \frac{2\sqrt{x}}{a} + \frac{(4i) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b} + (1 - \frac{a}{b})x^2} dx, x, i \tanh \left( \frac{1}{2} (c + d\sqrt{x}) \right) \right)}{ad} \\
&= \frac{2\sqrt{x}}{a} - \frac{4b \tan^{-1} \left( \frac{\sqrt{a-b} \tanh \left( \frac{1}{2} (c + d\sqrt{x}) \right)}{\sqrt{a+b}} \right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 69, normalized size = 1.01

$$\frac{2 \left( \frac{2b \tan^{-1} \left( \frac{(b-a) \tanh \left( \frac{1}{2} (c + d\sqrt{x}) \right)}{\sqrt{a^2 - b^2}} \right)}{d\sqrt{a^2 - b^2}} + \frac{c}{d} + \sqrt{x} \right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]\*(a + b\*Sech[c + d\*Sqrt[x]])),x]

[Out] (2\*(c/d + Sqrt[x] + (2\*b\*ArcTan[((-a + b)\*Tanh[(c + d\*Sqrt[x])/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]\*d)))/a

**fricas [B]** time = 0.43, size = 254, normalized size = 3.74

$$\left[ \frac{2 \left( (a^2 - b^2)d\sqrt{x} - \sqrt{-a^2 + b^2} b \log \left( \frac{ab + (b^2 + \sqrt{-a^2 + b^2} b) \cosh(d\sqrt{x} + c) + (a^2 - b^2 - \sqrt{-a^2 + b^2} b) \sinh(d\sqrt{x} + c) + \sqrt{-a^2 + b^2} a}{a \cosh(d\sqrt{x} + c) + b} \right) \right)}{(a^3 - ab^2)d}, \frac{2 \left( (a^2 - b^2)d\sqrt{x} - \sqrt{-a^2 + b^2} b \log \left( \frac{ab + (b^2 + \sqrt{-a^2 + b^2} b) \cosh(d\sqrt{x} + c) + (a^2 - b^2 - \sqrt{-a^2 + b^2} b) \sinh(d\sqrt{x} + c) + \sqrt{-a^2 + b^2} a}{a \cosh(d\sqrt{x} + c) + b} \right) \right)}{(a^3 - ab^2)d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sech(c+d\*x^(1/2)))/x^(1/2),x, algorithm="fricas")

[Out] [2\*((a^2 - b^2)\*d\*sqrt(x) - sqrt(-a^2 + b^2)\*b\*log((a\*b + (b^2 + sqrt(-a^2 + b^2)\*b)\*cosh(d\*sqrt(x) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2)\*b)\*sinh(d\*sqrt(x) + c) + sqrt(-a^2 + b^2)\*a)/(a\*cosh(d\*sqrt(x) + c) + b)))/((a^3 - a\*b^2)\*d), 2\*((a^2 - b^2)\*d\*sqrt(x) + 2\*sqrt(a^2 - b^2)\*b\*arctan(-(sqrt(a^2 - b^2)\*a\*cosh(d\*sqrt(x) + c) + sqrt(a^2 - b^2)\*a\*sinh(d\*sqrt(x) + c) + sqrt(a^2 - b^2)\*b)/(a^2 - b^2)))/((a^3 - a\*b^2)\*d)]

**giac [A]** time = 0.14, size = 61, normalized size = 0.90

$$-\frac{4b \arctan \left( \frac{ae^{(d\sqrt{x}+c)}+b}{\sqrt{a^2-b^2}} \right)}{\sqrt{a^2-b^2}ad} + \frac{2(d\sqrt{x}+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sech(c+d\*x^(1/2)))/x^(1/2),x, algorithm="giac")

[Out]  $-4*b*\arctan((a*e^{(d*\sqrt{x})} + c) + b)/\sqrt{(a^2 - b^2)} / (\sqrt{(a^2 - b^2)}*a*d) + 2*(d*\sqrt{x} + c)/(a*d)$

**maple** [A] time = 0.38, size = 95, normalized size = 1.40

$$-\frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 1\right)}{da} + \frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) + 1\right)}{da} - \frac{4b \arctan\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{da\sqrt{(a+b)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x)`

[Out]  $-2/d/a*\ln(\tanh(1/2*c+1/2*d*x^{(1/2)})-1)+2/d/a*\ln(\tanh(1/2*c+1/2*d*x^{(1/2)}))+1)-4/d/a*b/((a+b)*(a-b))^{(1/2)}*\arctan((a-b)*\tanh(1/2*c+1/2*d*x^{(1/2)})/((a+b)*(a-b))^{(1/2)})$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*b^2-4\*a^2>0)', see 'assume?' for more details) Is 4\*b^2-4\*a^2 positive or negative?

**mupad** [B] time = 1.78, size = 155, normalized size = 2.28

$$\frac{2\sqrt{x}}{a} + \frac{2b \ln\left(\frac{2be^{d\sqrt{x}}e^c}{a^2\sqrt{x}} - \frac{2b(a+be^{d\sqrt{x}}e^c)}{a^2\sqrt{x}\sqrt{a+b}\sqrt{b-a}}\right)}{ad\sqrt{a+b}\sqrt{b-a}} - \frac{2b \ln\left(\frac{2be^{d\sqrt{x}}e^c}{a^2\sqrt{x}} + \frac{2b(a+be^{d\sqrt{x}}e^c)}{a^2\sqrt{x}\sqrt{a+b}\sqrt{b-a}}\right)}{ad\sqrt{a+b}\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2)*(a + b/cosh(c + d*x^(1/2))))),x)`

[Out]  $(2*x^{(1/2)})/a + (2*b*\log((2*b*\exp(d*x^{(1/2)})*\exp(c))/(a^2*x^{(1/2)}) - (2*b*(a + b*\exp(d*x^{(1/2)})*\exp(c)))/(a^2*x^{(1/2)*(a + b)^{(1/2)*(b - a)^{(1/2))})})/(a*d*(a + b)^{(1/2)*(b - a)^{(1/2)}) - (2*b*\log((2*b*\exp(d*x^{(1/2)})*\exp(c))/(a^2*x^{(1/2)}) + (2*b*(a + b*\exp(d*x^{(1/2)})*\exp(c)))/(a^2*x^{(1/2)*(a + b)^{(1/2)*(b - a)^{(1/2))})})/(a*d*(a + b)^{(1/2)*(b - a)^{(1/2)})$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(c+d*x**(1/2)))/x**(1/2),x)`

[Out] `Integral(1/(sqrt(x)*(a + b*sech(c + d*sqrt(x))))), x)`

$$3.65 \quad \int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))}, x\right)$$

[Out] Unintegrable(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2))), x)

**Rubi** [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

[Out] Defer[Int][1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

Rubi steps

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

**Mathematica** [A] time = 8.83, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

[Out] Integrate[1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

**fricas** [A] time = 0.40, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x}}{bx^2 \operatorname{sech}(d\sqrt{x} + c) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(sqrt(x)/(b\*x^2\*sech(d\*sqrt(x) + c) + a\*x^2), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2))), x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*sqrt(x) + c) + a)\*x^(3/2)), x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-2b \int \frac{e^{(d\sqrt{x}+c)}}{a^2 x^{\frac{3}{2}} e^{(2d\sqrt{x}+2c)} + 2abx^{\frac{3}{2}} e^{(d\sqrt{x}+c)} + a^2 x^{\frac{3}{2}}} dx - \frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] -2\*b\*integrate(e^(d\*sqrt(x) + c)/(a^2\*x^(3/2)\*e^(2\*d\*sqrt(x) + 2\*c) + 2\*a\*b\*x^(3/2)\*e^(d\*sqrt(x) + c) + a^2\*x^(3/2)), x) - 2/(a\*sqrt(x))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(3/2)\*(a + b/cosh(c + d\*x^(1/2))))),x)

[Out] int(1/(x^(3/2)\*(a + b/cosh(c + d\*x^(1/2))))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(a+b\*sech(c+d\*x\*\*(1/2))),x)

[Out] Integral(1/(x\*\*(3/2)\*(a + b\*sech(c + d\*sqrt(x))))), x)

$$3.66 \quad \int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

**Optimal.** Leaf size=25

$$\operatorname{Int}\left(\frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))}, x\right)$$

[Out] Unintegrable(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2))), x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

[Out] Defer[Int][1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

Rubi steps

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

**Mathematica [A]** time = 9.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

[Out] Integrate[1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]])), x]

**fricas [A]** time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x}}{bx^3 \operatorname{sech}(d\sqrt{x} + c) + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2))), x, algorithm="fricas")

[Out] integral(sqrt(x)/(b\*x^3\*sech(d\*sqrt(x) + c) + a\*x^3), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)x^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2))), x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*sqrt(x) + c) + a)\*x^(5/2)), x)

**maple** [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2))),x)

[Out] int(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2))),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-2b \int \frac{e^{(d\sqrt{x}+c)}}{a^2x^{\frac{5}{2}}e^{(2d\sqrt{x}+2c)} + 2abx^{\frac{5}{2}}e^{(d\sqrt{x}+c)} + a^2x^{\frac{5}{2}}} dx - \frac{2}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2))),x, algorithm="maxima")

[Out] -2\*b\*integrate(e^(d\*sqrt(x) + c)/(a^2\*x^(5/2)\*e^(2\*d\*sqrt(x) + 2\*c) + 2\*a\*b\*x^(5/2)\*e^(d\*sqrt(x) + c) + a^2\*x^(5/2)), x) - 2/3/(a\*x^(3/2))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{5/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(5/2)\*(a + b/cosh(c + d\*x^(1/2))))),x)

[Out] int(1/(x^(5/2)\*(a + b/cosh(c + d\*x^(1/2))))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(a+b\*sech(c+d\*x\*\*(1/2))),x)

[Out] Integral(1/(x\*\*(5/2)\*(a + b\*sech(c + d\*sqrt(x))))), x)

$$3.67 \quad \int \frac{x^{3/2}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=1755

$$\frac{2x^2 \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{2x^2 \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{8x^{3/2} \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{8x^{3/2} \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - 24x$$

```
[Out] 48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-96*b*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)+96*b*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)-8*b^2*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+2*b^3*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-8*b^2*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-2*b^3*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-24*b^2*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+8*b^3*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-24*b^2*x*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-8*b^3*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-24*b^3*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+24*b^3*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+48*b*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-48*b*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+48*b^2*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^4+48*b^2*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^4-48*b^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^5-48*b^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^5-48*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+48*b^3*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+96*b*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-96*b*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)+2*b^2*x^2/a^2/(a^2-b^2)/d+2/5*x^(5/2)/a^2+2*b^2*x^2*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))
```

**Rubi [A]** time = 2.67, antiderivative size = 1755, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 11, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2531, 6609, 2282, 6589, 5562}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(a + b\*Sech[c + d\*Sqrt[x]])^2, x]

```
[Out] (2*b^2*x^2)/(a^2*(a^2 - b^2)*d) + (2*x^(5/2))/(5*a^2) - (8*b^2*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - (4*b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (8*b^2*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x^2*Log[1 + (
```

$$\begin{aligned}
& a^c E^{(c + d\sqrt{x})} / (b + \sqrt{-a^2 + b^2}) / (a^2(-a^2 + b^2)^{3/2}d) + \\
& (4b^2x^2 \text{Log}[1 + (a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2})]) / (a^2 \sqrt{-a^2 + b^2} d) - \\
& (24b^2x \text{PolyLog}[2, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2(a^2 - b^2)d^3) + \\
& (8b^3x^{3/2} \text{PolyLog}[2, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^2) - \\
& (16b^3x^{3/2} \text{PolyLog}[2, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^2) - \\
& (24b^2x \text{PolyLog}[2, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2(a^2 - b^2)d^3) - \\
& (8b^3x^{3/2} \text{PolyLog}[2, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^2) + \\
& (16b^3x^{3/2} \text{PolyLog}[2, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^2) + \\
& (48b^2 \sqrt{x} \text{PolyLog}[3, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (b - \sqrt{-a^2 + b^2}) / (a^2(a^2 - b^2)d^4) - \\
& (24b^3x \text{PolyLog}[3, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^3) + \\
& (48b^3x \text{PolyLog}[3, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^3) + \\
& (48b^2 \sqrt{x} \text{PolyLog}[3, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (b + \sqrt{-a^2 + b^2}) / (a^2(a^2 - b^2)d^4) + \\
& (24b^3x \text{PolyLog}[3, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^3) - \\
& (48b^3x \text{PolyLog}[3, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^3) - \\
& (48b^2 \text{PolyLog}[4, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (b - \sqrt{-a^2 + b^2}) / (a^2(a^2 - b^2)d^5) + \\
& (48b^3 \sqrt{x} \text{PolyLog}[4, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^4) - \\
& (96b \sqrt{x} \text{PolyLog}[4, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^4) - \\
& (48b^2 \text{PolyLog}[4, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (b + \sqrt{-a^2 + b^2}) / (a^2(a^2 - b^2)d^5) - \\
& (48b^3 \sqrt{x} \text{PolyLog}[4, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^4) + \\
& (96b \sqrt{x} \text{PolyLog}[4, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^4) - \\
& (48b^3 \text{PolyLog}[5, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^5) + \\
& (96b \text{PolyLog}[5, -((a^c E^{(c + d\sqrt{x})}) / (b - \sqrt{-a^2 + b^2}))]) / (a^2 \sqrt{-a^2 + b^2} d^5) + \\
& (48b^3 \text{PolyLog}[5, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (a^2(-a^2 + b^2)^{3/2}d^5) - \\
& (96b \text{PolyLog}[5, -((a^c E^{(c + d\sqrt{x})}) / (b + \sqrt{-a^2 + b^2}))]) / (b + \sqrt{-a^2 + b^2}) / (a^2 \sqrt{-a^2 + b^2} d^5) + \\
& (2b^2x^2 \text{Sinh}[c + d\sqrt{x}]) / (a(a^2 - b^2)d(b + a \text{Cosh}[c + d\sqrt{x}]))
\end{aligned}$$

#### Rule 2190

```

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

#### Rule 2264

```

Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

#### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

```

#### Rule 2531



```
Int[Log[1 + (e_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 3320

```
Int[((c_.) + (d_.)*(x_)^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 3324

```
Int[((c_.) + (d_.)*(x_)^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

#### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^((c_.) + (d_.)*(x_)^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

#### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

#### Rule 5562

```
Int[((e_.) + (f_.)*(x_)^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_.), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_
.)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{x^4}{(a + b \operatorname{sech}(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^4}{a^2} + \frac{b^2 x^4}{a^2 (b + a \cosh(c + dx))^2} - \frac{2bx^4}{a^2 (b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{2x^{5/2}}{5a^2} - \frac{(4b) \operatorname{Subst} \left( \int \frac{x^4}{b+a \cosh(c+dx)} dx, x, \sqrt{x} \right)}{a^2} + \frac{(2b^2) \operatorname{Subst} \left( \int \frac{x^4}{(b+a \cosh(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2x^{5/2}}{5a^2} + \frac{2b^2 x^2 \sinh(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cosh(c + d\sqrt{x}))} - \frac{(8b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^4}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{2b^2 x^2 \sinh(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cosh(c + d\sqrt{x}))} - \frac{(4b^3) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^4}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} - \frac{4bx^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x^2}{a^2(a^2 - b^2)d} + \frac{2x^{5/2}}{5a^2} - \frac{8b^2 x^{3/2} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x^2 \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 15.43, size = 1769, normalized size = 1.01

result too large to display

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (2\*(b + a\*Cosh[c + d\*Sqrt[x]])\*Sech[c + d\*Sqrt[x]]^2\*(x^(5/2)\*(b + a\*Cosh[c + d\*Sqrt[x]]) + (5\*b\*E^c\*(b + a\*Cosh[c + d\*Sqrt[x]])\*(2\*b\*E^c\*x^2 - ((1 +

$$\begin{aligned}
& E^{(2c)} * (4 * b * d^3 * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] * x^{(3/2)} * \text{Log}[1 + (a * E^{(2c)} + d * \text{Sqrt}[x])] / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) + 2 * a^2 * d^4 * E^c * x^2 * \text{Log}[1 + (a * E^{(2c)} + d * \text{Sqrt}[x])] / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) - b^2 * d^4 * E^c * x^2 * \text{Log}[1 + (a * E^{(2c)} + d * \text{Sqrt}[x])] / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) \\
& + 4 * b * d^3 * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] * x^{(3/2)} * \text{Log}[1 + (a * E^{(2c)} + d * \text{Sqrt}[x])] / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) - 2 * a^2 * d^4 * E^c * x^2 * \text{Log}[1 + (a * E^{(2c)} + d * \text{Sqrt}[x])] / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) + b^2 * d^4 * E^c * x^2 * \text{Log}[1 + (a * E^{(2c)} + d * \text{Sqrt}[x])] / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) + 4 * d^2 * (3 * b * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] + 2 * a^2 * d * E^c * \text{Sqrt}[x] - b^2 * d * E^c * \text{Sqrt}[x]) * x * \text{PolyLog}[2, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 4 * d^2 * (3 * b * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] - 2 * a^2 * d * E^c * \text{Sqrt}[x] + b^2 * d * E^c * \text{Sqrt}[x]) * x * \text{PolyLog}[2, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 24 * b * d * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] * \text{Sqrt}[x] * \text{PolyLog}[3, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 24 * a^2 * d^2 * E^c * x * \text{PolyLog}[3, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 12 * b^2 * d^2 * E^c * x * \text{PolyLog}[3, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 24 * b * d * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] * \text{Sqrt}[x] * \text{PolyLog}[3, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 24 * a^2 * d^2 * E^c * x * \text{PolyLog}[3, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 12 * b^2 * d^2 * E^c * x * \text{PolyLog}[3, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 24 * b * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] * \text{PolyLog}[4, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 48 * a^2 * d * E^c * \text{Sqrt}[x] * \text{PolyLog}[4, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 24 * b^2 * d * E^c * \text{Sqrt}[x] * \text{PolyLog}[4, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 24 * b * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}] * \text{PolyLog}[4, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 48 * a^2 * d * E^c * \text{Sqrt}[x] * \text{PolyLog}[4, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 24 * b^2 * d * E^c * \text{Sqrt}[x] * \text{PolyLog}[4, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 48 * a^2 * E^c * \text{PolyLog}[5, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 24 * b^2 * E^c * \text{PolyLog}[5, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c - \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] + 48 * a^2 * E^c * \text{PolyLog}[5, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] - 24 * b^2 * E^c * \text{PolyLog}[5, -((a * E^{(2c)} + d * \text{Sqrt}[x]) / (b * E^c + \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}]))] / (d^4 * E^c * \text{Sqrt}[(-a^2 + b^2) * E^{(2c)}])) / ((a^2 - b^2) * d * (1 + E^{(2c)})) + (5 * b^2 * x^2 * \text{Sech}[c] * (-b * \text{Sinh}[c]) + a * \text{Sinh}[d * \text{Sqrt}[x]]) / ((a - b) * (a + b) * d)) / (5 * a^2 * (a + b * \text{Sech}[c + d * \text{Sqrt}[x]])^2)
\end{aligned}$$

**fricas** [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^{\frac{3}{2}}}{b^2 \text{sech}(d\sqrt{x} + c)^2 + 2ab \text{sech}(d\sqrt{x} + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(x^(3/2)/(b^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sech(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(b \text{sech}(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(x^(3/2)/(b\*sech(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

[Out] `int(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2)))^2,x)`

[Out] `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2)))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2)/(a+b*sech(c+d*x**(1/2)))**2,x)`

[Out] `Integral(x**(3/2)/(a + b*sech(c + d*sqrt(x)))**2, x)`

$$3.68 \quad \int \frac{\sqrt{x}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=1027

$$\frac{2x \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{2x \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{4\sqrt{x} \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{4\sqrt{x} \operatorname{Li}_2\left(-\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{4\operatorname{Li}_3\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} + \frac{4\operatorname{Li}_3\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2}$$

[Out]  $2*b^2*x/a^2/(a^2-b^2)/d+2/3*x^{(3/2)}/a^2+2*b^3*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d-2*b^3*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d-4*b^2*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^3-4*b^2*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^3-4*b^3*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^3+4*b^3*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^3+2*b^2*x*\sinh(c+d*x^{(1/2)})/a/(a^2-b^2)/d/(b+a*\cosh(c+d*x^{(1/2)}))-4*b*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/d/(-a^2+b^2)^{(1/2)}+4*b*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/d/(-a^2+b^2)^{(1/2)}+8*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/d^3/(-a^2+b^2)^{(1/2)}-8*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/d^3/(-a^2+b^2)^{(1/2)}-4*b^2*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(a^2-b^2)/d^2-4*b^2*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(a^2-b^2)/d^2+4*b^3*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^2-4*b^3*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^2-8*b*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/d^2/(-a^2+b^2)^{(1/2)}+8*b*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/d^2/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 1.90, antiderivative size = 1027, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$ , Rules used = {5436, 4191, 3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{2x \log\left(\frac{e^{c+d\sqrt{x}} a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{2x \log\left(\frac{e^{c+d\sqrt{x}} a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{4\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{4\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out]  $(2*b^2*x)/(a^2*(a^2 - b^2)*d) + (2*x^{(3/2)})/(3*a^2) - (4*b^2*Sqrt[x]*Log[1 + (a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (2*b^3*x*Log[1 + (a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d}) - (4*b*x*Log[1 + (a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (4*b^2*Sqrt[x]*Log[1 + (a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (2*b^3*x*Log[1 + (a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2])])/(b + Sqrt[-a^2 + b^2])/(a^2*(-a^2 + b^2)^{(3/2)*d}) + (4*b*x*Log[1 + (a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (4*b^2*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^{(3/2)*d^2}) - (8*b*Sqrt[x]*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (4*b^2*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) - (4*b^3*Sqrt[x]*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^{(3/2)*d^2}) + (8*b*Sqrt[x]*PolyLog[2, -((a*E^{(c + d*Sqrt[x])})/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^2)$

```
2]*d^2) - (4*b^3*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^3) + (8*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])))]/(a^2*Sqrt[-a^2 + b^2]*d^3) + (4*b^3*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])))]/(a^2*(-a^2 + b^2)^(3/2)*d^3) - (8*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])))]/(a^2*Sqrt[-a^2 + b^2]*d^3) + (2*b^2*x*Sinh[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*Sqrt[x]]))
```

#### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_)]/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

#### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)*(x_)]^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 3320

```
Int[((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*sin[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-(I*e) + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*(-(I*e) + f*fz*x)))/E^(2*I*k*Pi))], x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[(b*(c + d*x)^m*cos[e + f*x])/(f*(a^2 - b^2)*(a + b*sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*cos[e + f*x])/(a
+ b*sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

#### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

#### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

#### Rule 5562

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)])*(b_.) + (a_.), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{x^2}{(a + b \operatorname{sech}(c + dx))^2} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left( \int \left( \frac{x^2}{a^2} + \frac{b^2 x^2}{a^2 (b + a \cosh(c + dx))^2} - \frac{2bx^2}{a^2 (b + a \cosh(c + dx))} \right) dx, x, \sqrt{x} \right) \\
&= \frac{2x^{3/2}}{3a^2} - \frac{(4b) \operatorname{Subst} \left( \int \frac{x^2}{b+a \cosh(c+dx)} dx, x, \sqrt{x} \right)}{a^2} + \frac{(2b^2) \operatorname{Subst} \left( \int \frac{x^2}{(b+a \cosh(c+dx))^2} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2x^{3/2}}{3a^2} + \frac{2b^2 x \sinh(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cosh(c + d\sqrt{x}))} - \frac{(8b) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2b^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{2b^2 x \sinh(c + d\sqrt{x})}{a(a^2 - b^2)d(b + a \cosh(c + d\sqrt{x}))} - \frac{(4b^3) \operatorname{Subst} \left( \int \frac{e^{c+dx} x^2}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, \sqrt{x} \right)}{a^2} \\
&= \frac{2b^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} - \frac{4bx \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2 \sqrt{-a^2 + b^2} d} \\
&= \frac{2b^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d} \\
&= \frac{2b^2 x}{a^2(a^2 - b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2 \sqrt{x} \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(a^2 - b^2)d^2} + \frac{2b^3 x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{a^2(-a^2 + b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 13.45, size = 986, normalized size = 0.96

$$2(b + a \cosh(c + d\sqrt{x})) \operatorname{sech}^2(c + d\sqrt{x}) \frac{3x \operatorname{sech}(c)(b \sinh(c) - a \sinh(d\sqrt{x}))b^2}{(b^2 - a^2)d} + \frac{3e^c(b + a \cosh(c + d\sqrt{x})) \left( \frac{e^{-c(1+e^{2c})} 2d^2 e^c x \log \left( 1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}} \right)}{2be^c x} \right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(a + b\*Sech[c + d\*Sqrt[x]])^2,x]

[Out] (2\*(b + a\*Cosh[c + d\*Sqrt[x]])\*Sech[c + d\*Sqrt[x]]^2\*(x^(3/2)\*(b + a\*Cosh[c + d\*Sqrt[x]])) + (3\*b\*E^c\*(b + a\*Cosh[c + d\*Sqrt[x]]))\*(2\*b\*E^c\*x - ((1 + E^



$(2*c)) * (2*b*d*\text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]*\text{Sqrt}[x]*\text{Log}[1 + (a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c - \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])] + 2*a^2*d^2*E^c*x*\text{Log}[1 + (a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c - \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])] - b^2*d^2*E^c*x*\text{Log}[1 + (a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])] + 2*b*d*\text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]*\text{Sqrt}[x]*\text{Log}[1 + (a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])] - 2*a^2*d^2*E^c*x*\text{Log}[1 + (a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])] + b^2*d^2*E^c*x*\text{Log}[1 + (a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])] + 2*(b*\text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}] + 2*a^2*d*E^c*\text{Sqrt}[x] - b^2*d*E^c*\text{Sqrt}[x])*PolyLog[2, -((a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c - \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]))] + 2*(b*\text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}] - 2*a^2*d*E^c*\text{Sqrt}[x] + b^2*d*E^c*\text{Sqrt}[x])*PolyLog[2, -((a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]))] - 4*a^2*E^c*PolyLog[3, -((a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c - \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]))] + 2*b^2*E^c*PolyLog[3, -((a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c - \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]))] + 4*a^2*E^c*PolyLog[3, -((a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]))] - 2*b^2*E^c*PolyLog[3, -((a*E^{(2*c + d*\text{Sqrt}[x]))}/(b*E^c + \text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}]))]/(d^2*E^c*\text{Sqrt}[(-a^2 + b^2)*E^{(2*c)}])]/((a^2 - b^2)*d*(1 + E^{(2*c)})) + (3*b^2*x*Sech[c]*(b*Sinh[c] - a*Sinh[d*\text{Sqrt}[x]))]/((-a^2 + b^2)*d))/((3*a^2*(a + b*Sech[c + d*\text{Sqrt}[x]))^2)$

**fricas** [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{x}}{b^2 \text{sech}(d\sqrt{x} + c)^2 + 2ab \text{sech}(d\sqrt{x} + c) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*sech(d\*sqrt(x) + c) + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(b \text{sech}(d\sqrt{x} + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(sqrt(x)/(b\*sech(d\*sqrt(x) + c) + a)^2, x)

**maple** [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(a + b \text{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(x^(1/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details) Is a-b positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x}}{\left(a + \frac{b}{\cosh(c+d\sqrt{x})}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(a + b/cosh(c + d\*x^(1/2)))^2,x)

[Out] int(x^(1/2)/(a + b/cosh(c + d\*x^(1/2)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(sqrt(x)/(a + b\*sech(c + d\*sqrt(x)))\*\*2, x)

$$3.69 \quad \int \frac{1}{\sqrt{x} (a+b \operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Optimal.** Leaf size=127

$$-\frac{4b(2a^2 - b^2) \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{2b^2 \tanh(c+d\sqrt{x})}{ad(a^2 - b^2)(a+b \operatorname{sech}(c+d\sqrt{x}))} + \frac{2\sqrt{x}}{a^2}$$

[Out]  $-4*b*(2*a^2-b^2)*\arctan((a-b)^{(1/2)}*\tanh(1/2*c+1/2*d*x^{(1/2)})/(a+b)^{(1/2)})/a^2/(a-b)^{(3/2)}/(a+b)^{(3/2)}/d+2*x^{(1/2)}/a^2+2*b^2*\tanh(c+d*x^{(1/2)})/a/(a^2-b^2)/d/(a+b*\operatorname{sech}(c+d*x^{(1/2)}))$

**Rubi [A]** time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {5436, 3785, 3919, 3831, 2659, 208}

$$-\frac{4b(2a^2 - b^2) \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a^2 d(a-b)^{3/2}(a+b)^{3/2}} + \frac{2b^2 \tanh(c+d\sqrt{x})}{ad(a^2 - b^2)(a+b \operatorname{sech}(c+d\sqrt{x}))} + \frac{2\sqrt{x}}{a^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]\*(a + b\*Sech[c + d\*Sqrt[x]])^2), x]

[Out]  $(2*\operatorname{Sqrt}[x])/a^2 - (4*b*(2*a^2 - b^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[a - b]*\operatorname{Tanh}[(c + d*\operatorname{Sqrt}[x])/2])/(\operatorname{Sqrt}[a + b])]/(a^2*(a - b)^{(3/2)}*(a + b)^{(3/2)*d}) + (2*b^2*\operatorname{Tanh}[c + d*\operatorname{Sqrt}[x]])/(a*(a^2 - b^2)*d*(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]]))$

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3785

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.) + (a\_)^(n\_), x\_Symbol] := Simp[(b^2\*Cot[c + d\*x]\*(a + b\*Csc[c + d\*x])^(n + 1))/(a\*d\*(n + 1)\*(a^2 - b^2)), x] + Dist[1/(a\*(n + 1)\*(a^2 - b^2)), Int[(a + b\*Csc[c + d\*x])^(n + 1)\*Simp[(a^2 - b^2)\*(n + 1) - a\*b\*(n + 1)\*Csc[c + d\*x] + b^2\*(n + 2)\*Csc[c + d\*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3831

Int[csc[(e\_.) + (f\_.)\*(x\_)]/(csc[(e\_.) + (f\_.)\*(x\_)])\*(b\_.) + (a\_), x\_Symbol] := Dist[1/b, Int[1/(1 + (a\*Sin[e + f\*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx &= 2 \operatorname{Subst} \left( \int \frac{1}{(a + b \operatorname{sech}(c + dx))^2} dx, x, \sqrt{x} \right) \\ &= \frac{2b^2 \tanh(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \operatorname{sech}(c + d\sqrt{x}))} - \frac{2 \operatorname{Subst} \left( \int \frac{-a^2 + b^2 + ab \operatorname{sech}(c + dx)}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right)}{a(a^2 - b^2)} \\ &= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tanh(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \operatorname{sech}(c + d\sqrt{x}))} - \frac{(2b(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{\operatorname{sech}(c + dx)}{a + b \operatorname{sech}(c + dx)} dx, x, \sqrt{x} \right)}{a^2(a^2 - b^2)} \\ &= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tanh(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \operatorname{sech}(c + d\sqrt{x}))} - \frac{(2(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b \operatorname{sech}(c + dx)}} dx, x, \sqrt{x} \right)}{a^2(a^2 - b^2)} \\ &= \frac{2\sqrt{x}}{a^2} + \frac{2b^2 \tanh(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \operatorname{sech}(c + d\sqrt{x}))} + \frac{(4i(2a^2 - b^2)) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{a}{b \operatorname{sech}(c + dx)}} dx, x, \sqrt{x} \right)}{a^2} \\ &= \frac{2\sqrt{x}}{a^2} - \frac{4b(2a^2 - b^2) \tan^{-1} \left( \frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c + d\sqrt{x})\right)}{\sqrt{a+b}} \right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2 \tanh(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \operatorname{sech}(c + d\sqrt{x}))} \end{aligned}$$

**Mathematica [A]** time = 0.52, size = 232, normalized size = 1.83

$$\frac{2 \left( b \left( (a^2 - b^2)^{3/2} (c + d\sqrt{x}) + ab\sqrt{a^2 - b^2} \sinh(c + d\sqrt{x}) + (4a^2b - 2b^3) \tan^{-1} \left( \frac{(b-a) \tanh\left(\frac{1}{2}(c + d\sqrt{x})\right)}{\sqrt{a^2 - b^2}} \right) \right) \right) + a \cosh(c + d\sqrt{x})}{a^2 d (a - b) (a + b) \sqrt{a^2 - b^2} (a \cosh(c + d\sqrt{x}) + b \sinh(c + d\sqrt{x}))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2), x]
```

```
[Out] (2*(a*((a^2 - b^2)^(3/2)*(c + d*Sqrt[x]) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*Sqrt[x]] + b*((a^2 - b^2)^(3/2)*(c + d*Sqrt[x]) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*Sqrt[x]]))/((a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*(b + a*Cosh[c + d*Sqrt[x]]))
```

**fricas [B]** time = 0.48, size = 1387, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sech(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-2*(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x})*\cosh(d*\sqrt{x} + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x}*\sinh(d*\sqrt{x} + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x} + 2*(a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*\sqrt{x})*\cosh(d*\sqrt{x} + c) + ((2*a^3*b - a*b^3)*\sqrt{-a^2 + b^2}*\cosh(d*\sqrt{x} + c)^2 + (2*a^3*b - a*b^3)*\sqrt{-a^2 + b^2}*\sinh(d*\sqrt{x} + c)^2 + 2*(2*a^2*b^2 - b^4)*\sqrt{-a^2 + b^2}*\cosh(d*\sqrt{x} + c) + 2*((2*a^3*b - a*b^3)*\sqrt{-a^2 + b^2}*\cosh(d*\sqrt{x} + c) + (2*a^2*b^2 - b^4)*\sqrt{-a^2 + b^2})*\sinh(d*\sqrt{x} + c) + (2*a^3*b - a*b^3)*\sqrt{-a^2 + b^2})*\log((a*b + (b^2 + \sqrt{-a^2 + b^2})*b)*\cosh(d*\sqrt{x} + c) + (a^2 - b^2 - \sqrt{-a^2 + b^2})*b)*\sinh(d*\sqrt{x} + c) + \sqrt{-a^2 + b^2}*a)/(a*\cosh(d*\sqrt{x} + c) + b) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x})*\cosh(d*\sqrt{x} + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*\sqrt{x})*\sinh(d*\sqrt{x} + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*\cosh(d*\sqrt{x} + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*\sinh(d*\sqrt{x} + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*\cosh(d*\sqrt{x} + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*\cosh(d*\sqrt{x} + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*\sinh(d*\sqrt{x} + c)), -2*(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x})*\cosh(d*\sqrt{x} + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x}*\sinh(d*\sqrt{x} + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x} - 2*((2*a^3*b - a*b^3)*\sqrt{a^2 - b^2}*\cosh(d*\sqrt{x} + c)^2 + (2*a^3*b - a*b^3)*\sqrt{a^2 - b^2}*\sinh(d*\sqrt{x} + c)^2 + 2*(2*a^2*b^2 - b^4)*\sqrt{a^2 - b^2}*\cosh(d*\sqrt{x} + c) + 2*((2*a^3*b - a*b^3)*\sqrt{a^2 - b^2}*\cosh(d*\sqrt{x} + c) + (2*a^2*b^2 - b^4)*\sqrt{a^2 - b^2})*\sinh(d*\sqrt{x} + c) + (2*a^3*b - a*b^3)*\sqrt{a^2 - b^2})*\arctan(-(\sqrt{a^2 - b^2})*a*\cosh(d*\sqrt{x} + c) + \sqrt{a^2 - b^2})*a*\sinh(d*\sqrt{x} + c) + \sqrt{a^2 - b^2})*b)/(a^2 - b^2) + 2*(a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*\sqrt{x})*\cosh(d*\sqrt{x} + c) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d*\sqrt{x})*\cosh(d*\sqrt{x} + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*\sqrt{x})*\sinh(d*\sqrt{x} + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*\cosh(d*\sqrt{x} + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*\sinh(d*\sqrt{x} + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*\cosh(d*\sqrt{x} + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*\cosh(d*\sqrt{x} + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*\sinh(d*\sqrt{x} + c))]$$

**giac** [A] time = 0.16, size = 148, normalized size = 1.17

$$\frac{4(2a^2b - b^3) \arctan\left(\frac{ae^{(d\sqrt{x}+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^4d - a^2b^2d)\sqrt{a^2-b^2}} - \frac{4\left(b^3e^{(d\sqrt{x}+c)} + ab^2\right)}{(a^4d - a^2b^2d)\left(ae^{(2d\sqrt{x}+2c)} + 2be^{(d\sqrt{x}+c)} + a\right)} + \frac{2(d\sqrt{x} + c)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*sech(c+d\*x^(1/2)))^2/x^(1/2),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -4*(2*a^2*b - b^3)*\arctan((a*e^{(d*\sqrt{x} + c)} + b)/\sqrt{a^2 - b^2})/((a^4*d - a^2*b^2*d)*\sqrt{a^2 - b^2}) - 4*(b^3*e^{(d*\sqrt{x} + c)} + a*b^2)/((a^4*d - a^2*b^2*d)*(a*e^{(2*d*\sqrt{x} + 2*c)} + 2*b*e^{(d*\sqrt{x} + c)} + a)) + 2*(d*\sqrt{x} + c)/(a^2*d) \end{aligned}$$

**maple** [B] time = 0.42, size = 236, normalized size = 1.86

$$\frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 1\right)}{d a^2} + \frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) + 1\right)}{d a^2} + \frac{4 b^2 \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{d a (a^2 - b^2) \left(a \left(\tanh^2\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right) - \left(\tanh^2\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x)`

[Out] 
$$-2/d/a^2*\ln(\tanh(1/2*c+1/2*d*x^(1/2))-1)+2/d/a^2*\ln(\tanh(1/2*c+1/2*d*x^(1/2))+1)+4/d/a*b^2/(a^2-b^2)*\tanh(1/2*c+1/2*d*x^(1/2))/(a*\tanh(1/2*c+1/2*d*x^(1/2))^2-\tanh(1/2*c+1/2*d*x^(1/2))^2*b+a+b)-8/d*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*\arctan((a-b)*\tanh(1/2*c+1/2*d*x^(1/2))/((a+b)*(a-b))^(1/2))+4/d/a^2*b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*\arctan((a-b)*\tanh(1/2*c+1/2*d*x^(1/2))/((a+b)*(a-b))^(1/2))$$

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(4\*b^2-4\*a^2>0)', see `assume?` for more details)Is 4\*b^2-4\*a^2 positive or negative?

**mupad** [B] time = 1.85, size = 344, normalized size = 2.71

$$\frac{2\sqrt{x}}{a^2} - \frac{4b^2\sqrt{x}}{d(a^3\sqrt{x}-ab^2\sqrt{x})} + \frac{4b^3\sqrt{x}e^{c+d\sqrt{x}}}{ad(a^3\sqrt{x}-ab^2\sqrt{x})} + \frac{\ln\left(\frac{2e^{c+d\sqrt{x}}(2a^2b-b^3)}{a^3\sqrt{x}(a^2-b^2)} - \frac{(4a^2b-2b^3)(a+be^{c+d\sqrt{x}})}{a^3\sqrt{x}(a+b)^{3/2}(b-a)^{3/2}}\right)}{a^2d(a+b)^{3/2}(b-a)^{3/2}} - \frac{2b\ln\left(\frac{2e^{c+d\sqrt{x}}(2a^2b-b^3)}{a^3\sqrt{x}(a^2-b^2)} - \frac{(4a^2b-2b^3)(a+be^{c+d\sqrt{x}})}{a^3\sqrt{x}(a+b)^{3/2}(b-a)^{3/2}}\right)}{a^2d(a+b)^{3/2}(b-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2)*(a + b/cosh(c + d*x^(1/2))))^2,x)`

[Out] 
$$(2*x^(1/2))/a^2 - ((4*b^2*x^(1/2))/(d*(a^3*x^(1/2) - a*b^2*x^(1/2))) + (4*b^3*x^(1/2)*\exp(c + d*x^(1/2)))/(a*d*(a^3*x^(1/2) - a*b^2*x^(1/2))))/(a + 2*b*\exp(c + d*x^(1/2)) + a*\exp(2*c + 2*d*x^(1/2))) + (\log((2*\exp(c + d*x^(1/2)))*(2*a^2*b - b^3))/(a^3*x^(1/2)*(a^2 - b^2)) - ((4*a^2*b - 2*b^3)*(a + b*\exp(c + d*x^(1/2)))/(a^3*x^(1/2)*(a + b)^(3/2)*(b - a)^(3/2)))*(4*a^2*b - 2*b^3))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2)) - (2*b*\log((2*\exp(c + d*x^(1/2)))*(2*a^2*b - b^3))/(a^3*x^(1/2)*(a^2 - b^2)) + (2*b*(a + b*\exp(c + d*x^(1/2)))*(2*a^2 - b^2))/(a^3*x^(1/2)*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sech(c+d*x**(1/2)))**2/x**(1/2),x)`

[Out] `Integral(1/(sqrt(x)*(a + b*sech(c + d*sqrt(x)))**2), x)`

$$3.70 \quad \int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2}, x\right)$$

[Out] Unintegrable(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2, x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

[Out] Defer[Int][1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

Rubi steps

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Mathematica [A]** time = 66.93, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

[Out] Integrate[1/(x^(3/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

**fricas [A]** time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x}}{b^2x^2\operatorname{sech}(d\sqrt{x}+c)^2+2abx^2\operatorname{sech}(d\sqrt{x}+c)+a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2, x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2\*x^2\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^2\*sech(d\*sqrt(x) + c) + a^2\*x^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\operatorname{sech}(d\sqrt{x}+c)+a)^2 x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*sech(d\*sqrt(x) + c) + a)^2\*x^(3/2)), x)

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( 2ab^2 + (a^3de^{2c} - ab^2de^{2c})\sqrt{x}e^{2d\sqrt{x}} + 2(b^3e^c + (a^2bde^c - b^3de^c)\sqrt{x})e^{d\sqrt{x}} + (a^3d - ab^2d)\sqrt{x} \right)}{(a^5de^{2c} - a^3b^2de^{2c})xe^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)xe^{d\sqrt{x}} + (a^5d - a^3b^2d)x} \int \frac{1}{(a^5d - a^3b^2d)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] -2\*(2\*a\*b^2 + (a^3\*d\*e^(2\*c) - a\*b^2\*d\*e^(2\*c))\*sqrt(x)\*e^(2\*d\*sqrt(x)) + 2\*(b^3\*e^c + (a^2\*b\*d\*e^c - b^3\*d\*e^c)\*sqrt(x))\*e^(d\*sqrt(x)) + (a^3\*d - a\*b^2\*d)\*sqrt(x))/((a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*x\*e^(2\*d\*sqrt(x)) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*x\*e^(d\*sqrt(x)) + (a^5\*d - a^3\*b^2\*d)\*x) - integrate(2\*(2\*a\*b^2\*sqrt(x) + (2\*b^3\*sqrt(x)\*e^c + (2\*a^2\*b\*d\*e^c - b^3\*d\*e^c)\*x)\*e^(d\*sqrt(x)))/((a^5\*d\*e^(2\*c) - a^3\*b^2\*d\*e^(2\*c))\*x^(5/2)\*e^(2\*d\*sqrt(x)) + 2\*(a^4\*b\*d\*e^c - a^2\*b^3\*d\*e^c)\*x^(5/2)\*e^(d\*sqrt(x)) + (a^5\*d - a^3\*b^2\*d)\*x^(5/2)), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(3/2)\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

[Out] int(1/(x^(3/2)\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(3/2)/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(1/(x\*\*(3/2)\*(a + b\*sech(c + d\*sqrt(x)))\*\*2), x)



$$3.71 \quad \int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2}, x\right)$$

[Out] Unintegrable(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2)))^2, x)

**Rubi** [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

[Out] Defer[Int][1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

Rubi steps

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**Mathematica** [A] time = 67.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

[Out] Integrate[1/(x^(5/2)\*(a + b\*Sech[c + d\*Sqrt[x]]))^2, x]

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{x}}{b^2x^3\operatorname{sech}(d\sqrt{x}+c)^2+2abx^3\operatorname{sech}(d\sqrt{x}+c)+a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2\*x^3\*sech(d\*sqrt(x) + c)^2 + 2\*a\*b\*x^3\*sech(d\*sqrt(x) + c) + a^2\*x^3), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

*sage0x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="giac")

[Out] sage0\*x

**maple** [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x)

[Out] int(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \left( 6ab^2 + (a^3de^{2c} - ab^2de^{2c})\sqrt{x}e^{2d\sqrt{x}} + 2(3b^3e^c + (a^2bde^c - b^3de^c)\sqrt{x})e^{d\sqrt{x}} + (a^3d - ab^2d)\sqrt{x} \right)}{3 \left( (a^5de^{2c} - a^3b^2de^{2c})x^2e^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)x^2e^{d\sqrt{x}} + (a^5d - a^3b^2d)x^2 \right)} \int \frac{1}{(a^5de^{2c} - a^3b^2de^{2c})x^2e^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)x^2e^{d\sqrt{x}} + (a^5d - a^3b^2d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(a+b\*sech(c+d\*x^(1/2)))^2,x, algorithm="maxima")

[Out] 
$$-\frac{2}{3} \left( 6a^3b^2e^{2c} + (a^5d - a^3b^2d)x^2e^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)x^2e^{d\sqrt{x}} + (a^5d - a^3b^2d)x^2 \right) \int \frac{1}{(a^5de^{2c} - a^3b^2de^{2c})x^2e^{2d\sqrt{x}} + 2(a^4bde^c - a^2b^3de^c)x^2e^{d\sqrt{x}} + (a^5d - a^3b^2d)x^2} dx$$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{5/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(5/2)\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

[Out] int(1/(x^(5/2)\*(a + b/cosh(c + d\*x^(1/2))))^2,x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*(5/2)/(a+b\*sech(c+d\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(1/(x\*\*(5/2)\*(a + b\*sech(c + d\*sqrt(x)))\*\*2), x)

### 3.72 $\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$

Optimal. Leaf size=32

$$x^{-m}(ex)^m \operatorname{Int}\left(x^m (a + b \operatorname{sech}(c + dx^n))^p, x\right)$$

[Out]  $(e*x)^m * \operatorname{Unintegrable}(x^m * (a + b * \operatorname{sech}(c + d*x^n))^p, x) / (x^m)$

**Rubi** [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(e*x)^m * (a + b * \operatorname{Sech}[c + d*x^n])^p, x]$

[Out]  $((e*x)^m * \operatorname{Defer}[\operatorname{Int}[x^m * (a + b * \operatorname{Sech}[c + d*x^n])^p, x]]) / x^m$

Rubi steps

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx = (x^{-m}(ex)^m) \int x^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

Mathematica [A] time = 18.34, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[(e*x)^m * (a + b * \operatorname{Sech}[c + d*x^n])^p, x]$

[Out]  $\operatorname{Integrate}[(e*x)^m * (a + b * \operatorname{Sech}[c + d*x^n])^p, x]$

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((ex)^m (b \operatorname{sech}(dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((e*x)^m * (a + b * \operatorname{sech}(c + d*x^n))^p, x, \text{algorithm}="fricas")$

[Out]  $\operatorname{integral}((e*x)^m * (b * \operatorname{sech}(d*x^n + c) + a)^p, x)$

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b \operatorname{sech}(dx^n + c) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((e*x)^m * (a + b * \operatorname{sech}(c + d*x^n))^p, x, \text{algorithm}="giac")$

[Out]  $\operatorname{integrate}((e*x)^m * (b * \operatorname{sech}(d*x^n + c) + a)^p, x)$

maple [A] time = 0.70, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^m*(a+b*sech(c+d*x^n))^p,x)`

[Out] `int((e*x)^m*(a+b*sech(c+d*x^n))^p,x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b \operatorname{sech}(dx^n + c) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="maxima")`

[Out] `integrate((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \left( a + \frac{b}{\cosh(c + dx^n)} \right)^p (ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/cosh(c + d*x^n))^p*(e*x)^m,x)`

[Out] `int((a + b/cosh(c + d*x^n))^p*(e*x)^m, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**m*(a+b*sech(c+d*x**n))**p,x)`

[Out] `Integral((e*x)**m*(a + b*sech(c + d*x**n))**p, x)`

### 3.73 $\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx$

Optimal. Leaf size=44

$$\frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den}$$

[Out] a\*(e\*x)^n/e/n+b\*(e\*x)^n\*arctan(sinh(c+d\*x^n))/d/e/n/(x^n)

**Rubi [A]** time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {14, 5440, 5436, 3770}

$$\frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + n)\*(a + b\*Sech[c + d\*x^n]), x]

[Out] (a\*(e\*x)^n)/(e\*n) + (b\*(e\*x)^n\*ArcTan[Sinh[c + d\*x^n]])/(d\*e\*n\*x^n)

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 3770

Int[csc[(c\_.) + (d\_)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 5436

Int[(x\_)^(m\_)\*((a\_.) + (b\_)\*Sech[(c\_.) + (d\_)\*(x\_)^(n\_)])^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rule 5440

Int[((e\_)\*(x\_))^(m\_)\*((a\_.) + (b\_)\*Sech[(c\_.) + (d\_)\*(x\_)^(n\_)])^(p\_), x\_Symbol] :> Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sech[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

#### Rubi steps

$$\begin{aligned} \int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx &= \int (a(ex)^{-1+n} + b(ex)^{-1+n} \operatorname{sech}(c + dx^n)) dx \\ &= \frac{a(ex)^n}{en} + b \int (ex)^{-1+n} \operatorname{sech}(c + dx^n) dx \\ &= \frac{a(ex)^n}{en} + \frac{(bx^{-n}(ex)^n) \int x^{-1+n} \operatorname{sech}(c + dx^n) dx}{e} \\ &= \frac{a(ex)^n}{en} + \frac{(bx^{-n}(ex)^n) \operatorname{Subst}\left(\int \operatorname{sech}(c + dx) dx, x, x^n\right)}{en} \\ &= \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 41, normalized size = 0.93

$$\frac{x^{-n}(ex)^n \left( a(c + dx^n) + b \tan^{-1}(\sinh(c + dx^n)) \right)}{den}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1+n)\*(a+b\*Sech[c+d\*x^n]),x]

[Out] ((e\*x)^n\*(a\*(c+d\*x^n)+b\*ArcTan[Sinh[c+d\*x^n]]))/(d\*e\*n\*x^n)

**fricas [B]** time = 0.41, size = 122, normalized size = 2.77

$$\frac{ad \cosh((n-1)\log(e)) \cosh(n \log(x)) + ad \cosh(n \log(x)) \sinh((n-1)\log(e)) + 2(b \cosh((n-1)\log(e)) + b \sinh((n-1)\log(e))) \operatorname{arctan}(\cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)) + (a*d*\cosh((n-1)*\log(e)) + a*d*\sinh((n-1)*\log(e)))*\sinh(n*\log(x))}{(d*n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="fricas")

[Out] (a\*d\*cosh((n-1)\*log(e))\*cosh(n\*log(x)) + a\*d\*cosh(n\*log(x))\*sinh((n-1)\*log(e)) + 2\*(b\*cosh((n-1)\*log(e)) + b\*sinh((n-1)\*log(e)))\*arctan(cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)) + (a\*d\*cosh((n-1)\*log(e)) + a\*d\*sinh((n-1)\*log(e)))\*sinh(n\*log(x)))/(d\*n)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^n + c) + a) (ex)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^n + c) + a)\*(e\*x)^(n-1), x)

**maple [C]** time = 0.73, size = 155, normalized size = 3.52

$$\frac{ax e^{\frac{(-1+n)(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e))}{2}}}{n} + \frac{2 \operatorname{arctan}(e^{c+dx^n}) e^n b e^{\frac{i\pi \operatorname{csgn}(iex)(-1+n)}{2}}}{den}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)\*(a+b\*sech(c+d\*x^n)),x)

[Out] a/n\*x\*exp(1/2\*(-1+n)\*(-I\*Pi\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x)+I\*Pi\*csgn(I\*e)\*csgn(I\*e\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*e\*x)^2-I\*Pi\*csgn(I\*e\*x)^3+2\*ln(x)+2\*ln(e)))+2\*arctan(exp(c+d\*x^n))/d/e\*e^n/n\*b\*exp(1/2\*I\*Pi\*csgn(I\*e\*x)\*(-1+n)\*(csgn(I\*e\*x)-csgn(I\*x))\*(-csgn(I\*e\*x)+csgn(I\*e)))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{(ex)^{n-1}}{e^{(dx^n+c)} + e^{(-dx^n-c)}} dx + \frac{(ex)^n a}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="maxima")

[Out] 2\*b\*integrate((e\*x)^(n-1)/(e^(d\*x^n+c)+e^(-d\*x^n-c)),x)+ (e\*x)^n\*a/(e\*n)

**mupad [B]** time = 1.40, size = 110, normalized size = 2.50

$$\frac{2 \operatorname{atan}\left(\frac{b x e^{d x^n} e^c (e x)^{n-1} \sqrt{d^2 n^2 x^{2n}}}{d n x^n \sqrt{b^2 x^2 (e x)^{2n-2}}}\right) \sqrt{b^2 x^2 (e x)^{2n-2}}}{\sqrt{d^2 n^2 x^{2n}}} + \frac{a x (e x)^{n-1}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^n))\*(e\*x)^(n - 1), x)

[Out] (2\*atan((b\*x\*exp(d\*x^n)\*exp(c)\*(e\*x)^(n - 1)\*(d^2\*n^2\*x^(2\*n))^(1/2))/(d\*n\*x^n\*(b^2\*x^2\*(e\*x)^(2\*n - 2))^(1/2)))\*(b^2\*x^2\*(e\*x)^(2\*n - 2))^(1/2))/(d^2\*n^2\*x^(2\*n))^(1/2) + (a\*x\*(e\*x)^(n - 1))/n

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (e x)^{n-1} (a + b \operatorname{sech}(c + d x^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)\*(a+b\*sech(c+d\*x\*\*n)), x)

[Out] Integral((e\*x)\*\*(n - 1)\*(a + b\*sech(c + d\*x\*\*n)), x)

### 3.74 $\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n)) dx$

**Optimal.** Leaf size=135

$$\frac{a(ex)^{2n}}{2en} - \frac{ibx^{-2n}(ex)^{2n}\operatorname{Li}_2(-ie^{dx^n+c})}{d^2en} + \frac{ibx^{-2n}(ex)^{2n}\operatorname{Li}_2(ie^{dx^n+c})}{d^2en} + \frac{2bx^{-n}(ex)^{2n}\tan^{-1}(e^{c+dx^n})}{den}$$

[Out]  $1/2*a*(e*x)^{(2*n)}/e/n+2*b*(e*x)^{(2*n)}*\arctan(\exp(c+d*x^n))/d/e/n/(x^n)-I*b*(e*x)^{(2*n)}*\operatorname{polylog}(2,-I*\exp(c+d*x^n))/d^2/e/n/(x^{(2*n)})+I*b*(e*x)^{(2*n)}*\operatorname{polylog}(2,I*\exp(c+d*x^n))/d^2/e/n/(x^{(2*n)})$

**Rubi [A]** time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {14, 5440, 5436, 4180, 2279, 2391}

$$-\frac{ibx^{-2n}(ex)^{2n}\operatorname{PolyLog}(2,-ie^{c+dx^n})}{d^2en} + \frac{ibx^{-2n}(ex)^{2n}\operatorname{PolyLog}(2,ie^{c+dx^n})}{d^2en} + \frac{a(ex)^{2n}}{2en} + \frac{2bx^{-n}(ex)^{2n}\tan^{-1}(e^{c+dx^n})}{den}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+2*n}*(a + b*\operatorname{Sech}[c + d*x^n]),x]$

[Out]  $(a*(e*x)^{(2*n)})/(2*e*n) + (2*b*(e*x)^{(2*n)}*\operatorname{ArcTan}[E^{(c + d*x^n)}])/(d*e*n*x^n) - (I*b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x^n)}])/(d^2*e*n*x^{(2*n)}) + (I*b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2, I*E^{(c + d*x^n)}])/(d^2*e*n*x^{(2*n)})$

#### Rule 14

$\operatorname{Int}[(u_)*((c_)*(x_))^{(m_)}, x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /; \operatorname{FreeQ}\{c, m\}, x \ \&\& \operatorname{SumQ}[u] \ \&\& \ !\operatorname{LinearQ}[u, x] \ \&\& \ !\operatorname{MatchQ}[u, (a_ + (b_)*(v_)) /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \ \operatorname{InverseFunctionQ}[v]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_ + (b_)*((F_)^{((e_)*((c_.) + (d_)*(x_)))^{(n_)}), x\_Symbol] := \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_.) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] := -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x \ \&\& \ \operatorname{EqQ}[c*d, 1]$

#### Rule 4180

$\operatorname{Int}[\operatorname{csc}[(e_.) + \operatorname{Pi}*(k_.) + (\operatorname{Complex}[0, fz_])*(f_)*(x_)]*((c_.) + (d_)*(x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}])/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \ \operatorname{IntegerQ}[2*k] \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 5436

$\operatorname{Int}[(x_)^{(m_)*((a_.) + (b_)*\operatorname{Sech}[(c_.) + (d_)*(x_)^{(n_)}])^{(p_)}, x\_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)*(a + b*\operatorname{Sech}[c + d*x])^p}, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x \ \&\& \ \operatorname{IGtQ}[\operatorname{Simplify}[(m+1)/n], 0] \ \&\& \ \operatorname{IntegerQ}[p]$

#### Rule 5440





```
*log(e))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n
*log(x)) + d*sinh(n*log(x)) + c) - I) + (-2*I*b*d*cosh((2*n - 1)*log(e))*co
sh(n*log(x)) - 2*I*b*c*cosh((2*n - 1)*log(e)) + (-2*I*b*d*cosh(n*log(x)) -
2*I*b*c)*sinh((2*n - 1)*log(e)) + (-2*I*b*d*cosh((2*n - 1)*log(e)) - 2*I*b*
d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(I*cosh(d*cosh(n*log(x)) + d*s
inh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) +
(2*I*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*I*b*c*cosh((2*n - 1)*lo
g(e)) + (2*I*b*d*cosh(n*log(x)) + 2*I*b*c)*sinh((2*n - 1)*log(e)) + (2*I*b*
d*cosh((2*n - 1)*log(e)) + 2*I*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*
log(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c) + 1) + 2*(a*d^2*cosh((2*n - 1)*log(e))*cosh(n*1
og(x)) + a*d^2*cosh(n*log(x))*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))/(d^2*
n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^n + c) + a) (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^n + c) + a)\*(e\*x)^(2\*n - 1), x)

**maple** [C] time = 0.81, size = 368, normalized size = 2.73

$$\frac{ax e^{(-1+2n)\left(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}}{2n} + 2b e^{-i\pi n \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)} e^{i\pi n \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n)),x)

```
[Out] 1/2*a/n*x*exp(1/2*(-1+2*n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn
(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)
+2*ln(e)))+2*b*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*csgn
(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I
*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I
*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn
(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(-1/2*(-exp(2*c))^(1/2)*x^n*d*(ln(1+exp(d
*x^n)*(-exp(2*c))^(1/2))-ln(1-exp(d*x^n)*(-exp(2*c))^(1/2)))*exp(-2*c)-1/2*
(-exp(2*c))^(1/2)*(dilog(1+exp(d*x^n)*(-exp(2*c))^(1/2))-dilog(1-exp(d*x^n)
*(-exp(2*c))^(1/2)))*exp(-2*c))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{(ex)^{2n-1}}{e^{(dx^n+c)} + e^{(-dx^n-c)}} dx + \frac{(ex)^{2n} a}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="maxima")

```
[Out] 2*b*integrate((e*x)^(2*n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + 1/2*(e
*x)^(2*n)*a/(e*n)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + \frac{b}{\cosh(c + dx^n)} \right) (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x^n))*(e*x)^(2*n - 1), x)
```

```
[Out] int((a + b/cosh(c + d*x^n))*(e*x)^(2*n - 1), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2n-1} (a + b \operatorname{sech}(c + dx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*sech(c+d*x**n)), x)
```

```
[Out] Integral((e*x)**(2*n - 1)*(a + b*sech(c + d*x**n)), x)
```

### 3.75 $\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx$

**Optimal.** Leaf size=217

$$\frac{a(ex)^{3n}}{3en} + \frac{2ibx^{-3n}(ex)^{3n}\operatorname{Li}_3(-ie^{dx^n+c})}{d^3en} - \frac{2ibx^{-3n}(ex)^{3n}\operatorname{Li}_3(ie^{dx^n+c})}{d^3en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{Li}_2(-ie^{dx^n+c})}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{Li}_2(ie^{dx^n+c})}{d^2en}$$

```
[Out] 1/3*a*(e*x)^(3*n)/e/n+2*b*(e*x)^(3*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)-2*I*b*(e*x)^(3*n)*polylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n)*polylog(2,I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n)*polylog(3,-I*exp(c+d*x^n))/d^3/e/n/(x^(3*n))-2*I*b*(e*x)^(3*n)*polylog(3,I*exp(c+d*x^n))/d^3/e/n/(x^(3*n))
```

**Rubi [A]** time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$ , Rules used = {14, 5440, 5436, 4180, 2531, 2282, 6589}

$$\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}(3, -ie^{c+dx^n})}{d^3en} - \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}(3, ie^{c+dx^n})}{d^3en} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}(2, -ie^{c+dx^n})}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n}\operatorname{PolyLog}(2, ie^{c+dx^n})}{d^2en}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(3*n))/(3*e*n) + (2*b*(e*x)^(3*n)*ArcTan[E^(c + d*x^n)])/(d*e*n*x^n) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, (-I)*E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, I*E^(c + d*x^n)])/(d^2*e*n*x^(2*n)) + ((2*I)*b*(e*x)^(3*n)*PolyLog[3, (-I)*E^(c + d*x^n)])/(d^3*e*n*x^(3*n)) - ((2*I)*b*(e*x)^(3*n)*PolyLog[3, I*E^(c + d*x^n)])/(d^3*e*n*x^(3*n))
```

#### Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_)+(b_)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_)+(b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m-1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

#### Rule 4180

```
Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x]
;/; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 5440

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x]
;/; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x]
;/; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx &= \int (a(ex)^{-1+3n} + b(ex)^{-1+3n}\operatorname{sech}(c + dx^n)) dx \\
&= \frac{a(ex)^{3n}}{3en} + b \int (ex)^{-1+3n}\operatorname{sech}(c + dx^n) dx \\
&= \frac{a(ex)^{3n}}{3en} + \frac{(bx^{-3n}(ex)^{3n}) \int x^{-1+3n}\operatorname{sech}(c + dx^n) dx}{e} \\
&= \frac{a(ex)^{3n}}{3en} + \frac{(bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2\operatorname{sech}(c + dx) dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{(2ibx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x \log\right)}{den} \\
&= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{Li}_2(-ie^{c+dx^n})}{d^2en} + \dots \\
&= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{Li}_2(-ie^{c+dx^n})}{d^2en} + \dots \\
&= \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{2ibx^{-2n}(ex)^{3n}\operatorname{Li}_2(-ie^{c+dx^n})}{d^2en} + \dots
\end{aligned}$$

**Mathematica [F]** time = 10.53, size = 0, normalized size = 0.00

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sech[c + d\*x^n]), x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sech[c + d\*x^n]), x]

**fricas [C]** time = 0.49, size = 1072, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="fricas")

```
[Out] 1/3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + a*d^3*cosh(n*log(x))^3
*sinh((3*n - 1)*log(e)) + (a*d^3*cosh((3*n - 1)*log(e)) + a*d^3*sinh((3*n -
1)*log(e)))*sinh(n*log(x))^3 + 3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(
x)) + a*d^3*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (6*I*
b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + 6*I*b*d*cosh(n*log(x))*sinh((3*
n - 1)*log(e)) + (6*I*b*d*cosh((3*n - 1)*log(e)) + 6*I*b*d*sinh((3*n - 1)*l
og(e)))*sinh(n*log(x)))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) +
c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + (-6*I*b*d*cosh((3*n
- 1)*log(e))*cosh(n*log(x)) - 6*I*b*d*cosh(n*log(x))*sinh((3*n - 1)*log(e)
) + (-6*I*b*d*cosh((3*n - 1)*log(e)) - 6*I*b*d*sinh((3*n - 1)*log(e)))*sinh
(n*log(x)))*dilog(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh
(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + (3*I*b*c^2*cosh((3*n - 1)*log(
e)) + 3*I*b*c^2*sinh((3*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I) + (-3*I
*b*c^2*cosh((3*n - 1)*log(e)) - 3*I*b*c^2*sinh((3*n - 1)*log(e)))*log(cosh(
d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c) - I) + (-3*I*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 +
3*I*b*c^2*cosh((3*n - 1)*log(e)) + (-3*I*b*d^2*cosh((3*n - 1)*log(e)) - 3*I
*b*d^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (-3*I*b*d^2*cosh(n*log(x)
)^2 + 3*I*b*c^2)*sinh((3*n - 1)*log(e)) + (-6*I*b*d^2*cosh((3*n - 1)*log(e)
)*cosh(n*log(x)) - 6*I*b*d^2*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*
log(x))*log(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cos
h(n*log(x)) + d*sinh(n*log(x)) + c) + 1) + (3*I*b*d^2*cosh((3*n - 1)*log(e)
)*cosh(n*log(x))^2 - 3*I*b*c^2*cosh((3*n - 1)*log(e)) + (3*I*b*d^2*cosh((3*
n - 1)*log(e)) + 3*I*b*d^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (3*I*
b*d^2*cosh(n*log(x))^2 - 3*I*b*c^2)*sinh((3*n - 1)*log(e)) + (6*I*b*d^2*cos
h((3*n - 1)*log(e))*cosh(n*log(x)) + 6*I*b*d^2*cosh(n*log(x))*sinh((3*n - 1
)*log(e)))*sinh(n*log(x))*log(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) + (-6*I*b*cosh(
(3*n - 1)*log(e)) - 6*I*b*sinh((3*n - 1)*log(e)))*polylog(3, I*cosh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(
x)) + c)) + (6*I*b*cosh((3*n - 1)*log(e)) + 6*I*b*sinh((3*n - 1)*log(e)))*p
olylog(3, -I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(
n*log(x)) + d*sinh(n*log(x)) + c)) + 3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n
*log(x))^2 + a*d^3*cosh(n*log(x))^2*sinh((3*n - 1)*log(e)))*sinh(n*log(x))
/(d^3*n)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^n + c) + a) (ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

**maple** [F] time = 1.16, size = 0, normalized size = 0.00

$$\int (ex)^{3n-1} (a + b \operatorname{sech}(c + dx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3*n-1)*(a+b*sech(c+d*x^n)),x)
```

```
[Out] int((e*x)^(3*n-1)*(a+b*sech(c+d*x^n)),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2b \int \frac{(ex)^{3n-1}}{e^{(dx^n+c)} + e^{(-dx^n-c)}} dx + \frac{(ex)^{3n} a}{3en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sech(c+d\*x^n)),x, algorithm="maxima")

[Out] 2\*b\*integrate((e\*x)^(3\*n - 1)/(e^(d\*x^n + c) + e^(-d\*x^n - c)), x) + 1/3\*(e\*x)^(3\*n)\*a/(e\*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + \frac{b}{\cosh(c + d x^n)} \right) (e x)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^n))\*(e\*x)^(3\*n - 1),x)

[Out] int((a + b/cosh(c + d\*x^n))\*(e\*x)^(3\*n - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e x)^{3n-1} (a + b \operatorname{sech}(c + d x^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+3\*n)\*(a+b\*sech(c+d\*x\*\*n)),x)

[Out] Integral((e\*x)\*\*(3\*n - 1)\*(a + b\*sech(c + d\*x\*\*n)), x)

### 3.76 $\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx$

Optimal. Leaf size=79

$$\frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tanh(c + dx^n)}{den}$$

[Out]  $a^2*(e*x)^n/e/n+2*a*b*(e*x)^n*\arctan(\sinh(c+d*x^n))/d/e/n/(x^n)+b^2*(e*x)^n*\tanh(c+d*x^n)/d/e/n/(x^n)$

**Rubi [A]** time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {5440, 5436, 3773, 3770, 3767, 8}

$$\frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tanh(c + dx^n)}{den}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + n)\*(a + b\*Sech[c + d\*x^n])^2,x]

[Out]  $(a^2*(e*x)^n)/(e*n) + (2*a*b*(e*x)^n*\text{ArcTan}[\text{Sinh}[c + d*x^n]])/(d*e*n*x^n) + (b^2*(e*x)^n*\text{Tanh}[c + d*x^n])/(d*e*n*x^n)$

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3773

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_.))^2, x\_Symbol] := Simp[a^2\*x, x] + (Dist[2\*a\*b, Int[Csc[c + d\*x], x], x] + Dist[b^2, Int[Csc[c + d\*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rule 5440

Int[((e)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sech[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

#### Rubi steps



$$\begin{aligned}
\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx &= \frac{(x^{-n}(ex)^n) \int x^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx}{e} \\
&= \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int (a + b \operatorname{sech}(c + dx))^2 dx, x, x^n\right)}{en} \\
&= \frac{a^2(ex)^n}{en} + \frac{(2abx^{-n}(ex)^n) \operatorname{Subst}\left(\int \operatorname{sech}(c + dx) dx, x, x^n\right)}{en} + \frac{(b^2x^{-n}(ex)^n) \operatorname{Subst}\left(\int \operatorname{sech}^2(c + dx) dx, x, x^n\right)}{en} \\
&= \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den} + \frac{(b^2x^{-n}(ex)^n) \operatorname{Subst}\left(\int \operatorname{sech}^2(c + dx) dx, x, x^n\right)}{den} \\
&= \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \tan^{-1}(\sinh(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tanh(c + dx^n)}{den}
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 57, normalized size = 0.72

$$\frac{x^{-n}(ex)^n \left( a \left( a(c + dx^n) + 2b \tan^{-1}(\sinh(c + dx^n)) \right) + b^2 \tanh(c + dx^n) \right)}{den}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)\*(a + b\*Sech[c + d\*x^n])^2,x]

[Out] ((e\*x)^n\*(a\*(a\*(c + d\*x^n) + 2\*b\*ArcTan[Sinh[c + d\*x^n]]) + b^2\*Tanh[c + d\*x^n]))/(d\*e\*n\*x^n)

**fricas [B]** time = 0.42, size = 646, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)\*(a+b\*sech(c+d\*x^n))^2,x, algorithm="fricas")

[Out] (a^2\*d\*cosh((n - 1)\*log(e))\*cosh(n\*log(x)) + (a^2\*d\*cosh((n - 1)\*log(e))\*cosh(n\*log(x)) + a^2\*d\*cosh(n\*log(x))\*sinh((n - 1)\*log(e)) + (a^2\*d\*cosh((n - 1)\*log(e)) + a^2\*d\*sinh((n - 1)\*log(e))\*sinh(n\*log(x)))\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 - 2\*b^2\*cosh((n - 1)\*log(e)) + 2\*(a^2\*d\*cosh((n - 1)\*log(e))\*cosh(n\*log(x)) + a^2\*d\*cosh(n\*log(x))\*sinh((n - 1)\*log(e)) + (a^2\*d\*cosh((n - 1)\*log(e)) + a^2\*d\*sinh((n - 1)\*log(e))\*sinh(n\*log(x)))\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + (a^2\*d\*cosh((n - 1)\*log(e))\*cosh(n\*log(x)) + a^2\*d\*cosh(n\*log(x))\*sinh((n - 1)\*log(e)) + (a^2\*d\*cosh((n - 1)\*log(e)) + a^2\*d\*sinh((n - 1)\*log(e))\*sinh(n\*log(x)))\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 + 4\*((a\*b\*cosh((n - 1)\*log(e)) + a\*b\*sinh((n - 1)\*log(e))\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 + a\*b\*cosh((n - 1)\*log(e)) + 2\*(a\*b\*cosh((n - 1)\*log(e)) + a\*b\*sinh((n - 1)\*log(e))\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + (a\*b\*cosh((n - 1)\*log(e)) + a\*b\*sinh((n - 1)\*log(e))\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 + a\*b\*sinh((n - 1)\*log(e))\*arctan(cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)) + (a^2\*d\*cosh(n\*log(x)) - 2\*b^2)\*sinh((n - 1)\*log(e)) + (a^2\*d\*cosh((n - 1)\*log(e)) + a^2\*d\*sinh((n - 1)\*log(e))\*sinh(n\*log(x)))/(d\*n\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 + 2\*d\*n\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + d\*n\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 + d\*n)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(n - 1), x)
```

**maple** [C] time = 0.66, size = 271, normalized size = 3.43

$$\frac{a^2 x e^{\frac{(-1+n)(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e))}{2}}}{n} - \frac{2 x x^{-n} b^2 e^{\frac{(-1+n)(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex))}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x)
```

```
[Out] a^2/n*x*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*ln(e)))-2/d/n*x/(x^n)*b^2*exp(1/2*(-1+n)*(-I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+I*Pi*csgn(I*e)*csgn(I*e*x)^2+I*Pi*csgn(I*x)*csgn(I*e*x)^2-I*Pi*csgn(I*e*x)^3+2*ln(x)+2*ln(e)))/(exp(2*c+2*d*x^n)+1)+4*arctan(exp(c+d*x^n))/d/e*e^n/n*a*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4 a b e^n \int \frac{e^{(dx^n+n \log(x)+c)}}{e x e^{(2 dx^n+2 c)}+e x} d x - \frac{2 b^2 e^n}{d e n e^{(2 dx^n+2 c)}+d e n} + \frac{(e x)^n a^2}{e n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] 4*a*b*e^n*integrate(e^(d*x^n + n*log(x) + c)/(e*x*e^(2*d*x^n + 2*c) + e*x), x) - 2*b^2*e^n/(d*e*n*e^(2*d*x^n + 2*c) + d*e*n) + (e*x)^n*a^2/(e*n)
```

**mupad** [B] time = 1.42, size = 158, normalized size = 2.00

$$\frac{4 \operatorname{atan}\left(\frac{a b x e^{d x^n} e^c (e x)^{n-1} \sqrt{d^2 n^2 x^{2 n}}}{d n x^n \sqrt{a^2 b^2 x^2 (e x)^{2 n-2}}}\right) \sqrt{a^2 b^2 x^2 (e x)^{2 n-2}}}{\sqrt{d^2 n^2 x^{2 n}}} + \frac{a^2 x (e x)^{n-1}}{n} - \frac{2 b^2 x (e x)^{n-1}}{d n x^n \left(e^{2 c+2 d x^n}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cosh(c + d*x^n))^2*(e*x)^(n - 1), x)
```

```
[Out] (4*atan((a*b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(d^2*n^2*x^(2*n))^(1/2))/(d*n*x^n*(a^2*b^2*x^2*(e*x)^(2*n - 2))^(1/2)))+(a^2*b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(d^2*n^2*x^(2*n))^(1/2) + (a^2*x*(e*x)^(n - 1))/n - (2*b^2*x*(e*x)^(n - 1))/(d*n*x^n*(exp(2*c + 2*d*x^n) + 1))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*sech(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(n - 1)*(a + b*sech(c + d*x**n))**2, x)
```

### 3.77 $\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx$

**Optimal.** Leaf size=208

$$\frac{a^2(ex)^{2n}}{2en} - \frac{2iabx^{-2n}(ex)^{2n}\operatorname{Li}_2(-ie^{dx^n+c})}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\operatorname{Li}_2(ie^{dx^n+c})}{d^2en} + \frac{4abx^{-n}(ex)^{2n}\tan^{-1}(e^{c+dx^n})}{den} - \frac{b^2x^{-2n}(ex)^{2n}}{den}$$

[Out]  $1/2*a^2*(e*x)^{(2*n)}/e/n+4*a*b*(e*x)^{(2*n)}*\arctan(\exp(c+d*x^n))/d/e/n/(x^n)-b^2*(e*x)^{(2*n)}*\ln(\cosh(c+d*x^n))/d^2/e/n/(x^{(2*n)})-2*I*a*b*(e*x)^{(2*n)}*\operatorname{polylog}(2,-I*\exp(c+d*x^n))/d^2/e/n/(x^{(2*n)})+2*I*a*b*(e*x)^{(2*n)}*\operatorname{polylog}(2,I*\exp(c+d*x^n))/d^2/e/n/(x^{(2*n)})+b^2*(e*x)^{(2*n)}*\tanh(c+d*x^n)/d/e/n/(x^n)$

**Rubi [A]** time = 0.21, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5440, 5436, 4190, 4180, 2279, 2391, 4184, 3475}

$$-\frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}(2,-ie^{c+dx^n})}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\operatorname{PolyLog}(2,ie^{c+dx^n})}{d^2en} + \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n}\tan^{-1}(e^{c+dx^n})}{den}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+2n}*(a+b*\operatorname{Sech}[c+d*x^n])^2,x]$

[Out]  $(a^2*(e*x)^{(2*n)})/(2*e*n) + (4*a*b*(e*x)^{(2*n)}*\operatorname{ArcTan}[E^{(c+d*x^n)}])/(d*e*n*x^n) - (b^2*(e*x)^{(2*n)}*\operatorname{Log}[\operatorname{Cosh}[c+d*x^n]])/(d^2*e*n*x^{(2*n)}) - ((2*I)*a*b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2,(-I)*E^{(c+d*x^n)}])/(d^2*e*n*x^{(2*n)}) + ((2*I)*a*b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2,I*E^{(c+d*x^n)}])/(d^2*e*n*x^{(2*n)}) + (b^2*(e*x)^{(2*n)}*\operatorname{Tanh}[c+d*x^n])/(d*e*n*x^n)$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x\_Symbol]$   
 $\rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \ \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\operatorname{FreeQ}\{c, d, e, n, x\} \ \&\& \ \operatorname{EqQ}[c*d, 1]$

#### Rule 3475

$\operatorname{Int}[\tan[(c_)+(d_)*(x_)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d*x], x]]/d, x] /;$   $\operatorname{FreeQ}\{c, d, x\}$

#### Rule 4180

$\operatorname{Int}[\operatorname{csc}[(e_)+\operatorname{Pi}*(k_)+(Complex[0, fz_])*(f_)*(x_)]*((c_)+(d_)*(x_))^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)})/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)}], x], x)] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\& \ \operatorname{IntegerQ}[2*k] \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_)+(f_)*(x_)]^2*((c_)+(d_)*(x_))^{(m_)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cot}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Cot}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 4190

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_.)])^(p_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 5440

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_.)])^(p_.),
x_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx &= \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx}{e} \\ &= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x(a + b \operatorname{sech}(c + dx))^2 dx, x, x^n\right)}{en} \\ &= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int (a^2x + 2abx \operatorname{sech}(c + dx) + b^2x \operatorname{sech}^2(c + dx)) dx, x, x^n\right)}{en} \\ &= \frac{a^2(ex)^{2n}}{2en} + \frac{(2abx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x \operatorname{sech}(c + dx) dx, x, x^n\right)}{en} + \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x \operatorname{sech}^2(c + dx) dx, x, x^n\right)}{en} \\ &= \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n} \tan^{-1}\left(e^{c+dx^n}\right)}{den} + \frac{b^2x^{-n}(ex)^{2n} \tanh(c + dx^n)}{den} - \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int x \operatorname{sech}^2(c + dx) dx, x, x^n\right)}{d^2en} \\ &= \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n} \tan^{-1}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-2n}(ex)^{2n} \log(\cosh(c + dx^n))}{d^2en} \\ &= \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n} \tan^{-1}\left(e^{c+dx^n}\right)}{den} - \frac{b^2x^{-2n}(ex)^{2n} \log(\cosh(c + dx^n))}{d^2en} \end{aligned}$$

**Mathematica [B]** time = 2.88, size = 501, normalized size = 2.41

$$\operatorname{csch}^5(c)x^{-2n}(ex)^{2n} \operatorname{sech}(c + dx^n) \left( -a^2d^2\sqrt{-\operatorname{csch}^2(c)x^{2n} \sinh(dx^n)} + a^2d^2\sqrt{-\operatorname{csch}^2(c)x^{2n} \sinh(2c + dx^n)} + 8ab \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^(2*n)*Csch[c]^5*Sech[c + d*x^n]*(-2*b^2*d*x^n*Cosh[d*x^n]*Sqrt[-Csch[c]^2] + 2*b^2*d*x^n*Cosh[2*c + d*x^n]*Sqrt[-Csch[c]^2] + 8*a*b*d*x^n*Cosh[c + d*x^n]*Log[1 - E^(-(d*x^n) - ArcTanh[Coth[c]])] + 8*a*b*ArcTanh[Coth[c]]*Cosh[c + d*x^n]*Log[1 - E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*d*x^n*Cosh[c + d*x^n]*Log[1 + E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*ArcTanh[Coth[c]]*Cosh[c + d*x^n]*Log[1 + E^(-(d*x^n) - ArcTanh[Coth[c]])] + 8*a*b*Cosh[c + d*x^n]*PolyLog[2, -E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*Cosh[c + d*x^n])
```

$$x^n * \text{PolyLog}[2, E^{-(d*x^n) - \text{ArcTanh}[\text{Coth}[c]]}] - a^2*d^2*x^{(2*n)}*\text{Sqrt}[-\text{Csch}[c]^2*\text{Sinh}[d*x^n] + 8*a*b*\text{ArcTan}[\text{Sinh}[c] + \text{Cosh}[c]*\text{Tanh}[(d*x^n)/2]]*\text{ArcTanh}[\text{Coth}[c]]*\text{Sqrt}[-\text{Csch}[c]^2*\text{Sinh}[d*x^n] + 2*b^2*\text{Sqrt}[-\text{Csch}[c]^2]*\text{Log}[\text{Cosh}[c + d*x^n]]*\text{Sinh}[d*x^n] + a^2*d^2*x^{(2*n)}*\text{Sqrt}[-\text{Csch}[c]^2*\text{Sinh}[2*c + d*x^n] - 8*a*b*\text{ArcTan}[\text{Sinh}[c] + \text{Cosh}[c]*\text{Tanh}[(d*x^n)/2]]*\text{ArcTanh}[\text{Coth}[c]]*\text{Sqrt}[-\text{Csch}[c]^2*\text{Sinh}[2*c + d*x^n] - 2*b^2*\text{Sqrt}[-\text{Csch}[c]^2]*\text{Log}[\text{Cosh}[c + d*x^n]]*\text{Sinh}[2*c + d*x^n]])/(4*d^2*e^n*x^{(2*n)}*(-\text{Csch}[c]^2)^{(5/2)})$$

**fricas** [B] time = 0.54, size = 2964, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
[Out] 1/2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + ((4*I*a*b*cosh((2*n - 1)*log(e)) + 4*I*a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 4*I*a*b*cosh((2*n - 1)*log(e)) + (8*I*a*b*cosh((2*n - 1)*log(e)) + 8*I*a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (4*I*a*b*cosh((2*n - 1)*log(e)) + 4*I*a*b*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 4*I*a*b*sinh((2*n - 1)*log(e))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + ((-4*I*a*b*cosh((2*n - 1)*log(e)) - 4*I*a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - 4*I*a*b*cosh((2*n - 1)*log(e)) + (-8*I*a*b*cosh((2*n - 1)*log(e)) - 8*I*a*b*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (-4*I*a*b*cosh((2*n - 1)*log(e)) - 4*I*a*b*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 - 4*I*a*b*sinh((2*n - 1)*log(e))*dilog(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(((2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(((2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + ((2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e))
```

```

n - 1)*log(e)) + (2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e))*log(cosh(d*cosh(
n*log(x) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x) + d*sinh(n*log(x)
) + c) + I) - 2*((-2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (-2*I*a*b*c +
b^2)*sinh((2*n - 1)*log(e))*cosh(d*cosh(n*log(x) + d*sinh(n*log(x) + c)
^2 + 2*((-2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (-2*I*a*b*c + b^2)*sinh
((2*n - 1)*log(e))*cosh(d*cosh(n*log(x) + d*sinh(n*log(x) + c)*sinh(d*co
sh(n*log(x) + d*sinh(n*log(x) + c) + ((-2*I*a*b*c + b^2)*cosh((2*n - 1)*l
og(e)) + (-2*I*a*b*c + b^2)*sinh((2*n - 1)*log(e))*sinh(d*cosh(n*log(x) +
d*sinh(n*log(x) + c)^2 + (-2*I*a*b*c + b^2)*cosh((2*n - 1)*log(e)) + (-2*
I*a*b*c + b^2)*sinh((2*n - 1)*log(e))*log(cosh(d*cosh(n*log(x) + d*sinh(n
*log(x) + c) + sinh(d*cosh(n*log(x) + d*sinh(n*log(x) + c) - I) + (-4*I*
a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x) - 4*I*a*b*c*cosh((2*n - 1)*log(
e)) + (-4*I*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x) - 4*I*a*b*c*cosh((2
*n - 1)*log(e)) + (-4*I*a*b*d*cosh(n*log(x) - 4*I*a*b*c)*sinh((2*n - 1)*lo
g(e)) + (-4*I*a*b*d*cosh((2*n - 1)*log(e)) - 4*I*a*b*d*sinh((2*n - 1)*log(e
)))*sinh(n*log(x))*cosh(d*cosh(n*log(x) + d*sinh(n*log(x) + c)^2 + (-8*I
*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x) - 8*I*a*b*c*cosh((2*n - 1)*log
(e)) + (-8*I*a*b*d*cosh(n*log(x) - 8*I*a*b*c)*sinh((2*n - 1)*log(e)) + (-8
*I*a*b*d*cosh((2*n - 1)*log(e)) - 8*I*a*b*d*sinh((2*n - 1)*log(e))*sinh(n*
log(x))*cosh(d*cosh(n*log(x) + d*sinh(n*log(x) + c)*sinh(d*cosh(n*log(x)
) + d*sinh(n*log(x) + c) + (-4*I*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x
)) - 4*I*a*b*c*cosh((2*n - 1)*log(e)) + (-4*I*a*b*d*cosh(n*log(x) - 4*I*a*
b*c)*sinh((2*n - 1)*log(e)) + (-4*I*a*b*d*cosh((2*n - 1)*log(e)) - 4*I*a*b*
d*sinh((2*n - 1)*log(e))*sinh(n*log(x))*sinh(d*cosh(n*log(x) + d*sinh(n*
log(x) + c)^2 + (-4*I*a*b*d*cosh(n*log(x) - 4*I*a*b*c)*sinh((2*n - 1)*log
(e)) + (-4*I*a*b*d*cosh((2*n - 1)*log(e)) - 4*I*a*b*d*sinh((2*n - 1)*log(e
)))*sinh(n*log(x))*log(I*cosh(d*cosh(n*log(x) + d*sinh(n*log(x) + c) + I*
sinh(d*cosh(n*log(x) + d*sinh(n*log(x) + c) + 1) + (4*I*a*b*d*cosh((2*n -
1)*log(e))*cosh(n*log(x) + 4*I*a*b*c*cosh((2*n - 1)*log(e)) + (4*I*a*b*d*
cosh((2*n - 1)*log(e))*cosh(n*log(x) + 4*I*a*b*c*cosh((2*n - 1)*log(e)) +
(4*I*a*b*d*cosh(n*log(x) + 4*I*a*b*c)*sinh((2*n - 1)*log(e)) + (4*I*a*b*d*
cosh((2*n - 1)*log(e)) + 4*I*a*b*d*sinh((2*n - 1)*log(e))*sinh(n*log(x))*
cosh(d*cosh(n*log(x) + d*sinh(n*log(x) + c)^2 + (8*I*a*b*d*cosh((2*n - 1)
*log(e))*cosh(n*log(x) + 8*I*a*b*c*cosh((2*n - 1)*log(e)) + (8*I*a*b*d*cos
h(n*log(x) + 8*I*a*b*c)*sinh((2*n - 1)*log(e)) + (8*I*a*b*d*cosh((2*n - 1)
*log(e)) + 8*I*a*b*d*sinh((2*n - 1)*log(e))*sinh(n*log(x))*cosh(d*cosh(n*
log(x) + d*sinh(n*log(x) + c)*sinh(d*cosh(n*log(x) + d*sinh(n*log(x) +
c) + (4*I*a*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x) + 4*I*a*b*c*cosh((2*n
- 1)*log(e)) + (4*I*a*b*d*cosh(n*log(x) + 4*I*a*b*c)*sinh((2*n - 1)*log(e
)) + (4*I*a*b*d*cosh((2*n - 1)*log(e)) + 4*I*a*b*d*sinh((2*n - 1)*log(e))*
sinh(n*log(x))*sinh(d*cosh(n*log(x) + d*sinh(n*log(x) + c)^2 + (4*I*a*b*
d*cosh(n*log(x) + 4*I*a*b*c)*sinh((2*n - 1)*log(e)) + (4*I*a*b*d*cosh((2*n
- 1)*log(e)) + 4*I*a*b*d*sinh((2*n - 1)*log(e))*sinh(n*log(x))*log(-I*co
sh(d*cosh(n*log(x) + d*sinh(n*log(x) + c) - I*sinh(d*cosh(n*log(x) + d*s
inh(n*log(x) + c) + 1) + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*c)*sinh((2*n -
1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x) + a^2*d^2*cos
h(n*log(x))*sinh((2*n - 1)*log(e))*sinh(n*log(x)))/(d^2*n*cosh(d*cosh(n*lo
g(x) + d*sinh(n*log(x) + c)^2 + 2*d^2*n*cosh(d*cosh(n*log(x) + d*sinh(n*
log(x) + c)*sinh(d*cosh(n*log(x) + d*sinh(n*log(x) + c) + d^2*n*sinh(d*c
osh(n*log(x) + d*sinh(n*log(x) + c)^2 + d^2*n)

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^n + c) + a)^2\*(e\*x)^(2\*n - 1), x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n))^2,x)

[Out] int((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4abe^{2n} \int \frac{e^{(dx^n+2n\log(x)+c)}}{exe^{(2dx^n+2c)}+ex} dx + b^2 \left( \frac{2e^{2n}e^{(2dx^n+n\log(x)+2c)}}{dene^{(2dx^n+2c)}+den} - \frac{e^{2n-1} \log\left(\left(e^{(2dx^n+2c)}+1\right)e^{(-2c)}\right)}{d^{2n}} \right) + \frac{(ex)^{2n} a^2}{2en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)\*(a+b\*sech(c+d\*x^n))^2,x, algorithm="maxima")

[Out] 4\*a\*b\*e^(2\*n)\*integrate(e^(d\*x^n + 2\*n\*log(x) + c)/(e\*x\*e^(2\*d\*x^n + 2\*c) + e\*x), x) + b^2\*(2\*e^(2\*n)\*e^(2\*d\*x^n + n\*log(x) + 2\*c)/(d\*e\*n\*e^(2\*d\*x^n + 2\*c) + d\*e\*n) - e^(2\*n - 1)\*log((e^(2\*d\*x^n + 2\*c) + 1)\*e^(-2\*c))/(d^2\*n)) + 1/2\*(e\*x)^(2\*n)\*a^2/(e\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + \frac{b}{\cosh(c + dx^n)} \right)^2 (ex)^{2n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^n))^2\*(e\*x)^(2\*n - 1),x)

[Out] int((a + b/cosh(c + d\*x^n))^2\*(e\*x)^(2\*n - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+2\*n)\*(a+b\*sech(c+d\*x\*\*n))\*\*2,x)

[Out] Integral((e\*x)\*\*(2\*n - 1)\*(a + b\*sech(c + d\*x\*\*n))\*\*2, x)

### 3.78 $\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n))^2 dx$

**Optimal.** Leaf size=363

$$\frac{a^2(ex)^{3n}}{3en} + \frac{4iabx^{-3n}(ex)^{3n}\operatorname{Li}_3(-ie^{dx^n+c})}{d^3en} - \frac{4iabx^{-3n}(ex)^{3n}\operatorname{Li}_3(ie^{dx^n+c})}{d^3en} - \frac{4iabx^{-2n}(ex)^{3n}\operatorname{Li}_2(-ie^{dx^n+c})}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\operatorname{Li}_2(ie^{dx^n+c})}{d^2en}$$

[Out]  $\frac{1}{3}a^2(e^x)^{(3n)}/e/n + b^2(e^x)^{(3n)}/d/e/n/(x^n) + 4a*b*(e^x)^{(3n)}*\arctan(\exp(c+d*x^n))/d/e/n/(x^n) - 2*b^2*(e^x)^{(3n)}*\ln(1+\exp(2*c+2*d*x^n))/d^2/e/n/(x^{(2*n)}) - 4*I*a*b*(e^x)^{(3n)}*\operatorname{polylog}(2, -I*\exp(c+d*x^n))/d^2/e/n/(x^{(2*n)}) + 4*I*a*b*(e^x)^{(3n)}*\operatorname{polylog}(2, I*\exp(c+d*x^n))/d^2/e/n/(x^{(2*n)}) - b^2*(e^x)^{(3n)}*\operatorname{polylog}(2, -\exp(2*c+2*d*x^n))/d^3/e/n/(x^{(3*n)}) + 4*I*a*b*(e^x)^{(3n)}*\operatorname{polylog}(3, -I*\exp(c+d*x^n))/d^3/e/n/(x^{(3*n)}) - 4*I*a*b*(e^x)^{(3n)}*\operatorname{polylog}(3, I*\exp(c+d*x^n))/d^3/e/n/(x^{(3*n)}) + b^2*(e^x)^{(3n)}*\tanh(c+d*x^n)/d/e/n/(x^n)$

**Rubi [A]** time = 0.40, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5440, 5436, 4190, 4180, 2531, 2282, 6589, 4184, 3718, 2190, 2279, 2391}

$$\frac{4iabx^{-3n}(ex)^{3n}\operatorname{PolyLog}(3, -ie^{c+dx^n})}{d^3en} - \frac{4iabx^{-3n}(ex)^{3n}\operatorname{PolyLog}(3, ie^{c+dx^n})}{d^3en} - \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}(2, -ie^{c+dx^n})}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}(2, ie^{c+dx^n})}{d^2en}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e^x)^{-1+3n}*(a + b*\operatorname{Sech}[c + d*x^n])^2, x]$

[Out]  $(a^2*(e^x)^{(3n)})/(3*e^n) + (b^2*(e^x)^{(3n)})/(d*e^n*x^n) + (4*a*b*(e^x)^{(3n)}*\operatorname{ArcTan}[E^{(c + d*x^n)}])/(d*e^n*x^n) - (2*b^2*(e^x)^{(3n)}*\operatorname{Log}[1 + E^{(2*(c + d*x^n))}])/(d^2*e^n*x^{(2*n)}) - ((4*I)*a*b*(e^x)^{(3n)}*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x^n)}])/(d^2*e^n*x^{(2*n)}) + ((4*I)*a*b*(e^x)^{(3n)}*\operatorname{PolyLog}[2, I*E^{(c + d*x^n)}])/(d^2*e^n*x^{(2*n)}) - (b^2*(e^x)^{(3n)}*\operatorname{PolyLog}[2, -E^{(2*(c + d*x^n))}])/(d^3*e^n*x^{(3*n)}) + ((4*I)*a*b*(e^x)^{(3n)}*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x^n)}])/(d^3*e^n*x^{(3*n)}) - ((4*I)*a*b*(e^x)^{(3n)}*\operatorname{PolyLog}[3, I*E^{(c + d*x^n)}])/(d^3*e^n*x^{(3*n)}) + (b^2*(e^x)^{(3n)}*\operatorname{Tanh}[c + d*x^n])/(d*e^n*x^n)$

#### Rule 2190

$\operatorname{Int}[(((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_)}})/((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a]]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + (b*(F^{(g*(e + f*x)))^n)/a]], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_))})^{(n_)})], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /;$  FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2282

$\operatorname{Int}[u, x\_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$  FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^{(n\_)})^{(m\_)} /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^{((c\_)\*((a\_)\*(b\_)\*x))}\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2391



Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 3718

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

#### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-I\*e) + f\*fz\*x]/E^(I\*k\*Pi)))/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-I\*e) + f\*fz\*x]/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-I\*e) + f\*fz\*x]/E^(I\*k\*Pi)], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 4190

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Csc[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rule 5440

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] := Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sech[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
 \int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx &= \frac{(x^{-3n}(ex)^{3n}) \int x^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx}{e} \\
 &= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2(a + b\operatorname{sech}(c + dx))^2 dx, x, x^n\right)}{en} \\
 &= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int (a^2x^2 + 2abx^2\operatorname{sech}(c + dx) + b^2x^2\operatorname{sech}^2(c + dx)) dx\right)}{en} \\
 &= \frac{a^2(ex)^{3n}}{3en} + \frac{(2abx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2\operatorname{sech}(c + dx) dx, x, x^n\right)}{en} + \frac{(b^2x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int x^2\operatorname{sech}^2(c + dx) dx, x, x^n\right)}{en} \\
 &= \frac{a^2(ex)^{3n}}{3en} + \frac{4abx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} + \frac{b^2x^{-n}(ex)^{3n} \tanh(c + dx^n)}{den} - \frac{2b^2x^{-2n}(ex)^{3n} \operatorname{sech}(c + dx^n)}{d^2en} \\
 &= \frac{a^2(ex)^{3n}}{3en} + \frac{b^2x^{-n}(ex)^{3n}}{den} + \frac{4abx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{4iabx^{-2n}(ex)^{3n} \operatorname{sech}(c + dx^n)}{d^2en} \\
 &= \frac{a^2(ex)^{3n}}{3en} + \frac{b^2x^{-n}(ex)^{3n}}{den} + \frac{4abx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{2b^2x^{-2n}(ex)^{3n} \operatorname{sech}(c + dx^n)}{d^2en} \\
 &= \frac{a^2(ex)^{3n}}{3en} + \frac{b^2x^{-n}(ex)^{3n}}{den} + \frac{4abx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{2b^2x^{-2n}(ex)^{3n} \operatorname{sech}(c + dx^n)}{d^2en} \\
 &= \frac{a^2(ex)^{3n}}{3en} + \frac{b^2x^{-n}(ex)^{3n}}{den} + \frac{4abx^{-n}(ex)^{3n} \tan^{-1}(e^{c+dx^n})}{den} - \frac{2b^2x^{-2n}(ex)^{3n} \operatorname{sech}(c + dx^n)}{d^2en}
 \end{aligned}$$

**Mathematica** [F] time = 70.76, size = 0, normalized size = 0.00

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sech[c + d\*x^n])^2,x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)\*(a + b\*Sech[c + d\*x^n])^2, x]

**fricas** [C] time = 0.61, size = 5493, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sech(c+d\*x^n))^2,x, algorithm="fricas")

[Out] 1/3\*(a^2\*d^3\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x))^3 - 6\*b^2\*c^2\*cosh((3\*n - 1)\*log(e)) + (a^2\*d^3\*cosh((3\*n - 1)\*log(e)) + a^2\*d^3\*sinh((3\*n - 1)\*log(e)))\*sinh(n\*log(x))^3 + (a^2\*d^3\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x))^3 + 6\*b^2\*d^2\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x))^2 - 6\*b^2\*c^2\*cosh((3\*n - 1)\*log(e)) + (a^2\*d^3\*cosh((3\*n - 1)\*log(e)) + a^2\*d^3\*sinh((3\*n - 1)\*log(e)))\*sinh(n\*log(x))^3 + 3\*(a^2\*d^3\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x)) + 2\*b^2\*d^2\*cosh((3\*n - 1)\*log(e)) + (a^2\*d^3\*cosh(n\*log(x)) + 2\*b^2\*d^2)\*sinh((3\*n - 1)\*log(e)))\*sinh(n\*log(x))^2 + (a^2\*d^3\*cosh(n\*log(x))^3 + 6\*b^2\*d^2\*cosh(n\*log(x))^2 - 6\*b^2\*c^2)\*sinh((3\*n - 1)\*log(e)) + 3\*(a^2\*d^3\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x))^2 + 4\*b^2\*d^2\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x)) + (a^2\*d^3\*cosh(n\*log(x))^2 + 4\*b^2\*d^2\*cosh(n\*log(x)))\*sinh((3\*n - 1)\*log(e)))\*sinh(n\*log(x)))\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c)^2 + 2\*(a^2\*d^3\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x))^3 + 6\*b^2\*d^2\*cosh((3\*n - 1)\*log(e))\*cosh(n\*log(x))^2 - 6\*b^2\*c^2\*cosh((3\*n - 1)\*log(e)) + (a^2\*d^3\*cosh((3\*n - 1)\*log(e)) + a^2\*d^3\*sinh((3\*n - 1)\*log(e)))\*sinh(n\*log(x))^2

$$\begin{aligned}
& 3 + 3*(a^2*d^3*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + 2*b^2*d^2*\cosh((3*n \\
& - 1)*\log(e)) + (a^2*d^3*\cosh(n*\log(x)) + 2*b^2*d^2)*\sinh((3*n - 1)*\log(e))) \\
& *\sinh(n*\log(x))^2 + (a^2*d^3*\cosh(n*\log(x))^3 + 6*b^2*d^2*\cosh(n*\log(x))^2 \\
& - 6*b^2*c^2)*\sinh((3*n - 1)*\log(e)) + 3*(a^2*d^3*\cosh((3*n - 1)*\log(e))*\cos \\
& h(n*\log(x))^2 + 4*b^2*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (a^2*d^3* \\
& \cosh(n*\log(x))^2 + 4*b^2*d^2*\cosh(n*\log(x)))*\sinh((3*n - 1)*\log(e)))*\sinh(n \\
& *\log(x))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)*\sinh(d*\cosh(n*\log(x) \\
& )) + d*\sinh(n*\log(x)) + c) + (a^2*d^3*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) \\
& ^3 + 6*b^2*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x))^2 - 6*b^2*c^2*\cosh((3* \\
& n - 1)*\log(e)) + (a^2*d^3*\cosh((3*n - 1)*\log(e)) + a^2*d^3*\sinh((3*n - 1)* \\
& \log(e)))*\sinh(n*\log(x))^3 + 3*(a^2*d^3*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) \\
& + 2*b^2*d^2*\cosh((3*n - 1)*\log(e)) + (a^2*d^3*\cosh(n*\log(x)) + 2*b^2*d^2)* \\
& \sinh((3*n - 1)*\log(e)))*\sinh(n*\log(x))^2 + (a^2*d^3*\cosh(n*\log(x))^3 + 6*b^ \\
& 2*d^2*\cosh(n*\log(x))^2 - 6*b^2*c^2)*\sinh((3*n - 1)*\log(e)) + 3*(a^2*d^3*\cos \\
& h((3*n - 1)*\log(e))*\cosh(n*\log(x))^2 + 4*b^2*d^2*\cosh((3*n - 1)*\log(e))*\cos \\
& h(n*\log(x)) + (a^2*d^3*\cosh(n*\log(x))^2 + 4*b^2*d^2*\cosh(n*\log(x)))*\sinh((3 \\
& *n - 1)*\log(e)))*\sinh(n*\log(x))*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + \\
& c)^2 + 3*(a^2*d^3*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + a^2*d^3*\cosh(n*\log(x) \\
& )*\sinh((3*n - 1)*\log(e)))*\sinh(n*\log(x))^2 + (12*I*a*b*d*\cosh((3*n - 1) \\
& )*\log(e))*\cosh(n*\log(x)) + (12*I*a*b*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x) \\
& ) - 6*b^2*\cosh((3*n - 1)*\log(e)) - 6*(-2*I*a*b*d*\cosh(n*\log(x)) + b^2)*\sinh \\
& ((3*n - 1)*\log(e)) + (12*I*a*b*d*\cosh((3*n - 1)*\log(e)) + 12*I*a*b*d*\sinh(( \\
& 3*n - 1)*\log(e)))*\sinh(n*\log(x)))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) \\
& + c)^2 - 6*b^2*\cosh((3*n - 1)*\log(e)) + (24*I*a*b*d*\cosh((3*n - 1)*\log(e))* \\
& \cosh(n*\log(x)) - 12*b^2*\cosh((3*n - 1)*\log(e)) - 12*(-2*I*a*b*d*\cosh(n*\log( \\
& x)) + b^2)*\sinh((3*n - 1)*\log(e)) + (24*I*a*b*d*\cosh((3*n - 1)*\log(e)) + 24 \\
& *I*a*b*d*\sinh((3*n - 1)*\log(e)))*\sinh(n*\log(x)))*\cosh(d*\cosh(n*\log(x)) + d* \\
& \sinh(n*\log(x)) + c)*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c) + (12*I*a \\
& *b*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) - 6*b^2*\cosh((3*n - 1)*\log(e)) - \\
& 6*(-2*I*a*b*d*\cosh(n*\log(x)) + b^2)*\sinh((3*n - 1)*\log(e)) + (12*I*a*b*d*c \\
& \osh((3*n - 1)*\log(e)) + 12*I*a*b*d*\sinh((3*n - 1)*\log(e)))*\sinh(n*\log(x)))* \\
& \sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)^2 - 6*(-2*I*a*b*d*\cosh(n*\log( \\
& x)) + b^2)*\sinh((3*n - 1)*\log(e)) + (12*I*a*b*d*\cosh((3*n - 1)*\log(e)) + 12 \\
& *I*a*b*d*\sinh((3*n - 1)*\log(e)))*\sinh(n*\log(x)))*\operatorname{dilog}(I*\cosh(d*\cosh(n*\log( \\
& x)) + d*\sinh(n*\log(x)) + c) + I*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + \\
& c)) + (-12*I*a*b*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (-12*I*a*b*d*\cos \\
& h((3*n - 1)*\log(e))*\cosh(n*\log(x)) - 6*b^2*\cosh((3*n - 1)*\log(e)) - 6*(2*I* \\
& a*b*d*\cosh(n*\log(x)) + b^2)*\sinh((3*n - 1)*\log(e)) + (-12*I*a*b*d*\cosh((3*n \\
& - 1)*\log(e)) - 12*I*a*b*d*\sinh((3*n - 1)*\log(e)))*\sinh(n*\log(x)))*\cosh(d*c \\
& \osh(n*\log(x)) + d*\sinh(n*\log(x)) + c)^2 - 6*b^2*\cosh((3*n - 1)*\log(e)) + (- \\
& 24*I*a*b*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) - 12*b^2*\cosh((3*n - 1)*\lo \\
& g(e)) - 12*(2*I*a*b*d*\cosh(n*\log(x)) + b^2)*\sinh((3*n - 1)*\log(e)) + (-24*I \\
& *a*b*d*\cosh((3*n - 1)*\log(e)) - 24*I*a*b*d*\sinh((3*n - 1)*\log(e)))*\sinh(n* \\
& \log(x)))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)*\sinh(d*\cosh(n*\log(x)) \\
& + d*\sinh(n*\log(x)) + c) + (-12*I*a*b*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x) \\
& )) - 6*b^2*\cosh((3*n - 1)*\log(e)) - 6*(2*I*a*b*d*\cosh(n*\log(x)) + b^2)*\sinh \\
& ((3*n - 1)*\log(e)) + (-12*I*a*b*d*\cosh((3*n - 1)*\log(e)) - 12*I*a*b*d*\sinh( \\
& (3*n - 1)*\log(e)))*\sinh(n*\log(x)))*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) \\
& + c)^2 - 6*(2*I*a*b*d*\cosh(n*\log(x)) + b^2)*\sinh((3*n - 1)*\log(e)) + (-12* \\
& I*a*b*d*\cosh((3*n - 1)*\log(e)) - 12*I*a*b*d*\sinh((3*n - 1)*\log(e)))*\sinh(n* \\
& \log(x)))*\operatorname{dilog}(-I*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c) - I*\sinh(d* \\
& \cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)) + (((6*I*a*b*c^2 + 6*b^2*c)*\cosh((3 \\
& *n - 1)*\log(e)) + (6*I*a*b*c^2 + 6*b^2*c)*\sinh((3*n - 1)*\log(e)))*\cosh(d*\co \\
& sh(n*\log(x)) + d*\sinh(n*\log(x)) + c)^2 + ((12*I*a*b*c^2 + 12*b^2*c)*\cosh((3 \\
& *n - 1)*\log(e)) + (12*I*a*b*c^2 + 12*b^2*c)*\sinh((3*n - 1)*\log(e)))*\cosh(d* \\
& \cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log \\
& (x)) + c) + ((6*I*a*b*c^2 + 6*b^2*c)*\cosh((3*n - 1)*\log(e)) + (6*I*a*b*c^2 \\
& + 6*b^2*c)*\sinh((3*n - 1)*\log(e)))*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) \\
& + c)^2 + (6*I*a*b*c^2 + 6*b^2*c)*\cosh((3*n - 1)*\log(e)) + (6*I*a*b*c^2 + 6
\end{aligned}$$

$$\begin{aligned}
& *b^2*c)*\sinh((3*n - 1)*\log(e))*\log(\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) \\
& ) + c) + \sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c) + I) + (((-6*I*a*b*c \\
& ^2 + 6*b^2*c)*\cosh((3*n - 1)*\log(e)) + (-6*I*a*b*c^2 + 6*b^2*c)*\sinh((3*n - \\
& 1)*\log(e))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)^2 + ((-12*I*a*b* \\
& c^2 + 12*b^2*c)*\cosh((3*n - 1)*\log(e)) + (-12*I*a*b*c^2 + 12*b^2*c)*\sinh((3 \\
& *n - 1)*\log(e))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)*\sinh(d*\cosh( \\
& n*\log(x)) + d*\sinh(n*\log(x)) + c) + ((-6*I*a*b*c^2 + 6*b^2*c)*\cosh((3*n - 1 \\
& )*\log(e)) + (-6*I*a*b*c^2 + 6*b^2*c)*\sinh((3*n - 1)*\log(e))*\sinh(d*\cosh(n* \\
& \log(x)) + d*\sinh(n*\log(x)) + c)^2 + (-6*I*a*b*c^2 + 6*b^2*c)*\cosh((3*n - 1) \\
& *\log(e)) + (-6*I*a*b*c^2 + 6*b^2*c)*\sinh((3*n - 1)*\log(e))*\log(\cosh(d*\cosh \\
& (n*\log(x)) + d*\sinh(n*\log(x)) + c) + \sinh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x) \\
& )) + c) - I) + (-6*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x))^2 - 6*b^ \\
& 2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (-6*I*a*b*d^2*\cosh((3*n - 1)*lo \\
& g(e))*\cosh(n*\log(x))^2 - 6*b^2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (- \\
& 6*I*a*b*d^2*\cosh((3*n - 1)*\log(e)) - 6*I*a*b*d^2*\sinh((3*n - 1)*\log(e))*\si \\
& nh(n*\log(x))^2 + (6*I*a*b*c^2 - 6*b^2*c)*\cosh((3*n - 1)*\log(e)) + (-6*I*a*b \\
& *d^2*\cosh(n*\log(x))^2 + 6*I*a*b*c^2 - 6*b^2*d*\cosh(n*\log(x)) - 6*b^2*c)*\sin \\
& h((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) \\
& - 6*b^2*d*\cosh((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh(n*\log(x)) - 6*b^2*d) \\
& *\sinh((3*n - 1)*\log(e))*\sinh(n*\log(x)))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(nl \\
& og(x)) + c)^2 + (-12*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x))^2 - 12 \\
& *b^2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (-12*I*a*b*d^2*\cosh((3*n - 1) \\
& )*\log(e)) - 12*I*a*b*d^2*\sinh((3*n - 1)*\log(e))*\sinh(n*\log(x))^2 + (12*I*a \\
& *b*c^2 - 12*b^2*c)*\cosh((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh(n*\log(x))^2 \\
& + 12*I*a*b*c^2 - 12*b^2*d*\cosh(n*\log(x)) - 12*b^2*c)*\sinh((3*n - 1)*\log(e) \\
& ) + (-24*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) - 12*b^2*d*\cosh((3 \\
& *n - 1)*\log(e)) + (-24*I*a*b*d^2*\cosh(n*\log(x)) - 12*b^2*d)*\sinh((3*n - 1)* \\
& \log(e))*\sinh(n*\log(x))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c)*\sinh \\
& (d*\cosh(n*\log(x)) + d*\sinh(n*\log(x)) + c) + (-6*I*a*b*d^2*\cosh((3*n - 1)*lo \\
& g(e))*\cosh(n*\log(x))^2 - 6*b^2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (- \\
& 6*I*a*b*d^2*\cosh((3*n - 1)*\log(e)) - 6*I*a*b*d^2*\sinh((3*n - 1)*\log(e))*\si \\
& nh(n*\log(x))^2 + (6*I*a*b*c^2 - 6*b^2*c)*\cosh((3*n - 1)*\log(e)) + (-6*I*a*b \\
& *d^2*\cosh(n*\log(x))^2 + 6*I*a*b*c^2 - 6*b^2*d*\cosh(n*\log(x)) - 6*b^2*c)*\sin \\
& h((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) \\
& - 6*b^2*d*\cosh((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh(n*\log(x)) - 6*b^2*d) \\
& *\sinh((3*n - 1)*\log(e))*\sinh(n*\log(x))*\sinh(d*\cosh(n*\log(x)) + d*\sinh(nl \\
& og(x)) + c)^2 + (-6*I*a*b*d^2*\cosh((3*n - 1)*\log(e)) - 6*I*a*b*d^2*\sinh((3* \\
& n - 1)*\log(e))*\sinh(n*\log(x))^2 + (6*I*a*b*c^2 - 6*b^2*c)*\cosh((3*n - 1)*l \\
& og(e)) + (-6*I*a*b*d^2*\cosh(n*\log(x))^2 + 6*I*a*b*c^2 - 6*b^2*d*\cosh(n*\log( \\
& x)) - 6*b^2*c)*\sinh((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh((3*n - 1)*\log(e) \\
& )*\cosh(n*\log(x)) - 6*b^2*d*\cosh((3*n - 1)*\log(e)) + (-12*I*a*b*d^2*\cosh(n* \\
& \log(x)) - 6*b^2*d)*\sinh((3*n - 1)*\log(e))*\sinh(n*\log(x))*\log(I*\cosh(d*cos \\
& h(n*\log(x)) + d*\sinh(n*\log(x)) + c) + I*\sinh(d*\cosh(n*\log(x)) + d*\sinh(n*lo \\
& g(x)) + c) + 1) + (6*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x))^2 - 6* \\
& b^2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (6*I*a*b*d^2*\cosh((3*n - 1)*l \\
& og(e))*\cosh(n*\log(x))^2 - 6*b^2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + ( \\
& 6*I*a*b*d^2*\cosh((3*n - 1)*\log(e)) + 6*I*a*b*d^2*\sinh((3*n - 1)*\log(e))*\si \\
& nh(n*\log(x))^2 + (-6*I*a*b*c^2 - 6*b^2*c)*\cosh((3*n - 1)*\log(e)) + (6*I*a*b \\
& *d^2*\cosh(n*\log(x))^2 - 6*I*a*b*c^2 - 6*b^2*d*\cosh(n*\log(x)) - 6*b^2*c)*\sin \\
& h((3*n - 1)*\log(e)) + (12*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) - \\
& 6*b^2*d*\cosh((3*n - 1)*\log(e)) + (12*I*a*b*d^2*\cosh(n*\log(x)) - 6*b^2*d)*s \\
& inh((3*n - 1)*\log(e))*\sinh(n*\log(x))*\cosh(d*\cosh(n*\log(x)) + d*\sinh(n*log \\
& (x)) + c)^2 + (12*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x))^2 - 12*b^ \\
& 2*d*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) + (12*I*a*b*d^2*\cosh((3*n - 1)*lo \\
& g(e)) + 12*I*a*b*d^2*\sinh((3*n - 1)*\log(e))*\sinh(n*\log(x))^2 + (-12*I*a*b* \\
& c^2 - 12*b^2*c)*\cosh((3*n - 1)*\log(e)) + (12*I*a*b*d^2*\cosh(n*\log(x))^2 - 1 \\
& 2*I*a*b*c^2 - 12*b^2*d*\cosh(n*\log(x)) - 12*b^2*c)*\sinh((3*n - 1)*\log(e)) + \\
& (24*I*a*b*d^2*\cosh((3*n - 1)*\log(e))*\cosh(n*\log(x)) - 12*b^2*d*\cosh((3*n - \\
& 1)*\log(e)) + (24*I*a*b*d^2*\cosh(n*\log(x)) - 12*b^2*d)*\sinh((3*n - 1)*\log(e)
\end{aligned}$$

$$\begin{aligned} &)) * \sinh(n \log(x)) * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (6 * I * a * b * d^2 * \cosh((3 * n - 1) * \log(e)) * \cosh(n \log(x))^2 - 6 * b^2 * d * \cosh((3 * n - 1) * \log(e)) * \cosh(n \log(x)) + (6 * I * a * b * d^2 * \cosh((3 * n - 1) * \log(e)) + 6 * I * a * b * d^2 * \sinh((3 * n - 1) * \log(e))) * \sinh(n \log(x))^2 + (-6 * I * a * b * c^2 - 6 * b^2 * c) * \cosh((3 * n - 1) * \log(e)) + (6 * I * a * b * d^2 * \cosh(n \log(x))^2 - 6 * I * a * b * c^2 - 6 * b^2 * d * \cosh(n \log(x)) - 6 * b^2 * c) * \sinh((3 * n - 1) * \log(e)) + (12 * I * a * b * d^2 * \cosh((3 * n - 1) * \log(e)) * \cosh(n \log(x)) - 6 * b^2 * d * \cosh((3 * n - 1) * \log(e)) + (12 * I * a * b * d^2 * \cosh(n \log(x)) - 6 * b^2 * d) * \sinh((3 * n - 1) * \log(e))) * \sinh(n \log(x)) * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 + (6 * I * a * b * d^2 * \cosh((3 * n - 1) * \log(e)) + 6 * I * a * b * d^2 * \sinh((3 * n - 1) * \log(e))) * \sinh(n \log(x))^2 + (-6 * I * a * b * c^2 - 6 * b^2 * c) * \cosh((3 * n - 1) * \log(e)) + (6 * I * a * b * d^2 * \cosh(n \log(x))^2 - 6 * I * a * b * c^2 - 6 * b^2 * d * \cosh(n \log(x)) - 6 * b^2 * c) * \sinh((3 * n - 1) * \log(e)) + (12 * I * a * b * d^2 * \cosh((3 * n - 1) * \log(e)) * \cosh(n \log(x)) - 6 * b^2 * d * \cosh((3 * n - 1) * \log(e)) + (12 * I * a * b * d^2 * \cosh(n \log(x)) - 6 * b^2 * d) * \sinh((3 * n - 1) * \log(e))) * \sinh(n \log(x)) * \log(-I * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) - I * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + 1) + ((-12 * I * a * b * \cosh((3 * n - 1) * \log(e)) - 12 * I * a * b * \sinh((3 * n - 1) * \log(e))) * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 - 12 * I * a * b * \cosh((3 * n - 1) * \log(e)) + (-24 * I * a * b * \cosh((3 * n - 1) * \log(e)) - 24 * I * a * b * \sinh((3 * n - 1) * \log(e))) * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (-12 * I * a * b * \cosh((3 * n - 1) * \log(e)) - 12 * I * a * b * \sinh((3 * n - 1) * \log(e))) * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 - 12 * I * a * b * \sinh((3 * n - 1) * \log(e)) * \text{polylog}(3, I * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + I * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)) + ((12 * I * a * b * \cosh((3 * n - 1) * \log(e)) + 12 * I * a * b * \sinh((3 * n - 1) * \log(e))) * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 + 12 * I * a * b * \cosh((3 * n - 1) * \log(e)) + (24 * I * a * b * \cosh((3 * n - 1) * \log(e)) + 24 * I * a * b * \sinh((3 * n - 1) * \log(e))) * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (12 * I * a * b * \cosh((3 * n - 1) * \log(e)) + 12 * I * a * b * \sinh((3 * n - 1) * \log(e))) * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 + 12 * I * a * b * \sinh((3 * n - 1) * \log(e)) * \text{polylog}(3, -I * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) - I * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)) + (a^2 * d^3 * \cosh(n \log(x))^3 - 6 * b^2 * c^2) * \sinh((3 * n - 1) * \log(e)) + 3 * (a^2 * d^3 * \cosh((3 * n - 1) * \log(e)) * \cosh(n \log(x))^2 + a^2 * d^3 * \cosh(n \log(x))^2 * \sinh((3 * n - 1) * \log(e)) * \sinh(n \log(x))) / (d^3 * n * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 + 2 * d^3 * n * \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + d^3 * n * \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)^2 + d^3 * n) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sech(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((b\*sech(d\*x^n + c) + a)^2\*(e\*x)^(3\*n - 1), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int (ex)^{3n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(3\*n-1)\*(a+b\*sech(c+d\*x^n))^2,x)

[Out] int((e\*x)^(3\*n-1)\*(a+b\*sech(c+d\*x^n))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2b^2e^{3n}x^{2n}}{\operatorname{dene}(2dx^n+2c)+\operatorname{den}} + \frac{(ex)^{3n}a^2}{3en} + \int \frac{4(abde^{3n}e^{(dx^n+3n\log(x)+c)} + b^2e^{3n}x^{2n})}{\operatorname{dex}e^{(2dx^n+2c)} + \operatorname{dex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)\*(a+b\*sech(c+d\*x^n))^2,x, algorithm="maxima")

[Out] -2\*b^2\*e^(3\*n)\*x^(2\*n)/(d\*e\*n\*e^(2\*d\*x^n + 2\*c) + d\*e\*n) + 1/3\*(e\*x)^(3\*n)\*a^2/(e\*n) + integrate(4\*(a\*b\*d\*e^(3\*n)\*e^(d\*x^n + 3\*n\*log(x) + c) + b^2\*e^(3\*n)\*x^(2\*n))/(d\*e\*x\*e^(2\*d\*x^n + 2\*c) + d\*e\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + \frac{b}{\cosh(c + dx^n)} \right)^2 (ex)^{3n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cosh(c + d\*x^n))^2\*(e\*x)^(3\*n - 1), x)

[Out] int((a + b/cosh(c + d\*x^n))^2\*(e\*x)^(3\*n - 1), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{3n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+3\*n)\*(a+b\*sech(c+d\*x\*\*n))\*\*2,x)

[Out] Integral((e\*x)\*\*(3\*n - 1)\*(a + b\*sech(c + d\*x\*\*n))\*\*2, x)

$$3.79 \quad \int \frac{(ex)^{-1+n}}{a+b\operatorname{sech}(c+dx^n)} dx$$

Optimal. Leaf size=87

$$\frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{aden\sqrt{a-b}\sqrt{a+b}}$$

[Out]  $(e*x)^n/a/e/n-2*b*(e*x)^n*\arctan((a-b)^{(1/2)*\tanh(1/2*c+1/2*d*x^n)/(a+b)^{(1/2)})/a/d/e/n/(x^n)/(a-b)^{(1/2)/(a+b)^{(1/2)}}$

**Rubi [A]** time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$ , Rules used = {5440, 5436, 3783, 2659, 208}

$$\frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{aden\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + n)/(a + b\*Sech[c + d\*x^n]), x]

[Out]  $(e*x)^n/(a*e*n) - (2*b*(e*x)^n*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tanh}[(c + d*x^n)/2])/(\text{Sqrt}[a + b])]/(a*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d*e*n*x^n)$

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]]/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + b + (a - b)\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3783

Int[(csc[(c\_.) + (d\_.)\*(x\_)])\*(b\_.) + (a\_)^(-1), x\_Symbol] :> Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a\*Sin[c + d\*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

#### Rule 5440

Int[((e\_)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbol] :> Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sech[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx &= \frac{(x^{-n}(ex)^n) \int \frac{x^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx}{e} \\
&= \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int \frac{1}{a + b \operatorname{sech}(c + dx)} dx, x, x^n\right)}{en} \\
&= \frac{(ex)^n}{aen} - \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int \frac{1}{1 + \frac{a \cosh(c + dx)}{b}} dx, x, x^n\right)}{aen} \\
&= \frac{(ex)^n}{aen} + \frac{(2ix^{-n}(ex)^n) \operatorname{Subst}\left(\int \frac{1}{1 + \frac{a}{b} + \left(1 - \frac{a}{b}\right)x^2} dx, x, i \tanh\left(\frac{1}{2}(c + dx^n)\right)\right)}{aden} \\
&= \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c + dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}den}
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 80, normalized size = 0.92

$$\frac{(ex)^n \left( \frac{2bx^{-n} \tan^{-1}\left(\frac{(b-a) \tanh\left(\frac{1}{2}(c + dx^n)\right)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}} + cx^{-n} + d \right)}{aden}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)/(a + b\*Sech[c + d\*x^n]), x]

[Out] ((e\*x)^n\*(d + c/x^n + (2\*b\*ArcTan[(-a + b)\*Tanh[(c + d\*x^n)/2]]/Sqrt[a^2 - b^2]))/(Sqrt[a^2 - b^2]\*x^n))/(a\*d\*e^n)

**fricas [B]** time = 0.44, size = 511, normalized size = 5.87

$$\left[ \frac{(a^2 - b^2)d \cosh((n-1)\log(e)) \cosh(n \log(x)) + (a^2 - b^2)d \cosh(n \log(x)) \sinh((n-1)\log(e)) - \left(\sqrt{-a^2 + b^2}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n)), x, algorithm="fricas")

[Out] (((a^2 - b^2)\*d\*cosh((n - 1)\*log(e))\*cosh(n\*log(x)) + (a^2 - b^2)\*d\*cosh(n\*log(x))\*sinh((n - 1)\*log(e)) - (sqrt(-a^2 + b^2)\*b\*cosh((n - 1)\*log(e)) + sqrt(-a^2 + b^2)\*b\*sinh((n - 1)\*log(e)))\*log((a\*b + (b^2 + sqrt(-a^2 + b^2)\*b)\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2)\*b)\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + sqrt(-a^2 + b^2)\*a)/(a\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + b)) + ((a^2 - b^2)\*d\*cosh((n - 1)\*log(e)) + (a^2 - b^2)\*d\*sinh((n - 1)\*log(e))\*sinh(n\*log(x)))/(a^3 - a\*b^2)\*d\*n), ((a^2 - b^2)\*d\*cosh((n - 1)\*log(e))\*cosh(n\*log(x)) + (a^2 - b^2)\*d\*cosh(n\*log(x))\*sinh((n - 1)\*log(e)) + 2\*(sqrt(a^2 - b^2)\*b\*cosh((n - 1)\*log(e)) + sqrt(a^2 - b^2)\*b\*sinh((n - 1)\*log(e)))\*arctan(-(sqrt(a^2 - b^2)\*a\*cosh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + sqrt(a^2 - b^2)\*a\*sinh(d\*cosh(n\*log(x)) + d\*sinh(n\*log(x)) + c) + sqrt(a^2 - b^2)\*b)/(a^2 -



$b^2)) + ((a^2 - b^2)*d*\cosh((n - 1)*\log(e)) + (a^2 - b^2)*d*\sinh((n - 1)*\log(e)))*\sinh(n*\log(x)))/((a^3 - a*b^2)*d*n]$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((e\*x)^(n - 1)/(b\*sech(d\*x^n + c) + a), x)

**maple** [C] time = 0.56, size = 317, normalized size = 3.64

$$x e^{\frac{(-1+n)\left(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{an} - \frac{2b e^{-\frac{i\pi n \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)}{2}}}{e^{\frac{i\pi n \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)}{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n)),x)

[Out]  $\frac{1}{a} \frac{1}{n} x \exp\left(\frac{1}{2}(-1+n)*(-i\pi \operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)+i\pi \operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2+i\pi \operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2-i\pi \operatorname{csgn}(I*e*x)^3+2*\ln(x)+2*\ln(e)\right)-2/a*b/n*\exp(-1/2*I*Pi*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x))*\exp(1/2*I*Pi*n*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2)*\exp(1/2*I*Pi*n*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2)*\exp(-1/2*I*Pi*n*\operatorname{csgn}(I*e*x)^3)*\exp(1/2*I*Pi*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x))*\exp(-1/2*I*Pi*\operatorname{csgn}(I*e)*\operatorname{csgn}(I*e*x)^2)*\exp(-1/2*I*Pi*\operatorname{csgn}(I*x)*\operatorname{csgn}(I*e*x)^2)*\exp(1/2*I*Pi*\operatorname{csgn}(I*e*x)^3)*e^n/e*\exp(c)/d/(a^2*\exp(2*c)-b^2*\exp(2*c))^(1/2)*\arctan(1/2*(2*a*\exp(2*c+d*x^n)+2*\exp(c)*b)/(a^2*\exp(2*c)-b^2*\exp(2*c))^(1/2))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2be^n \int \frac{e^{(dx^n+n\log(x)+c)}}{a^2exe^{(2dx^n+2c)} + 2abexe^{(dx^n+c)} + a^2ex} dx + \frac{e^{n-1}x^n}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n)),x, algorithm="maxima")

[Out]  $-2*b*e^n*\int e^{(d*x^n + n*\log(x) + c)}/(a^2*e*x*e^{(2*d*x^n + 2*c)} + 2*a*b*e*x*e^{(d*x^n + c)} + a^2*e*x), x) + e^{(n - 1)}*x^n/(a*n)$

**mupad** [B] time = 2.10, size = 409, normalized size = 4.70

$$\frac{x (ex)^{n-1}}{an} \left( 2 \operatorname{atan} \left( \frac{a^2 e^{dx^n} e^c \left( \frac{2bx(ex)^{n-1}}{a^4 d n x^n \sqrt{b^2 x^2 (ex)^{2n-2}}} + \frac{2bdn x^n (ex)^{1-n} \sqrt{b^2 x^2 (ex)^{2n-2}}}{a^2 x \sqrt{a^4 d^2 n^2 x^{2n} - a^2 b^2 d^2 n^2 x^{2n}} \sqrt{a^2 d^2 n^2 x^{2n} (a^2 - b^2)}} \right)}{2} \right) \sqrt{a^4 d^2 n^2 x^{2n} - a^2 b^2 d^2 n^2 x^{2n}}}{\sqrt{a^4 d^2 n^2 x^{2n} - a^2 b^2 d^2 n^2 x^{2n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(n - 1)/(a + b/cosh(c + d\*x^n)),x)

[Out]  $(x*(e*x)^(n - 1))/(a*n) - ((2*\operatorname{atan}((a^2*\exp(d*x^n)*\exp(c))*((2*b*x*(e*x)^(n - 1))/(a^4*d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)) + (2*b*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)))/(a^2*x*(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2))))*(a^4*d^2$

```

*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2))/2 + (a*d*n*x^n*(e*x)^(1 - n)
*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(x*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2
))) + 2*atan((x*(e*x)^(n - 1)*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2))/(a*d
*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/
(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)/(a+b\*sech(c+d\*x\*\*n)),x)

[Out] Integral((e\*x)\*\*(n - 1)/(a + b\*sech(c + d\*x\*\*n)), x)

$$3.80 \quad \int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)} dx$$

Optimal. Leaf size=307

$$\frac{bx^{-2n}(ex)^{2n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{bx^{-2n}(ex)^{2n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{aden\sqrt{b^2-a^2}} + \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b+\sqrt{b^2-a^2}}+1\right)}{aden\sqrt{b^2-a^2}}$$

[Out]  $1/2*(e*x)^{(2*n)}/a/e/n-b*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+b*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-b*(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+b*(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}$

Rubi [A] time = 0.56, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5440, 5436, 4191, 3320, 2264, 2190, 2279, 2391}

$$\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{aden\sqrt{b^2-a^2}} + \frac{bx^{-n}(ex)^{2n}\log\left(\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}+1\right)}{aden\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1 + 2\*n)/(a + b\*Sech[c + d\*x^n]), x]

[Out]  $(e*x)^{(2*n)}/(2*a*e*n) - (b*(e*x)^{(2*n)}*\operatorname{Log}[1 + (a*E^{(c + d*x^n)})/(b - \operatorname{Sqrt}[-a^2 + b^2])])/(a*\operatorname{Sqrt}[-a^2 + b^2]*d*e*n*x^n) + (b*(e*x)^{(2*n)}*\operatorname{Log}[1 + (a*E^{(c + d*x^n)})/(b + \operatorname{Sqrt}[-a^2 + b^2])])/(a*\operatorname{Sqrt}[-a^2 + b^2]*d*e*n*x^n) - (b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2, -((a*E^{(c + d*x^n)})/(b - \operatorname{Sqrt}[-a^2 + b^2]))])/(a*\operatorname{Sqrt}[-a^2 + b^2]*d^2*e*n*x^{(2*n)}) + (b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2, -((a*E^{(c + d*x^n)})/(b + \operatorname{Sqrt}[-a^2 + b^2]))])/(a*\operatorname{Sqrt}[-a^2 + b^2]*d^2*e*n*x^{(2*n)})$

Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2264

Int[(((F\_)^(u\_)\*((f\_) + (g\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*(F\_)^(u\_) + (c\_)\*((F\_)^(v\_))), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)], x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3320

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*sin[(e_.) + Pi*(k_.) + (Complex[0, fz_]
*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-I*e) + f*fz*x))/
(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-I*e) + f*fz*x))/E^(I*Pi*(k - 1/2)) -
(b*E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi)), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x]
&& IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.),
x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x]^n)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :=
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /;
FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

Rule 5440

```
Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a + b*Sech[c + d*x^n])^p,
x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+2n}}{a + b \operatorname{sech}(c + dx^n)} dx &= \frac{(x^{-2n}(ex)^{2n}) \int \frac{x^{-1+2n}}{a+b \operatorname{sech}(c+dx^n)} dx}{e} \\
&= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{en} \\
&= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \left(\frac{x}{a} - \frac{bx}{a(b+a \cosh(c+dx))}\right) dx, x, x^n\right)}{en} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{(bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{b+a \cosh(c+dx)} dx, x, x^n\right)}{aen} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{(2bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, x^n\right)}{aen} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{(2bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^{c+dx}x}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2}en} + \frac{(2bx^{-2n}(ex)^{2n})}{\sqrt{-a^2+b^2}en} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{(bx)^{-n}}{a\sqrt{-a^2+b^2}den} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{(bx)^{-n}}{a\sqrt{-a^2+b^2}den} \\
&= \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{bx^{-n}}{a\sqrt{-a^2+b^2}den}
\end{aligned}$$

**Mathematica [C]** time = 2.10, size = 859, normalized size = 2.80

$$(ex)^{2n} (b + a \cosh(dx^n + c)) \left( \frac{2b \left( 2(dx^n + c) \tan^{-1}\left(\frac{(a+b) \coth\left(\frac{1}{2}(dx^n + c)\right)}{\sqrt{a^2 - b^2}}\right) + 2\left(c - i \cos^{-1}\left(-\frac{b}{a}\right)\right) \tan^{-1}\left(\frac{(a-b) \tanh\left(\frac{1}{2}(dx^n + c)\right)}{\sqrt{a^2 - b^2}}\right) + \left(\cos^{-1}\left(-\frac{b}{a}\right) + 2\left(\tan^{-1}\left(\frac{(a+b) \coth\left(\frac{1}{2}(dx^n + c)\right)}{\sqrt{a^2 - b^2}}\right) + \tan^{-1}\left(\frac{(a-b) \tanh\left(\frac{1}{2}(dx^n + c)\right)}{\sqrt{a^2 - b^2}}\right)\right)\right)}{\sqrt{a^2 - b^2}} \right)}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + 2\*n)/(a + b\*Sech[c + d\*x^n]), x]

[Out] ((e\*x)^(2\*n)\*(b + a\*Cosh[c + d\*x^n])\*(1 + (2\*b\*(2\*(c + d\*x^n)\*ArcTan[((a + b)\*Coth[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]] + 2\*(c - I\*ArcCos[-(b/a)])\*ArcTan[((a - b)\*Tanh[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] + 2\*(ArcTan[((a + b)\*Coth[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)\*Tanh[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]]))\*Log[(Sqrt[a^2 - b^2]\*E^(-1/2\*c - (d\*x^n)/2))/(Sqrt[2]\*Sqrt[a]\*Sqrt[b + a\*Cosh[c + d\*x^n]])] + (ArcCos[-(b/a)] - 2\*(ArcTan[((a + b)\*Coth[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)\*Tanh[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]]))\*Log[(Sqrt[a^2 - b^2]\*E^((c + d\*x^n)/2))/(Sqrt[2]\*Sqrt[a]\*Sqrt[b + a\*Cosh[c + d\*x^n]])] - (ArcCos[-(b/a)] + 2\*ArcTan[((a - b)\*Tanh[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]]\*Log[((a + b)\*(-a + b + I\*Sqrt[a^2 - b^2])\*(-1 + Tanh[(c + d\*x^n)/2]))/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^n)/2]))] - (ArcCos[-(b/a)] - 2\*ArcTan[((a - b)\*Tanh[(c + d\*x^n)/2])/Sqrt[a^2 - b^2]]\*Log[((a + b)\*(a - b + I\*Sqrt[a^2 - b^2])\*(1 + Tanh[(c + d\*x^n)/2]))/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^n)/2]))] + I\*(PolyLog[2, ((b - I\*Sqrt[a^2 - b^2])\*(a + b - I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^n)/2])/(a\*(a + b + I\*Sqrt[a^2 - b^2]\*Tanh[(c + d\*x^n)/2]))]))

$$\frac{\int (e^x)^{-1+2n} / (a + b + I \sqrt{a^2 - b^2} \operatorname{Tanh}[(c + d x^n)/2]) dx - \operatorname{PolyLog}[2, (b + I \sqrt{a^2 - b^2})(a + b - I \sqrt{a^2 - b^2} \operatorname{Tanh}[(c + d x^n)/2])]}{(a + b + I \sqrt{a^2 - b^2} \operatorname{Tanh}[(c + d x^n)/2])} \Big/ \left( \sqrt{a^2 - b^2} d^{2n} x^{2n} \right) \operatorname{Sech}[c + d x^n] / (2 a e^n (a + b \operatorname{Sech}[c + d x^n]))$$

**fricas [B]** time = 0.42, size = 1286, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n)),x, algorithm="fricas")

[Out]  $\frac{1}{2} \left( (a^2 - b^2) d^2 \cosh((2n - 1) \log(e)) \cosh(n \log(x))^2 + (a^2 - b^2) d^2 \cosh(n \log(x))^2 \sinh((2n - 1) \log(e)) + ((a^2 - b^2) d^2 \cosh((2n - 1) \log(e)) + (a^2 - b^2) d^2 \sinh((2n - 1) \log(e))) \sinh(n \log(x))^2 + 2(a b \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + a b \sqrt{-(a^2 - b^2)/a^2} \sinh((2n - 1) \log(e))) \operatorname{dilog}(-((a \sqrt{-(a^2 - b^2)/a^2} + b) \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (a \sqrt{-(a^2 - b^2)/a^2} + b) \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + a) / a + 1) - 2(a b \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + a b \sqrt{-(a^2 - b^2)/a^2} \sinh((2n - 1) \log(e))) \operatorname{dilog}(((a \sqrt{-(a^2 - b^2)/a^2} - b) \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (a \sqrt{-(a^2 - b^2)/a^2} - b) \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) - a) / a + 1) + 2(a b c \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + a b c \sqrt{-(a^2 - b^2)/a^2} \sinh((2n - 1) \log(e))) \log(2 a \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + 2 a \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + 2 a \sqrt{-(a^2 - b^2)/a^2} + 2 b) - 2(a b c \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + a b c \sqrt{-(a^2 - b^2)/a^2} \sinh((2n - 1) \log(e))) \log(2 a \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + 2 a \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) - 2 a \sqrt{-(a^2 - b^2)/a^2} + 2 b) + 2(a b d \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) \cosh(n \log(x)) + a b c \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + (a b d \sqrt{-(a^2 - b^2)/a^2} \cosh(n \log(x)) + a b c \sqrt{-(a^2 - b^2)/a^2}) \sinh((2n - 1) \log(e)) + (a b d \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + a b d \sqrt{-(a^2 - b^2)/a^2} \sinh((2n - 1) \log(e))) \sinh(n \log(x)) \log(((a \sqrt{-(a^2 - b^2)/a^2} + b) \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (a \sqrt{-(a^2 - b^2)/a^2} + b) \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + a) / a) - 2(a b d \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) \cosh(n \log(x)) + a b c \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + (a b d \sqrt{-(a^2 - b^2)/a^2} \cosh(n \log(x)) + a b c \sqrt{-(a^2 - b^2)/a^2}) \sinh((2n - 1) \log(e)) + (a b d \sqrt{-(a^2 - b^2)/a^2} \cosh((2n - 1) \log(e)) + a b d \sqrt{-(a^2 - b^2)/a^2} \sinh((2n - 1) \log(e))) \sinh(n \log(x)) \log(-((a \sqrt{-(a^2 - b^2)/a^2} - b) \cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + (a \sqrt{-(a^2 - b^2)/a^2} - b) \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) - a) / a) + 2((a^2 - b^2) d^2 \cosh((2n - 1) \log(e)) \cosh(n \log(x)) + (a^2 - b^2) d^2 \cosh(n \log(x)) \sinh((2n - 1) \log(e))) \sinh(n \log(x)) / ((a^3 - a b^2) d^{2n})$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((e\*x)^(2\*n - 1)/(b\*sech(d\*x^n + c) + a), x)

**maple [C]** time = 0.66, size = 587, normalized size = 1.91

$$x e^{\frac{(-1+2n)(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e))}{2}} 2b e^{-i\pi n \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)} e^{i\pi n}$$


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$$2an$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n)), x)

[Out] 1/2/a/n\*x\*exp(1/2\*(-1+2\*n)\*(-I\*Pi\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x)+I\*Pi\*csgn(I\*e)\*csgn(I\*e\*x)^2+I\*Pi\*csgn(I\*x)\*csgn(I\*e\*x)^2-I\*Pi\*csgn(I\*e\*x)^3+2\*ln(x)+2\*ln(e))-2/a\*b\*exp(-I\*Pi\*n\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x))\*exp(I\*Pi\*n\*csgn(I\*e)\*csgn(I\*e\*x)^2)\*exp(I\*Pi\*n\*csgn(I\*x)\*csgn(I\*e\*x)^2)\*exp(-I\*Pi\*n\*csgn(I\*e\*x)^3)\*exp(1/2\*I\*Pi\*csgn(I\*e)\*csgn(I\*x)\*csgn(I\*e\*x))\*exp(-1/2\*I\*Pi\*csgn(I\*e)\*csgn(I\*e\*x)^2)\*exp(-1/2\*I\*Pi\*csgn(I\*x)\*csgn(I\*e\*x)^2)\*exp(1/2\*I\*Pi\*csgn(I\*e\*x)^3)\*(e^n)^2/e\*exp(c)/n/d^2\*(1/2\*x^n\*d\*(ln((a\*exp(2\*c+d\*x^n)+exp(c))\*b-(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2)))/(exp(c)\*b-(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2)))-ln((a\*exp(2\*c+d\*x^n)+exp(c))\*b+(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2))/(exp(c)\*b+(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2)))/(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2)+1/2\*(dilog((a\*exp(2\*c+d\*x^n)+exp(c))\*b-(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2))/(exp(c)\*b-(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2))-dilog((a\*exp(2\*c+d\*x^n)+exp(c))\*b+(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2))/(exp(c)\*b+(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2)))/(b^2\*exp(2\*c)-a^2\*exp(2\*c))^(1/2))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-2 b e^{2n} \int \frac{e^{(dx^n+2n \log(x)+c)}}{a^2 e x e^{(2 dx^n+2c)} + 2 a b e x e^{(dx^n+c)} + a^2 e x} dx + \frac{e^{2n-1} x^{2n}}{2 a n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n)), x, algorithm="maxima")

[Out] -2\*b\*e^(2\*n)\*integrate(e^(d\*x^n + 2\*n\*log(x) + c)/(a^2\*e\*x\*e^(2\*d\*x^n + 2\*c) + 2\*a\*b\*e\*x\*e^(d\*x^n + c) + a^2\*e\*x), x) + 1/2\*e^(2\*n - 1)\*x^(2\*n)/(a\*n)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{2n-1}}{a + \frac{b}{\cosh(c+dx^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(2\*n - 1)/(a + b/cosh(c + d\*x^n)), x)

[Out] int((e\*x)^(2\*n - 1)/(a + b/cosh(c + d\*x^n)), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+2\*n)/(a+b\*sech(c+d\*x\*\*n)), x)

[Out] Integral((e\*x)\*\*(2\*n - 1)/(a + b\*sech(c + d\*x\*\*n)), x)

$$3.81 \quad \int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)} dx$$

**Optimal.** Leaf size=452

$$\frac{2bx^{-3n}(ex)^{3n}\operatorname{Li}_3\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{Li}_3\left(-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}}$$

[Out]  $1/3*(e*x)^{(3*n)}/a/e/n-b*(e*x)^{(3*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+b*(e*x)^{(3*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(3*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(3*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(3*n)}*\operatorname{polylog}(3,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d^3/e/n/(x^{(3*n)})/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(3*n)}*\operatorname{polylog}(3,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d^3/e/n/(x^{(3*n)})/(-a^2+b^2)^{(1/2)}$

**Rubi [A]** time = 0.86, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5440, 5436, 4191, 3320, 2264, 2190, 2531, 2282, 6589}

$$\frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2bx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+3*n}/(a+b*\operatorname{Sech}[c+d*x^n]),x]$

[Out]  $(e*x)^{(3*n)}/(3*a*e*n) - (b*(e*x)^{(3*n)}*\operatorname{Log}[1+(a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2])])/(a*\operatorname{Sqrt}[-a^2+b^2]*d*e*n*x^n) + (b*(e*x)^{(3*n)}*\operatorname{Log}[1+(a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2])])/(a*\operatorname{Sqrt}[-a^2+b^2]*d*e*n*x^n) - (2*b*(e*x)^{(3*n)}*\operatorname{PolyLog}[2, -((a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2]))])/(a*\operatorname{Sqrt}[-a^2+b^2]*d^2*e*n*x^{(2*n)}) + (2*b*(e*x)^{(3*n)}*\operatorname{PolyLog}[2, -((a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2]))])/(a*\operatorname{Sqrt}[-a^2+b^2]*d^2*e*n*x^{(2*n)}) + (2*b*(e*x)^{(3*n)}*\operatorname{PolyLog}[3, -((a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2]))])/(a*\operatorname{Sqrt}[-a^2+b^2]*d^3*e*n*x^{(3*n)}) - (2*b*(e*x)^{(3*n)}*\operatorname{PolyLog}[3, -((a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2]))])/(a*\operatorname{Sqrt}[-a^2+b^2]*d^3*e*n*x^{(3*n)})$

#### Rule 2190

$\operatorname{Int}[(((F_)^{(g_.)*((e_.)+(f_.)*(x_.)))^{(n_.)*((c_.)+(d_.)*(x_.))^{(m_.)})/((a_.)+(b_.)*((F_)^{(g_.)*((e_.)+(f_.)*(x_.)))^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Simp}[(c+d*x)^m*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n)/a)]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c+d*x)^{(m-1)}*\operatorname{Log}[1+(b*(F^{(g*(e+f*x)))^n)/a)], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2264

$\operatorname{Int}[((F_)^{(u_*)}*((f_.)+(g_.)*(x_.))^{(m_.)})/((a_.)+(b_.)*(F_)^{(u_*)}+(c_.)*(F_)^{(v_.)}), x\_Symbol] \rightarrow \operatorname{With}\{q = \operatorname{Rt}[b^2-4*a*c, 2]\}, \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f+g*x)^m*F^u/(b-q+2*c*F^u), x], x] - \operatorname{Dist}[(2*c)/q, \operatorname{Int}[(f+g*x)^m*F^u/(b+q+2*c*F^u), x], x] /; \operatorname{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \operatorname{EqQ}[v, 2*u] \&\& \operatorname{LinearQ}[u, x] \&\& \operatorname{NeQ}[b^2-4*a*c, 0] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_.)*(v_)^{(n_)})^{(m_)} /; \operatorname{FreeQ}[$



{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*x\_)))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)  
)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

#### Rule 3320

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + Pi\*(k\_.) + (Comple  
x[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) +  
f\*fz\*x))/(E^(I\*Pi\*(k - 1/2))\*(b + (2\*a\*E^(-(I\*e) + f\*fz\*x))/E^(I\*Pi\*(k - 1/  
2)) - (b\*E^(2\*(-(I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{a, b, c,  
d, e, f, fz}, x] && IntegerQ[2\*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 4191

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)^(m\_.))  
, x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, 1/(Sin[e + f\*x]^n/(b + a\*Si  
n[e + f\*x]^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt  
Q[m, 0]

#### Rule 5436

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.), x\_Symbo  
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Sech[c + d\*x])  
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m  
+ 1)/n], 0] && IntegerQ[p]

#### Rule 5440

Int[((e\_.)\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_)])^(p\_.),  
x\_Symbol] := Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(  
a + b\*Sech[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx &= \frac{(x^{-3n}(ex)^{3n}) \int \frac{x^{-1+3n}}{a+b \operatorname{sech}(c+dx^n)} dx}{e} \\
&= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{en} \\
&= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \left(\frac{x^2}{a} - \frac{bx^2}{a(b+a \cosh(c+dx))}\right) dx, x, x^n\right)}{en} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{(bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{b+a \cosh(c+dx)} dx, x, x^n\right)}{aen} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{(2bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{a+2be^{c+dx}+ae^{2(c+dx)}} dx, x, x^n\right)}{aen} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{(2bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2} en} + \frac{(2bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{2b+2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} + \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} + \frac{(2bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} + \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} - \frac{2bx^{-2n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} + \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} - \frac{2bx^{-2n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2} en} \\
&= \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} + \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2} den} - \frac{2bx^{-2n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{e^{c+dx} x^2}{2b-2\sqrt{-a^2+b^2}+2ae^{c+dx}} dx, x, x^n\right)}{\sqrt{-a^2+b^2} en}
\end{aligned}$$

**Mathematica** [F] time = 7.22, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sech[c + d\*x^n]), x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sech[c + d\*x^n]), x]

**fricas** [C] time = 0.45, size = 2005, normalized size = 4.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sech(c+d\*x^n)), x, algorithm="fricas")

[Out]  $\frac{1}{3} * ((a^2 - b^2) * d^3 * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x))^3 + (a^2 - b^2) * d^3 * \cosh(n * \log(x))^3 * \sinh((3*n - 1) * \log(e)) + ((a^2 - b^2) * d^3 * \cosh((3*n - 1) * \log(e)) + (a^2 - b^2) * d^3 * \sinh((3*n - 1) * \log(e))) * \sinh(n * \log(x))^3 + 3 * ((a^2 - b^2) * d^3 * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x)) + (a^2 - b^2) * d^3 * \cosh(n * \log(x)) * \sinh((3*n - 1) * \log(e))) * \sinh(n * \log(x))^2 + 6 * (a * b * d * \sqrt{-(a^2 - b^2) / a^2} * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x)) + a * b * d * \sqrt{-(a^2 - b^2) / a^2} * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x)))$

$$\begin{aligned} & /a^2) * \cosh(n * \log(x)) * \sinh((3*n - 1) * \log(e)) + (a*b*d*\sqrt{-(a^2 - b^2)}/a^2) \\ & * \cosh((3*n - 1) * \log(e)) + a*b*d*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) \\ & )) * \sinh(n * \log(x)) * \operatorname{dilog}(-((a*\sqrt{-(a^2 - b^2)}/a^2) + b) * \cosh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + (a*\sqrt{-(a^2 - b^2)}/a^2) + b) * \sinh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + a) / a + 1) - 6 * (a*b*d*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x)) \\ & + a*b*d*\sqrt{-(a^2 - b^2)}/a^2) * \cosh(n * \log(x)) * \sinh((3*n - 1) * \log(e)) + (a*b*d*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) \\ & + a*b*d*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x)) * \operatorname{dilog}(((a*\sqrt{-(a^2 - b^2)}/a^2) - b) * \cosh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + (a*\sqrt{-(a^2 - b^2)}/a^2) - b) * \sinh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) - a) / a + 1) - 3 * (a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) \\ & )) * \log(2*a*\cosh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c) + 2*a*\sinh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c) + 2*a*\sqrt{-(a^2 - b^2)}/a^2) + 2*b) + 3 * \\ & (a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) * \log(2*a*\cosh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) \\ & + c) + 2*a*\sinh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c) - 2*a*\sqrt{-(a^2 - b^2)}/a^2) + 2*b) + 3 * (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x))^2 \\ & - a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x))^2 \\ & + (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh(n * \log(x))^2 - a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) + 2 * (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x)) \\ & + a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh(n * \log(x)) * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x)) * \log(((a*\sqrt{-(a^2 - b^2)}/a^2) + b) * \cosh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + (a*\sqrt{-(a^2 - b^2)}/a^2) + b) * \sinh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c) + a) / a) - 3 * (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x))^2 \\ & - a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x))^2 \\ & + (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh(n * \log(x))^2 - a*b*c^2*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) + 2 * (a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x)) \\ & + a*b*d^2*\sqrt{-(a^2 - b^2)}/a^2) * \cosh(n * \log(x)) * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x)) * \log(-((a*\sqrt{-(a^2 - b^2)}/a^2) - b) * \cosh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + (a*\sqrt{-(a^2 - b^2)}/a^2) - b) * \sinh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c) - a) / a) - 6 * (a*b*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + a*b*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) * \operatorname{polylog}(3, -((a*\sqrt{-(a^2 - b^2)}/a^2) + b) * \cosh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + (a*\sqrt{-(a^2 - b^2)}/a^2) + b) * \sinh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c)) / a) + 6 * (a*b*\sqrt{-(a^2 - b^2)}/a^2) * \cosh((3*n - 1) * \log(e)) + a*b*\sqrt{-(a^2 - b^2)}/a^2) * \sinh((3*n - 1) * \log(e)) * \operatorname{polylog}(3, ((a*\sqrt{-(a^2 - b^2)}/a^2) - b) * \cosh(d * \cosh(n * \log(x)) \\ & + d * \sinh(n * \log(x)) + c) + (a*\sqrt{-(a^2 - b^2)}/a^2) - b) * \sinh(d * \cosh(n * \log(x)) + d * \sinh(n * \log(x)) + c)) / a) + 3 * ((a^2 - b^2) * d^3 * \cosh((3*n - 1) * \log(e)) * \cosh(n * \log(x))^2 \\ & + (a^2 - b^2) * d^3 * \cosh(n * \log(x))^2 * \sinh((3*n - 1) * \log(e)) * \sinh(n * \log(x))) / ((a^3 - a*b^2) * d^3 * n) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sech(c+d\*x^n)),x, algorithm="giac")

[Out] integrate((e\*x)^(3\*n - 1)/(b\*sech(d\*x^n + c) + a), x)

**maple** [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(3*n-1)/(a+b*sech(c+d*x^n)),x)`

[Out] `int((e*x)^(3*n-1)/(a+b*sech(c+d*x^n)),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-2be^{3n} \int \frac{e^{(dx^n+3n\log(x)+c)}}{a^2exe^{(2dx^n+2c)} + 2abexe^{(dx^n+c)} + a^2ex} dx + \frac{e^{3n-1}x^{3n}}{3an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

[Out] `-2*b*e^(3*n)*integrate(e^(d*x^n + 3*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + 1/3*e^(3*n - 1)*x^(3*n)/(a*n)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{3n-1}}{a + \frac{b}{\cosh(c+dx^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n)),x)`

[Out] `int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x)**(-1+3*n)/(a+b*sech(c+d*x**n)),x)`

[Out] `Integral((e*x)**(3*n - 1)/(a + b*sech(c + d*x**n)), x)`

$$3.82 \quad \int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

**Optimal.** Leaf size=157

$$\frac{2b(2a^2 - b^2)x^{-n}(ex)^n \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2 d e n (a-b)^{3/2} (a+b)^{3/2}} + \frac{b^2 x^{-n} (ex)^n \tanh(c+dx^n)}{a d e n (a^2 - b^2) (a+b\operatorname{sech}(c+dx^n))} + \frac{(ex)^n}{a^2 e n}$$

[Out] (e\*x)^n/a^2/e/n-2\*b\*(2\*a^2-b^2)\*(e\*x)^n\*arctan((a-b)^(1/2)\*tanh(1/2\*c+1/2\*d\*x^n)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d/e/n/(x^n)+b^2\*(e\*x)^n\*tanh(c+d\*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(a+b\*sech(c+d\*x^n))

**Rubi [A]** time = 0.30, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$ , Rules used = {5440, 5436, 3785, 3919, 3831, 2659, 208}

$$\frac{2b(2a^2 - b^2)x^{-n}(ex)^n \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a^2 d e n (a-b)^{3/2} (a+b)^{3/2}} + \frac{b^2 x^{-n} (ex)^n \tanh(c+dx^n)}{a d e n (a^2 - b^2) (a+b\operatorname{sech}(c+dx^n))} + \frac{(ex)^n}{a^2 e n}$$

Antiderivative was successfully verified.

[In] Int[(e\*x)^(-1+n)/(a+b\*Sech[c+d\*x^n])^2,x]

[Out] (e\*x)^n/(a^2\*e^n) - (2\*b\*(2\*a^2 - b^2)\*(e\*x)^n\*ArcTan[(Sqrt[a-b]\*Tanh[(c+d\*x^n)/2])/Sqrt[a+b]])/(a^2\*(a-b)^(3/2)\*(a+b)^(3/2)\*d\*e\*n\*x^n) + (b^2\*(e\*x)^n\*Tanh[c+d\*x^n])/(a\*(a^2 - b^2)\*d\*e\*n\*x^n\*(a+b\*Sech[c+d\*x^n]))

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 2659

Int[((a\_) + (b\_.)\*sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c+d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a+b+(a-b)\*e^2\*x^2), x], x, Tan[(c+d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3785

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.) + (a\_))^(n\_), x\_Symbol] := Simp[(b^2\*Cot[c+d\*x]\*(a+b\*Csc[c+d\*x])^(n+1))/(a\*d\*(n+1)\*(a^2 - b^2)), x] + Dist[1/(a\*(n+1)\*(a^2 - b^2)), Int[(a+b\*Csc[c+d\*x])^(n+1)\*Simp[(a^2 - b^2)\*(n+1) - a\*b\*(n+1)\*Csc[c+d\*x] + b^2\*(n+2)\*Csc[c+d\*x]^2, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2\*n]

#### Rule 3831

Int[csc[(e\_.) + (f\_.)\*(x\_)]/(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.) + (a\_)), x\_Symbol] := Dist[1/b, Int[1/(1+(a\*Sin[e+f\*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

#### Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### Rule 5440

```
Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(ex)^{-1+n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx &= \frac{(x^{-n}(ex)^n) \int \frac{x^{-1+n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx}{e} \\ &= \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int \frac{1}{(a + b \operatorname{sech}(c + dx))^2} dx, x, x^n\right)}{en} \\ &= \frac{b^2 x^{-n}(ex)^n \tanh(c + dx^n)}{a(a^2 - b^2) \operatorname{den}(a + b \operatorname{sech}(c + dx^n))} - \frac{(x^{-n}(ex)^n) \operatorname{Subst}\left(\int \frac{-a^2 + b^2 + ab \operatorname{sech}(c + dx)}{a + b \operatorname{sech}(c + dx)} dx, x, x^n\right)}{a(a^2 - b^2) en} \\ &= \frac{(ex)^n}{a^2 en} + \frac{b^2 x^{-n}(ex)^n \tanh(c + dx^n)}{a(a^2 - b^2) \operatorname{den}(a + b \operatorname{sech}(c + dx^n))} + \frac{\left((-a^2 b + b(-a^2 + b^2)) x^{-n}(ex)^n\right)}{a^2(a^2 - b^2)} \\ &= \frac{(ex)^n}{a^2 en} + \frac{b^2 x^{-n}(ex)^n \tanh(c + dx^n)}{a(a^2 - b^2) \operatorname{den}(a + b \operatorname{sech}(c + dx^n))} + \frac{\left((-a^2 b + b(-a^2 + b^2)) x^{-n}(ex)^n\right)}{a^2 b(a^2 - b^2)} \\ &= \frac{(ex)^n}{a^2 en} + \frac{b^2 x^{-n}(ex)^n \tanh(c + dx^n)}{a(a^2 - b^2) \operatorname{den}(a + b \operatorname{sech}(c + dx^n))} - \frac{\left(2i(-a^2 b + b(-a^2 + b^2)) x^{-n}(ex)^n\right)}{a^2(a^2 - b^2)} \\ &= \frac{(ex)^n}{a^2 en} - \frac{2b(2a^2 - b^2) x^{-n}(ex)^n \tan^{-1}\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c + dx^n)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2} \operatorname{den}} + \frac{b^2 x^{-n}(ex)^n \tanh(c + dx^n)}{a(a^2 - b^2) \operatorname{den}(a + b \operatorname{sech}(c + dx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.60, size = 233, normalized size = 1.48

$$\frac{x^{-n}(ex)^n \left( b \left( (a^2 - b^2)^{3/2} (c + dx^n) + ab \sqrt{a^2 - b^2} \sinh(c + dx^n) + (4a^2 b - 2b^3) \tan^{-1} \left( \frac{(b-a) \tanh\left(\frac{1}{2}(c + dx^n)\right)}{\sqrt{a^2 - b^2}} \right) \right) + a \cos(c + dx^n) \right)}{a^2 \operatorname{den}(a-b)(a+b) \sqrt{a^2 - b^2} (a \cosh(c + dx^n) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*x)^(-1 + n)/(a + b\*Sech[c + d\*x^n])^2, x]

```
[Out] ((e*x)^n*(a*((a^2 - b^2)^(3/2)*(c + d*x^n) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x^n] + b*((a^2 - b^2)^(3/2)*(c + d*x^n) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x^n]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*e*n*x^n*(b + a*Cosh[c + d*x^n]))
```

**fricas** [B] time = 0.50, size = 3547, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] [((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) - (a^2*b^3 - b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*(a^3*b^2 - a*b^4)*cosh((n - 1)*log(e)) - (((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)) + 2*((2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*((2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)) + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))*log((a*b + (b^2 + sqrt(-a^2 + b^2)*b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2)*b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + b)) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (a^2*b^3 - b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*n + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n)*
```

```

sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)), ((a^5 - 2*a^3*b^2 + a*b^4)*
d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((
n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*
sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) +
(a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(
(n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*log(x))
*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e)) +
(a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*c
osh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(((2*a^3*b - a*b^3)*sqrt(a^2 -
b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*sinh((n - 1)*
log(e)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((2*a^3*b - a*b^
3)*sqrt(a^2 - b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(a^2 - b^2)
*sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2
*a^3*b - a*b^3)*sqrt(a^2 - b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sq
rt(a^2 - b^2)*sinh((n - 1)*log(e)) + 2*((2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*c
osh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*sinh((n - 1)*log(e)
))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*((2*a^2*b^2 - b^4)*sq
rt(a^2 - b^2)*cosh((n - 1)*log(e)) + (2*a^2*b^2 - b^4)*sqrt(a^2 - b^2)*sinh(
(n - 1)*log(e)) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*cosh((n - 1)*log(e)) +
(2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*sinh((n - 1)*log(e)))*cosh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c))
*arctan(-sqrt(a^2 - b^2)*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) +
sqrt(a^2 - b^2)*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2
- b^2)*b)/(a^2 - b^2)) + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)
))*cosh(n*log(x)) - (a^2*b^3 - b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 -
(a^4*b - 2*a^2*b^3 + b^5)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b
- 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*si
nh((n - 1)*log(e)))*sinh(n*log(x))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)
) + c) - 2*(a^3*b^2 - a*b^4)*cosh((n - 1)*log(e)) + 2*((a^4*b - 2*a^2*b^3 +
b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*
cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(n*lo
g(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(
e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*cos
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - (a^2*b^3 - b^5)*cosh((n - 1)*l
og(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*cosh(n*log(x)))*sinh(
(n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e)) + (a^4*
b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))*sinh(d*cosh(n*
log(x)) + d*sinh(n*log(x)) + c) - (2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 +
a*b^4)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a*b^4)
*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*log(e))
*sinh(n*log(x)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) + d
*sinh(n*log(x)) + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sinh(d*cosh(n*log(
x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n*cosh(d*
cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*n +
2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d*n)*sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c)))]

```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((e\*x)^(n - 1)/(b\*sech(d\*x^n + c) + a)^2, x)



**maple [C]** time = 0.67, size = 491, normalized size = 3.13

$$x e^{\frac{(-1+n)\left(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{a^{2n}} - \frac{2b^2 e^{\frac{(-1+n)\left(-i\pi \operatorname{csgn}(ie)\operatorname{csgn}(ix)\operatorname{csgn}(iex)+i\pi \operatorname{csgn}(ie)\operatorname{csgn}(iex)^2+i\pi \operatorname{csgn}(ix)\operatorname{csgn}(iex)^2-i\pi \operatorname{csgn}(iex)^3+2\ln(x)+2\ln(e)\right)}{2}}}{a^2 \left(a^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n))^2,x)

[Out]  $\frac{1}{a^{2n}} x \exp\left(\frac{1}{2}(-1+n)\left(-i\pi \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x) + I\pi \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2 + I\pi \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2 - I\pi \operatorname{csgn}(Ie*x)^3 + 2\ln(x) + 2\ln(e)\right)\right) - 2b^2 \exp\left(\frac{1}{2}(-1+n)\left(-i\pi \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x) + I\pi \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2 + I\pi \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2 - I\pi \operatorname{csgn}(Ie*x)^3 + 2\ln(x) + 2\ln(e)\right)\right) * x * (b \exp(c+d*x^n) + a) / a^2 / (a^2 - b^2) / d / n / (x^n) / (2b \exp(c+d*x^n) + \exp(2*c+2*d*x^n) * a + a) - 2b/a^2 * (2a^2 - b^2) / (a^2 - b^2) / n * \exp(-1/2 * I\pi * n * \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)) * \exp(1/2 * I\pi * n * \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2) * \exp(1/2 * I\pi * n * \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2) * \exp(-1/2 * I\pi * n * \operatorname{csgn}(Ie*x)^3) * \exp(1/2 * I\pi * \operatorname{csgn}(Ie) \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)) * \exp(-1/2 * I\pi * \operatorname{csgn}(Ie) \operatorname{csgn}(Ie*x)^2) * \exp(-1/2 * I\pi * \operatorname{csgn}(Ix) \operatorname{csgn}(Ie*x)^2) * \exp(1/2 * I\pi * \operatorname{csgn}(Ie*x)^3) * e^n / \exp(c) / d / (a^2 \exp(2*c) - b^2 \exp(2*c))^{1/2} * \arctan(1/2 * (2a \exp(2*c+d*x^n) + 2 \exp(c) * b) / (a^2 \exp(2*c) - b^2 \exp(2*c))^{1/2})$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-2 \left(2 a^2 b e^n e^c - b^3 e^n e^c\right) \int \frac{e^{(dx^n + n \log(x))}}{\left(a^5 e e^{(2c)} - a^3 b^2 e e^{(2c)}\right) x e^{(2 dx^n)} + 2 \left(a^4 b e e^c - a^2 b^3 e e^c\right) x e^{(dx^n)} + \left(a^5 e - a^3 b^2 e\right) x} dx - \frac{2 a b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="maxima")

[Out]  $-2 * (2 * a^2 * b * e^n * e^c - b^3 * e^n * e^c) * \operatorname{integrate}\left(e^{(d*x^n + n*\log(x))} / ((a^5 * e * e^{(2*c)} - a^3 * b^2 * e * e^{(2*c)}) * x * e^{(2*d*x^n)} + 2 * (a^4 * b * e * e^c - a^2 * b^3 * e * e^c) * x * e^{(d*x^n)} + (a^5 * e - a^3 * b^2 * e) * x), x) - (2 * a * b^2 * e^n - (a^3 * d * e^n - a * b^2 * d * e^n) * x^n - (a^3 * d * e^n * e^{(2*c)} - a * b^2 * d * e^n * e^{(2*c)}) * e^{(2*d*x^n + n*\log(x))} + 2 * (b^3 * e^n * e^c - (a^2 * b * d * e^n * e^c - b^3 * d * e^n * e^c) * x^n) * e^{(d*x^n)}) / (a^5 * d * e * n - a^3 * b^2 * d * e * n + (a^5 * d * e * n * e^{(2*c)} - a^3 * b^2 * d * e * n * e^{(2*c)}) * e^{(2*d*x^n)} + 2 * (a^4 * b * d * e * n * e^c - a^2 * b^3 * d * e * n * e^c) * e^{(d*x^n)})$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex)^{n-1}}{\left(a + \frac{b}{\cosh(c+dx^n)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(n-1)/(a+b/cosh(c+d\*x^n))^2,x)

[Out] int((e\*x)^(n-1)/(a+b/cosh(c+d\*x^n))^2,x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+n)/(a+b\*sech(c+d\*x\*\*n))\*\*2,x)

[Out] Timed out

$$3.83 \quad \int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

**Optimal.** Leaf size=717

$$\frac{2bx^{-2n}(ex)^{2n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{2n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} - \frac{b^2x^{-2n}(ex)^{2n}\log(a\cosh(c+dx^n)+b)}{a^2d^2en(a^2-b^2)} - \frac{2bx^{-n}(ex)^{2n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}}$$

[Out]  $\frac{1}{2} \frac{(e*x)^{(2*n)}}{a^2/e/n-b^2} \frac{(e*x)^{(2*n)}*\ln(b+a*\cosh(c+d*x^n))}{a^2/(a^2-b^2)/d^2/e/n/(x^{(2*n)})+b^3} + \frac{(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))}{a^2/(-a^2+b^2)^{(3/2)}/d/e/n/(x^n)-b^3} + \frac{(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))}{a^2/(-a^2+b^2)^{(3/2)}/d/e/n/(x^n)+b^3} + \frac{(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))}{a^2/(-a^2+b^2)^{(3/2)}/d^2/e/n/(x^{(2*n)})-b^3} + \frac{(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))}{a^2/(-a^2+b^2)^{(3/2)}/d^2/e/n/(x^{(2*n)})+b^3} + \frac{(e*x)^{(2*n)}*\sinh(c+d*x^n)/a}{(a^2-b^2)/d/e/n/(x^n)/(b+a*\cosh(c+d*x^n))-2*b} + \frac{(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))}{a^2/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+2*b} + \frac{(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))}{a^2/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-2*b} + \frac{(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))}{a^2/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*b} + \frac{(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))}{a^2/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}-2*b}$

**Rubi [A]** time = 1.22, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$ , Rules used = {5440, 5436, 4191, 3324, 3320, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} + \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} + \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,-\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e*x)^{-1+2*n}/(a+b*\operatorname{Sech}[c+d*x^n])^2,x]$

[Out]  $(e*x)^{(2*n)}/(2*a^2*e^n) + (b^3*(e*x)^{(2*n)}*\operatorname{Log}[1+(a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2])])/(a^2*(-a^2+b^2)^{(3/2)}*d*e^n*x^n) - (2*b*(e*x)^{(2*n)}*\operatorname{Log}[1+(a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2])])/(a^2*\operatorname{Sqrt}[-a^2+b^2]*d*e^n*x^n) - (b^3*(e*x)^{(2*n)}*\operatorname{Log}[1+(a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2])])/(a^2*(-a^2+b^2)^{(3/2)}*d*e^n*x^n) + (2*b*(e*x)^{(2*n)}*\operatorname{Log}[1+(a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2])])/(a^2*\operatorname{Sqrt}[-a^2+b^2]*d*e^n*x^n) - (b^2*(e*x)^{(2*n)}*\operatorname{Log}[b+a*\operatorname{Cosh}[c+d*x^n]])/(a^2*(a^2-b^2)*d^2*e^n*x^{(2*n)}) + (b^3*(e*x)^{(2*n)}*\operatorname{PolyLog}[2,-((a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2])])/(a^2*(-a^2+b^2)^{(3/2)}*d^2*e^n*x^{(2*n)}) - (2*b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2,-((a*E^{(c+d*x^n)})/(b-\operatorname{Sqrt}[-a^2+b^2])])/(a^2*\operatorname{Sqrt}[-a^2+b^2]*d^2*e^n*x^{(2*n)}) - (b^3*(e*x)^{(2*n)}*\operatorname{PolyLog}[2,-((a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2])])/(a^2*(-a^2+b^2)^{(3/2)}*d^2*e^n*x^{(2*n)}) + (2*b*(e*x)^{(2*n)}*\operatorname{PolyLog}[2,-((a*E^{(c+d*x^n)})/(b+\operatorname{Sqrt}[-a^2+b^2])])/(a^2*\operatorname{Sqrt}[-a^2+b^2]*d^2*e^n*x^{(2*n)}) + (b^2*(e*x)^{(2*n)}*\operatorname{Sinh}[c+d*x^n])/(a*(a^2-b^2)*d*e^n*x^n*(b+a*\operatorname{Cosh}[c+d*x^n]))$

**Rule 31**

$\operatorname{Int}[(a_0 + (b_0)*(x_0))^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /; \operatorname{FreeQ}\{a, b\}, x]$

**Rule 2190**

$\operatorname{Int}[(F_0)^{(g_0)}*((e_0) + (f_0)*(x_0))^{(n_0)}*((c_0) + (d_0)*(x_0))^{(m_0)} / ((a_0) + (b_0)*((F_0)^{(g_0)}*((e_0) + (f_0)*(x_0))^{(n_0)})], x\_Symbol] \rightarrow \operatorname{Simp}$

$$\int \frac{((c + dx)^m \log[1 + (b(F^{g(e+fx)}))^n]/a)}{(bfgn \log[F])} dx - \text{Dist}[\frac{d^m}{bfgn \log[F]}, \int \frac{(c + dx)^{m-1} \log[1 + (b(F^{g(e+fx)}))^n]/a]}{x}, x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \text{IGtQ}[m, 0]$$

#### Rule 2264

$$\text{Int}[\frac{(F^u)((f) + (g)(x))^m}{(a) + (b)(F^u) + (c)(F^v)}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[\frac{2c}{q}, \text{Int}[\frac{(f + gx)^m F^u}{(b - q + 2cF^u)}, x], x] - \text{Dist}[\frac{2c}{q}, \text{Int}[\frac{(f + gx)^m F^u}{(b + q + 2cF^u)}, x], x] /; \text{FreeQ}\{F, a, b, c, f, g, x\} \&\& \text{EqQ}[v, 2u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 2279

$$\text{Int}[\text{Log}[(a) + (b)(F^{(e)(c) + (d)(x)})^n]], x\_Symbol] \rightarrow \text{Dist}[1/(de^n \log[F]), \text{Subst}[\text{Int}[\text{Log}[a + bx]/x], x, (F^{e(c+dx)})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n, x\} \&\& \text{GtQ}[a, 0]$$

#### Rule 2391

$$\text{Int}[\text{Log}[(c)(d) + (e)(x)^n]/(x)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(cex^n)]/n, x] /; \text{FreeQ}\{c, d, e, n, x\} \&\& \text{EqQ}[cd, 1]$$

#### Rule 2668

$$\text{Int}[\cos[(e) + (f)(x)]^{(p)}((a) + (b)\sin[(e) + (f)(x)])^m], x\_Symbol] \rightarrow \text{Dist}[1/(b^p f), \text{Subst}[\text{Int}[(a + x)^m (b^2 - x^2)^{(p-1)/2}], x], x, b \sin[e + fx]], x] /; \text{FreeQ}\{a, b, e, f, m, x\} \&\& \text{IntegerQ}[(p - 1)/2] \&\& \text{NeQ}[a^2 - b^2, 0]$$

#### Rule 3320

$$\text{Int}[\frac{(c) + (d)(x)^m}{(a) + (b)\sin[(e) + \text{Pi}(k) + (\text{Complex}[0, fz])f(x)]}], x\_Symbol] \rightarrow \text{Dist}[2, \text{Int}[\frac{(c + dx)^m E^{-Ie} + f fz x}{E^{I\text{Pi}(k - 1/2)}(b + (2aE^{-Ie} + f fz x))/E^{I\text{Pi}(k - 1/2)} - (bE^{2(-Ie) + f fz x})/E^{2I k \text{Pi}})}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, fz, x\} \&\& \text{IntegerQ}[2k] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 3324

$$\text{Int}[\frac{(c) + (d)(x)^m}{(a) + (b)\sin[(e) + (f)(x)]^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{b(c + dx)^m \cos[e + fx]}{f(a^2 - b^2)(a + b \sin[e + fx])}, x] + (\text{Dist}[a/(a^2 - b^2), \text{Int}[(c + dx)^m/(a + b \sin[e + fx])], x] - \text{Dist}[(b^m d)/(f(a^2 - b^2)), \text{Int}[(c + dx)^{m-1} \cos[e + fx]/(a + b \sin[e + fx])], x]) /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 4191

$$\text{Int}[(\text{csc}[(e) + (f)(x)](b) + (a))^{(n)}((c) + (d)(x))^m], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + dx)^m, 1/(\sin[e + fx]^n/(b + a \sin[e + fx])^n)], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 5436

$$\text{Int}[(x)^m((a) + (b)\text{Sech}[(c) + (d)(x)^n])^p], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)(a + b \text{Sech}[c + dx])^p}], x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, x\} \&\& \text{IGtQ}[\text{Simplify}[(m$$

+ 1)/n], 0] && IntegerQ[p]

Rule 5440

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*Sech[(c\_) + (d\_)\*(x\_)^(n\_)])^(p\_), x\_Symbol] :> Dist[(e^IntPart[m]\*(e\*x)^FracPart[m])/x^FracPart[m], Int[x^m\*(a + b\*Sech[c + d\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(ex)^{-1+2n}}{(a + b\operatorname{sech}(c + dx^n))^2} dx &= \frac{(x^{-2n}(ex)^{2n}) \int \frac{x^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx}{e} \\
 &= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{(a+b\operatorname{sech}(c+dx))^2} dx, x, x^n\right)}{en} \\
 &= \frac{(x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \left(\frac{x}{a^2} + \frac{bx}{a^2(b+a \cosh(c+dx))^2} - \frac{2bx}{a^2(b+a \cosh(c+dx))}\right) dx, x, x^n\right)}{en} \\
 &= \frac{(ex)^{2n}}{2a^2en} - \frac{(2bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{x}{b+a \cosh(c+dx)} dx, x, x^n\right)}{a^2en} + \frac{(b^2x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^c}{a+2be^{c+dx}} dx, x, x^n\right)}{a^2en} \\
 &= \frac{(ex)^{2n}}{2a^2en} + \frac{b^2x^{-n}(ex)^{2n} \sinh(c + dx^n)}{a(a^2 - b^2)den(b + a \cosh(c + dx^n))} - \frac{(4bx^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e^c}{a+2be^{c+dx}} dx, x, x^n\right)}{a^2en} \\
 &= \frac{(ex)^{2n}}{2a^2en} + \frac{b^2x^{-n}(ex)^{2n} \sinh(c + dx^n)}{a(a^2 - b^2)den(b + a \cosh(c + dx^n))} - \frac{(2b^3x^{-2n}(ex)^{2n}) \operatorname{Subst}\left(\int \frac{e}{a+2be^{c+dx}} dx, x, x^n\right)}{a^2(a^2 - b^2)den} \\
 &= \frac{(ex)^{2n}}{2a^2en} - \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2 + b^2}den} + \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2 + b^2}den} - \frac{b^3}{a^2} \\
 &= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2 + b^2)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2 + b^2}den} - \frac{b^3}{a^2} \\
 &= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2 + b^2)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2 + b^2}den} - \frac{b^3}{a^2} \\
 &= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2 + b^2)^{3/2}den} - \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2 + b^2}den} - \frac{b^3}{a^2}
 \end{aligned}$$

Mathematica [A] time = 30.82, size = 542, normalized size = 0.76

$$x^{-2n}(ex)^{2n} \operatorname{sech}^2(c + dx^n) (a \cosh(c + dx^n) + b) \left( \frac{2b(a \cosh(c+dx^n)+b) \left( \frac{2be^{2c}dx^n}{e^{2c}+1} - \frac{(2a^2-b^2)\operatorname{Li}_2\left(\frac{ae^{dx^n}+c}{\sqrt{b^2-a^2}-b}\right) + (b^2-2a^2)\operatorname{Li}_2\left(-\frac{ae^{dx^n}+c}{b+\sqrt{b^2-a^2}}\right) + 2a^2}{e^{2c}+1} \right)}{e^{2c}+1} \right)$$

Antiderivative was successfully verified.





$$\begin{aligned}
& *b - a^2*b^3)*c*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e))*\cosh(d*\cosh \\
& (n*\log(x)) + d*\sinh(n*\log(x)) + c) + (a^2*b^3 - b^5 + (2*a^3*b^2 - a*b^4)*c \\
& *\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (a^2*b^3 - b^5 + (2*a^3*b \\
& ^2 - a*b^4)*c*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e))*\sinh(d*\cosh(n \\
& *log(x)) + d*\sinh(n*log(x)) + c) + (a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*c \\
& *\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e))*\log(2*a*\cosh(d*\cosh(n*log( \\
& x)) + d*\sinh(n*log(x)) + c) + 2*a*\sinh(d*\cosh(n*log(x)) + d*\sinh(n*log(x)) \\
& + c) - 2*a*\sqrt{-(a^2 - b^2)/a^2} + 2*b) + 2*((2*a^4*b - a^2*b^3)*d*\sqrt{-(a \\
& ^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log(x)) + (2*a^4*b - a^2*b^3) \\
& *c*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d*s \\
& \sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log(x)) + (2*a^4*b - a^ \\
& 2*b^3)*c*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^ \\
& 3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh(n*log(x)) + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a \\
& ^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 \\
& - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2 \\
& )/a^2})*\sinh((2*n - 1)*\log(e))*\sinh(n*log(x)))*\cosh(d*\cosh(n*log(x)) + d*si \\
& nh(n*log(x)) + c)^2 + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2 \\
& *n - 1)*\log(e))*\cosh(n*log(x)) + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a^2 - b^2)/a^ \\
& 2})*\cosh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*c \\
& osh(n*log(x)) + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1 \\
& )*\log(e)) + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*lo \\
& g(e)) + (2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e)) \\
& )*\sinh(n*log(x)))*\sinh(d*\cosh(n*log(x)) + d*\sinh(n*log(x)) + c)^2 + 2*((2*a \\
& ^3*b^2 - a*b^4)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log( \\
& x)) + (2*a^3*b^2 - a*b^4)*c*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + \\
& ((2*a^3*b^2 - a*b^4)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh(n*log(x)) + (2*a^3*b^2 \\
& - a*b^4)*c*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e)) + ((2*a^3*b^2 - a \\
& *b^4)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (2*a^3*b^2 - a*b^4) \\
& *d*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e))*\sinh(n*log(x)))*\cosh(d*\co \\
& sh(n*log(x)) + d*\sinh(n*log(x)) + c) + 2*((2*a^3*b^2 - a*b^4)*d*\sqrt{-(a^2 \\
& - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log(x)) + (2*a^3*b^2 - a*b^4)*c*s \\
& \sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d*\sqrt{( \\
& -a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log(x)) + (2*a^4*b - a^2*b^ \\
& 3)*c*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d \\
& *\sqrt{-(a^2 - b^2)/a^2})*\cosh(n*log(x)) + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a^2 - \\
& b^2)/a^2})*\sinh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^ \\
& 2)/a^2})*\cosh((2*n - 1)*\log(e)) + (2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^ \\
& 2})*\sinh((2*n - 1)*\log(e))*\sinh(n*log(x)))*\cosh(d*\cosh(n*log(x)) + d*\sinh(n \\
& *log(x)) + c) + ((2*a^3*b^2 - a*b^4)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh(n*log(x) \\
& ) + (2*a^3*b^2 - a*b^4)*c*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e)) + \\
& ((2*a^3*b^2 - a*b^4)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (2*a \\
& ^3*b^2 - a*b^4)*d*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e))*\sinh(n*log \\
& (x)))*\sinh(d*\cosh(n*log(x)) + d*\sinh(n*log(x)) + c) + ((2*a^4*b - a^2*b^3)* \\
& d*\sqrt{-(a^2 - b^2)/a^2})*\cosh(n*log(x)) + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a^2 \\
& - b^2)/a^2})*\sinh((2*n - 1)*\log(e)) + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b \\
& ^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^ \\
& ^2})*\sinh((2*n - 1)*\log(e))*\sinh(n*log(x)))*\log(((a*\sqrt{-(a^2 - b^2)/a^2} \\
& + b)*\cosh(d*\cosh(n*log(x)) + d*\sinh(n*log(x)) + c) + (a*\sqrt{-(a^2 - b^2)/a \\
& ^2} + b)*\sinh(d*\cosh(n*log(x)) + d*\sinh(n*log(x)) + c) + a)/a) - 2*((2*a^4*b \\
& b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log(x)) \\
& + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (( \\
& 2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*1 \\
& og(x)) + (2*a^4*b - a^2*b^3)*c*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e) \\
& ) + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh(n*log(x)) + (2*a^4*b \\
& - a^2*b^3)*c*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e)) + ((2*a^4*b - \\
& a^2*b^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e)) + (2*a^4*b - a^2*b \\
& ^3)*d*\sqrt{-(a^2 - b^2)/a^2})*\sinh((2*n - 1)*\log(e))*\sinh(n*log(x)))*\cosh(d \\
& *\cosh(n*log(x)) + d*\sinh(n*log(x)) + c)^2 + ((2*a^4*b - a^2*b^3)*d*\sqrt{-(a \\
& ^2 - b^2)/a^2})*\cosh((2*n - 1)*\log(e))*\cosh(n*log(x)) + (2*a^4*b - a^2*b^3)*
\end{aligned}$$





$b^3 + a^2 b^5) d^{2n} \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((e\*x)^(2\*n - 1)/(b\*sech(d\*x^n + c) + a)^2, x)

**maple** [F] time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+2n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n))^2,x)

[Out] int((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4ab^2e^{2n}x^n - (a^3de^{2n} - ab^2de^{2n})x^{2n} - (a^3de^{2n}e^{2c} - ab^2de^{2n}e^{2c})e^{(2dx^n+2n\log(x))} + 2\left(2b^3e^{2n}e^{(n\log(x)+c)} - (a^5den - a^3b^2den + (a^5dene^{2c} - a^3b^2dene^{2c})e^{2dx^n} + 2(a^4bdene^c - a^2b^3dene^c)\right)}{2(a^5den - a^3b^2den + (a^5dene^{2c} - a^3b^2dene^{2c})e^{2dx^n} + 2(a^4bdene^c - a^2b^3dene^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+2\*n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="maxima")

[Out]  $-1/2*(4*a*b^2*e^{(2*n)}*x^n - (a^3*d*e^{(2*n)} - a*b^2*d*e^{(2*n)})*x^{(2*n)} - (a^3*d*e^{(2*n)}*e^{(2*c)} - a*b^2*d*e^{(2*n)}*e^{(2*c)})*e^{(2*d*x^n + 2*n*\log(x))} + 2*(2*b^3*e^{(2*n)}*e^{(n*\log(x) + c)} - (a^2*b*d*e^{(2*n)}*e^c - b^3*d*e^{(2*n)}*e^c)*x^{(2*n)})*e^{(d*x^n)})/(a^5*d*e^n - a^3*b^2*d*e*n + (a^5*d*e*n*e^{(2*c)} - a^3*b^2*d*e*n*e^{(2*c)})*e^{(2*d*x^n)} + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*e^{(d*x^n)}) - \operatorname{integrate}(-2*(a*b^2*e^{(2*n)}*x^n + (b^3*e^{(2*n)}*e^{(n*\log(x) + c)} - (2*a^2*b*d*e^{(2*n)}*e^c - b^3*d*e^{(2*n)}*e^c)*x^{(2*n)})*e^{(d*x^n)})/((a^5*d*e^n*e^{(2*c)} - a^3*b^2*d*e*n*e^{(2*c)})*x*e^{(2*d*x^n)} + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*x*e^{(d*x^n)} + (a^5*d*e^n - a^3*b^2*d*e)*x), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{2n-1}}{\left(a + \frac{b}{\cosh(c+dx^n)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(2\*n - 1)/(a + b/cosh(c + d\*x^n))^2,x)

[Out] int((e\*x)^(2\*n - 1)/(a + b/cosh(c + d\*x^n))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{2n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)\*\*(-1+2\*n)/(a+b\*sech(c+d\*x\*\*n))\*\*2,x)

[Out] Integral((e\*x)\*\*(2\*n - 1)/(a + b\*sech(c + d\*x\*\*n))\*\*2, x)

**3.84**  $\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

**Optimal.** Leaf size=1284

$$\frac{2b^2(ex)^{3n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(a^2-b^2)d^3en} - \frac{2b^2(ex)^{3n}\operatorname{Li}_2\left(-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(a^2-b^2)d^3en} + \frac{4b(ex)^{3n}\operatorname{Li}_3\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en} - \frac{2b^3(ex)^{3n}\operatorname{Li}_3\left(-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)d^3en}$$

```
[Out] 1/3*(e*x)^(3*n)/a^2/e/n+b^2*(e*x)^(3*n)/a^2/(a^2-b^2)/d/e/n/(x^n)-2*b^2*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))+b^3*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-2*b^2*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))-b^3*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-2*b^2*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))+2*b^3*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-2*b^2*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))-2*b^3*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-2*b^3*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))+2*b^3*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))+b^2*(e*x)^(3*n)*sinh(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*cosh(c+d*x^n))-2*b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+2*b*(e*x)^(3*n)*ln(1+a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-4*b*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+4*b*(e*x)^(3*n)*polylog(2,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+4*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)-4*b*(e*x)^(3*n)*polylog(3,-a*exp(c+d*x^n)/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)
```

**Rubi [A]** time = 2.26, antiderivative size = 1284, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$ , Rules used = {5440, 5436, 4191, 3324, 3320, 2264, 2190, 2531, 2282, 6589, 5562, 2279, 2391}

$$\frac{2b^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(a^2-b^2)d^3en} - \frac{2b^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(a^2-b^2)d^3en} + \frac{4b(ex)^{3n}\operatorname{PolyLog}\left(3,-\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}d^3en}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(3*n)/(3*a^2*e*n) + (b^2*(e*x)^(3*n))/(a^2*(a^2 - b^2)*d*e*n*x^n) - (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) + (b^3*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) - (2*b*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) - (b^3*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + (2*b*(e*x)^(3*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (2*b^2*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3*e*n*x^(3*n)) + (2*b^3*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) - (4*b*(e*x)^(3*n)*PolyLog[2, -((a*E^(c + d
```

$$\frac{x^n)}{(b - \sqrt{-a^2 + b^2})})]/(a^2 \sqrt{-a^2 + b^2} d^2 e^n x^{(2n)}) - (2 * b^2 * (e*x)^{(3n)} * \text{PolyLog}[2, -((a * E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}))]) / (a^2 * (a^2 - b^2) * d^3 * e^n * x^{(3n)}) - (2 * b^3 * (e*x)^{(3n)} * \text{PolyLog}[2, -((a * E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}))]) / (a^2 * (-a^2 + b^2)^{(3/2)} * d^2 * e^n * x^{(2n)}) + (4 * b * (e*x)^{(3n)} * \text{PolyLog}[2, -((a * E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}))]) / (a^2 * \sqrt{-a^2 + b^2} * d^2 * e^n * x^{(2n)}) - (2 * b^3 * (e*x)^{(3n)} * \text{PolyLog}[3, -((a * E^{(c + d*x^n)})/(b - \sqrt{-a^2 + b^2}))]) / (a^2 * (-a^2 + b^2)^{(3/2)} * d^3 * e^n * x^{(3n)}) + (4 * b * (e*x)^{(3n)} * \text{PolyLog}[3, -((a * E^{(c + d*x^n)})/(b - \sqrt{-a^2 + b^2}))]) / (a^2 * \sqrt{-a^2 + b^2} * d^3 * e^n * x^{(3n)}) + (2 * b^3 * (e*x)^{(3n)} * \text{PolyLog}[3, -((a * E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}))]) / (a^2 * (-a^2 + b^2)^{(3/2)} * d^3 * e^n * x^{(3n)}) - (4 * b * (e*x)^{(3n)} * \text{PolyLog}[3, -((a * E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}))]) / (a^2 * \sqrt{-a^2 + b^2} * d^3 * e^n * x^{(3n)}) + (b^2 * (e*x)^{(3n)} * \text{Sinh}[c + d*x^n]) / (a * (a^2 - b^2) * d * e^n * x^n * (b + a * \text{Cosh}[c + d*x^n]))$$

#### Rule 2190

$$\text{Int}[\frac{((F_)^{((g_.) * ((e_.) + (f_.) * (x_)))})^{(n_.)} * ((c_.) + (d_.) * (x_))^{(m_.)})}{((a_.) + (b_.) * ((F_)^{((g_.) * ((e_.) + (f_.) * (x_)))})^{(n_.)})}, x\_Symbol] := \text{Simp}[\frac{(c + d*x)^m * \text{Log}[1 + (b * (F^{(g * (e + f*x))))^n] / a]}{(b * f * g * n * \text{Log}[F])}, x] - \text{Dist}[\frac{(d * m)}{(b * f * g * n * \text{Log}[F])}, \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b * (F^{(g * (e + f*x))))^n] / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$$

#### Rule 2264

$$\text{Int}[\frac{(F_)^{(u_)} * ((f_.) + (g_.) * (x_))^{(m_.)}}{((a_.) + (b_.) * (F_)^{(u_)} + (c_.) * (F_)^{(v_)}), x\_Symbol] := \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[\frac{(f + g*x)^m * F^u}{(b - q + 2*c * F^u)}, x], x] - \text{Dist}[(2*c)/q, \text{Int}[\frac{(f + g*x)^m * F^u}{(b + q + 2*c * F^u)}, x], x]] /; \text{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$$

#### Rule 2279

$$\text{Int}[\text{Log}[(a_.) + (b_.) * ((F_)^{((e_.) * ((c_.) + (d_.) * (x_)))})^{(n_.)}], x\_Symbol] := \text{Dist}[1 / (d * e^n * \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x] / x, x], x, (F^{(e * (c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$$

#### Rule 2282

$$\text{Int}[u_, x\_Symbol] := \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v / D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x] / x, x], x, v], x]] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_.) * ((a_.) * (v_))^{(n_.)}]^{(m_.)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m * n] \&\& !\text{MatchQ}[u, E^{((c_.) * ((a_.) + (b_.) * x))} * (F_)^{(v_)}] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$$

#### Rule 2391

$$\text{Int}[\text{Log}[(c_.) * ((d_.) + (e_.) * (x_))^{(n_.)}] / (x_), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, -(c * e * x^n)] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c * d, 1]$$

#### Rule 2531

$$\text{Int}[\text{Log}[1 + (e_.) * ((F_)^{((c_.) * ((a_.) + (b_.) * (x_)))})^{(n_.)}] * ((f_.) + (g_.) * (x_))^{(m_.)}, x\_Symbol] := -\text{Simp}[\frac{(f + g*x)^m * \text{PolyLog}[2, -(e * (F^{(c * (a + b*x))))^n]}{(b * c * n * \text{Log}[F])}, x] + \text{Dist}[(g * m) / (b * c * n * \text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)} * \text{PolyLog}[2, -(e * (F^{(c * (a + b*x))))^n}], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$$

#### Rule 3320

$$\text{Int}[\frac{(c_.) + (d_.) * (x_))^{(m_.)}}{(a_.) + (b_.) * \sin[(e_.) + \text{Pi} * (k_.) + (\text{Comple$$

```
x[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-I*e) +
f*fz*x))/(E^(I*Pi*(k - 1/2))*(b + (2*a*E^(-I*e) + f*fz*x))/E^(I*Pi*(k - 1/
2)) - (b*E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi)), x], x] /; FreeQ[{a, b, c,
d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

### Rule 4191

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x]^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

### Rule 5436

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### Rule 5440

```
Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
x_Symbol] := Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(
a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### Rule 5562

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx &= \frac{(x^{-3n}(ex)^{3n}) \int \frac{x^{-1+3n}}{(a+b \operatorname{sech}(c+dx^n))^2} dx}{e} \\
&= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{(a+b \operatorname{sech}(c+dx))^2} dx, x, x^n\right)}{en} \\
&= \frac{(x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \left(\frac{x^2}{a^2} + \frac{b^2 x^2}{a^2(b+a \cosh(c+dx))^2} - \frac{2bx^2}{a^2(b+a \cosh(c+dx))}\right) dx, x, x^n\right)}{en} \\
&= \frac{(ex)^{3n}}{3a^2 en} - \frac{(2bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{b+a \cosh(c+dx)} dx, x, x^n\right)}{a^2 en} + \frac{(b^2 x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n} \sinh(c + dx^n)}{a(a^2 - b^2) den (b + a \cosh(c + dx^n))} - \frac{(4bx^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n}}{a^2(a^2 - b^2) den} + \frac{b^2 x^{-n}(ex)^{3n} \sinh(c + dx^n)}{a(a^2 - b^2) den (b + a \cosh(c + dx^n))} - \frac{(2b^3 x^{-3n}(ex)^{3n}) \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2 en} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n}}{a^2(a^2 - b^2) den} - \frac{2b^2 x^{-2n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2(a^2 - b^2) d^2 en} - \frac{2bx^{-n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2 \sqrt{-a^2 + b^2}} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n}}{a^2(a^2 - b^2) den} - \frac{2b^2 x^{-2n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2(a^2 - b^2) d^2 en} + \frac{b^3 x^{-n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2(-a^2 + b^2)} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n}}{a^2(a^2 - b^2) den} - \frac{2b^2 x^{-2n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2(a^2 - b^2) d^2 en} + \frac{b^3 x^{-n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2(-a^2 + b^2)} \\
&= \frac{(ex)^{3n}}{3a^2 en} + \frac{b^2 x^{-n}(ex)^{3n}}{a^2(a^2 - b^2) den} - \frac{2b^2 x^{-2n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2(a^2 - b^2) d^2 en} + \frac{b^3 x^{-n}(ex)^{3n} \operatorname{Subst}\left(\int \frac{x^2}{a+b \operatorname{sech}(c+dx)} dx, x, x^n\right)}{a^2(-a^2 + b^2)}
\end{aligned}$$

**Mathematica [F]** time = 117.91, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sech[c + d\*x^n])^2, x]

[Out] Integrate[(e\*x)^(-1 + 3\*n)/(a + b\*Sech[c + d\*x^n])^2, x]

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="giac")

[Out] integrate((e\*x)^(3\*n - 1)/(b\*sech(d\*x^n + c) + a)^2, x)

**maple** [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(3\*n-1)/(a+b\*sech(c+d\*x^n))^2,x)

[Out] int((e\*x)^(3\*n-1)/(a+b\*sech(c+d\*x^n))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{6ab^2e^{3n}x^{2n} - (a^3de^{3n} - ab^2de^{3n})x^{3n} - (a^3de^{3n}e^{2c} - ab^2de^{3n}e^{2c})e^{(2dx^n+3n\log(x))} + 2(3b^3e^{3n}e^{(2n\log(x)+c)} - (a^5den - a^3b^2den + (a^5dene^{2c} - a^3b^2dene^{2c})e^{2dx^n} + 2(a^4bdene^c - a^2b^3dene^c)e^{2dx^n})}{3(a^5den - a^3b^2den + (a^5dene^{2c} - a^3b^2dene^{2c})e^{2dx^n} + 2(a^4bdene^c - a^2b^3dene^c)e^{2dx^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x)^(-1+3\*n)/(a+b\*sech(c+d\*x^n))^2,x, algorithm="maxima")

[Out] -1/3\*(6\*a\*b^2\*e^(3\*n)\*x^(2\*n) - (a^3\*d\*e^(3\*n) - a\*b^2\*d\*e^(3\*n))\*x^(3\*n) - (a^3\*d\*e^(3\*n)\*e^(2\*c) - a\*b^2\*d\*e^(3\*n)\*e^(2\*c))\*e^(2\*d\*x^n + 3\*n\*log(x)) + 2\*(3\*b^3\*e^(3\*n)\*e^(2\*n\*log(x) + c) - (a^2\*b\*d\*e^(3\*n)\*e^c - b^3\*d\*e^(3\*n)\*e^c)\*x^(3\*n))/((a^5\*d\*e\*n - a^3\*b^2\*d\*e\*n + (a^5\*d\*e\*n\*e^(2\*c) - a^3\*b^2\*d\*e\*n\*e^(2\*c))\*e^(2\*d\*x^n) + 2\*(a^4\*b\*d\*e\*n\*e^c - a^2\*b^3\*d\*e\*n\*e^c)\*e^(d\*x^n)) - integrate(-2\*(2\*a\*b^2\*e^(3\*n)\*x^(2\*n) + (2\*b^3\*e^(3\*n)\*e^(2\*n\*log(x) + c) - (2\*a^2\*b\*d\*e^(3\*n)\*e^c - b^3\*d\*e^(3\*n)\*e^c)\*x^(3\*n))\*e^(d\*x^n))/((a^5\*d\*e\*e^(2\*c) - a^3\*b^2\*d\*e\*e^(2\*c))\*x\*e^(2\*d\*x^n) + 2\*(a^4\*b\*d\*e\*e^c - a^2\*b^3\*d\*e\*e^c)\*x\*e^(d\*x^n) + (a^5\*d\*e - a^3\*b^2\*d\*e)\*x), x)

**mpad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex)^{3n-1}}{\left(a + \frac{b}{\cosh(c+dx^n)}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x)^(3\*n - 1)/(a + b/cosh(c + d\*x^n))^2,x)

[Out] int((e\*x)^(3\*n - 1)/(a + b/cosh(c + d\*x^n))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{3n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+3*n)/(a+b*sech(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(3*n - 1)/(a + b*sech(c + d*x**n))**2, x)
```





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
    If[Head[expn]===RootSum,
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp,Log,
    Sin,Cos,Tan,Cot,Sec,Csc,
    ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
    Sinh,Cosh,Tanh,Coth,Sech,Csch,
    ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
  },func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  },func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```



```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```